OLIVIER DARRIGOL

A HISTORY OF OPTICS

From Greek Antiquity to the Nineteenth Century

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PREFACE

For the last few years, I have taught the history of optics to successive students at the Université Denis Diderot and at the Ecole Normale Supérieure in Paris. While preparing this course, I was impressed by the abundance and depth of recent research on specific aspects of this history. At the same time I noticed the lack of a concise, long-term history that would integrate the results of this research. The present book is aimed at filling this gap. Its broad scope and its conceptual orientation are meant to appeal to physicists and opticians who are interested in the history of their discipline, to historians who wish to grasp or teach the essentials of an attractive domain of the history of science, and to philosophers who know that optics often informed the philosophy of knowledge. No special mathematical competence is required from the reader, except for a few sections of the three last chapters covering the inherently more mathematical optics of the nineteenth century.

Nowadays, optics is commonly defined as the science of light and vision. It comprises a variety of topics including the fundamental physics of light, ocular vision, optical instruments, and the business of corrective lenses. Going back in time, the chain of back-references leads us to the ancient Greeks, who defined optics as a theory of vision. It is therefore natural to begin a history of Western optics with the Greek contributions, even though Greek optics did not have light as its main protagonist. It is also natural to end the narrative in the late nineteenth century, since classical wave optics reached its maturity around that time. In the two first millenaries of this long evolution, optics remained mainly a theory of vision and its history should therefore be recounted as such. For the next three centuries, following Kepler's intervention, there existed a fairly autonomous domain of physics concerned with the behavior of light. Most of the present book focuses on this narrower optics, with incidental references to relevant developments in acoustics, photochemistry, color perception, and spectroscopy.[1]

Even with this significant restriction, it is impossible to write a compact long-term history of optics without taking further measures of economy. A first difficulty comes from the enormous variations of the cultural and disciplinary contexts in which optical knowledge was produced. I provide the necessary minimum of information about these contexts, and refer the reader to secondary literature for further examination. For instance, I take the seventeenth-century mechanization of the world picture as a given, and I do not enter the debates on its various causes. I save further space by avoiding meta-historical questions, such as: Was Alhazen a revolutionary? Was Kepler the last of the ancients or the first of the moderns? Did Thomas Young deserve the contemporary neglect of his undulatory theory? When dealing with nineteenth-century optics, I save ink by being very brief on aspects that are usually covered in histories of electrodynamics, relativity, or

[1] On the history of post-Keplerian theories of vision, cf. Helmholtz 1867, Hirschberg 1899–1908, Pastore 1971, Hoorn 1972, Wade 1998.

quantum theory, such as the optics of moving bodies, thermal radiation, or the electron theories. In describing the reception of important optical systems or theories, I sample revealing commentaries and refer to the secondary literature for more systematic coverage. I give details of optical experiments only when they matter to the conceptual developments.[2] In presenting mathematical developments, I use concise modern notation as long as this does not alter the structure of the historical actors' arguments.

By trimming the narrative in these ways, I aim to highlight conceptual contents and interconnections. I apply two other strategies to this end. First, I pay more attention to the actors' long-term memory than could be done in studies confined to short periods of time. The main optical writers usually situated themselves within or against earlier traditions that spanned many generations. Their innovations largely depended on their creative reading of masters of a remote past. For example, seventeenth-century optical writers frequently referred to Greek philosophy and optics; Euler's theory of light largely reproduced anterior neo-Cartesian views on light; and Young's innovations depended on his erudite knowledge of the physics of sound and light from the Greeks to his time. Secondly, I examine the history of acoustics and the extent to which the leading opticians explored and deployed analogies between sound and light. Contrary to a well-spread opinion, the basic concept of sound as a compression wave was not available until the last third of the seventeenth century. In earlier times there were several concepts of sounds deriving from the ancient Greek notion of sound as a breath of air. This lack of a consensus on the nature of sound made it difficult to construct optics by analogy with acoustics. More generally, wave concepts that are now regarded as basic only entered acoustics a little before or even at the same time as they entered optics. Keeping this fact in mind helps clarify the timing and contents of the various wave theories of light.

Chapter 1 covers the period extending from the dawn of Greek philosophy to Kepler's optics. It describes highly diverse concepts of vision, including the ancient notion of visual rays probing the objects, the atomist idea of material simulacra emanating from the object, Aristotle's concept of forms propagated through a medium, Alhazen's idea of light rays traveling from each point of the object to the center of the eye, and Kepler's modern concept of the eye as a dioptric imaging device. The unity of this first chapter derives from the fact that during this whole period Greek theories remained a necessary reference or departure point (Kepler himself spent much time demolishing scholasticism). Chapter 2 deals with the mechanical medium theories of optics that flourished in the seventeenth century, from Descartes's to Huygens's. Special attention is here given to the gradual introduction of analogies with the propagation of sound, and a digression is made on the early theory of optical instruments. Chapter 3 presents Newton's highly influential optics and reveals its multiple levels, ranging from bare observation to long-hidden corpuscular and ethereal speculation. Chapter 4 describes various trends of eighteenth-century optics: a phenomenological approach including photometry and the development of achromatic objectives; a Newtonian approach that presumed light corpuscles and their attraction by matter; a mostly French, neo-Cartesian approach based on contact action in the ethereal

[2]For proper accounts of the instrumental dimension of optics, see for instance Buchwald 1989 and Chen 2000 (in the first half of the 19th century); Hentschel 2002 (in the context of spectroscopy); Staley 2008, chap. 1 (interferometry and metrology in the late 19th century).

medium; and a mostly German, Eulerian approach based on direct and improved analogy with acoustics. Chapter 5 recounts Thomas Young's attempt to revive the wave theory with his principle of interference, the subsequent Laplacian consolidation of the Newtonian theory based on the corpuscular interpretation of polarized light, and Fresnel's impressive demonstration of the superiority of the wave theory in explaining diffraction, polarization, and propagation in crystals. Chapter 6 describes a few of the many nineteenth-century attempts to provide Fresnel's ether with a proper mechanical foundation; Maxwell's electromagnetic theory of light and its competition with elastic-medium theories of light; theories of the coupling between ether and matter in the contexts of optical dispersion, optical rotation, and the optics of moving body. Chapter 7 describes some highly mathematical contributions to nineteenth-century optics that did not depend on the deeper constitution of the ether: Hamilton's theory of systems of rays, Kirchhoff's and others' diffraction theories, and Gouy's and others' introduction of Fourier analysis in discussing optical coherence and the nature of white light.

In preparing this book, I found much inspiration in the works of a number of historians of optics including David Lindberg, Gérard Simon, A. Mark Smith, and Roshdi Rashed on Greek and Arab optics; Abd al-Hamid Ibrahim Sabra on Alhazen's and Descartes's optics; Catherine Chevalley on Kepler's optics; Alan Shapiro on many aspects of seventeenth-century optics; Michel Blay on Newton's optics and on rainbow theory; Fokko Jan Dijksterhuis on Huygens's optics; Geoffrey Cantor on British eighteenth-century optics; Casper Hakfoort on Euler's optics; Fabrice Ferlin on d'Alembert's optics; Jean Eisenstaedt on some aspects of Newtonian optics; Nahum Kipnis on Young's principle of interference; Jed Buchwald on early nineteenth-century optics; André Chappert on Malus and on post-Fresnel optics; and Edmund Whittaker on mechanical theories of the ether. There are of course many more relevant secondary sources, a significant fraction of which is cited in the footnotes and listed in the bibliography. When many authors cover the same topic, contradictory viewpoints and interpretations are inevitable, and for this reason, among others, I have anchored my own analysis on the reading of primary sources. I found this imperative for treating some aspects of this long history, such as French neo-Cartesian optics, optics in the second half of the nineteenth century, or the connection of optics with acoustics where secondary sources are much less abundant, or weaker.

This work was done within the REHSEIS research team of Centre National de la Recherche Scientifique, with the friendly encouragements of its directors Karine Chemla and David Rabouin. I have also benefitted from the hospitality of Michèle Leduc at the Institut Francilien des Atomes Froids, from the hospitality of Cathryn Carson and Massimo Mazzotti at Berkeley's Office for the History of Science and Technology, from the help of the librarians at the Ecole Normale Supérieure, from the reactions of students who heard unripe versions of this history, and from Ed Jurkowitz's comments on some of the text. Lastly, two anonymous reviewers and my sponsoring editor at OUP, Sonke Adlung, suggested a few improvements to the manuscript. Like anyone teaching history, I ask my students to read the most important sources in the primary and secondary literature. Although this book tells the story of light and vision in more detail than my oral lectures did, it is still meant as an incentive to read more of the cited literature. Hopefully some of my readers will be inspired to go and investigate one of the many intriguing aspects of this history.

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CONTENTS

CONVENTIONS AND NOTATIONS

α, β, γ	angles, or direction cosines
c	velocity of light (in vacuum)
$\mathbf{B}$	magnetic induction field
$\mathbf{D}$	Fresnel's or Maxwell's displacement.
$\mathrm{d}\mathbf{S}$, $\mathrm{d}\Omega$, $\mathrm{d}\tau$	elements of surface, solid angle, and volume (respectively)
δ	variation
$\delta(x)$	Dirac's delta function
δ_{ij}	unit tensor (0 for $i \neq j$; 1 for $i = j$)
e_{ij}	strain tensor
$\mathbf{E}$	Fresnel's optical force or Maxwell's electric force
ε	dielectric permittivity
$\mathbf{F}$	mechanical force
$\mathbf{H}$	magnetic force field
i	$\sqrt{-1}$
$\mathbf{k}$	wave vector ($k = 2\pi / \lambda$)
K	elastic constant
K	Fresnel's elasticity operator
λ	wavelength
m	mass of a particle
μ	magnetic permeability
n	optical index
$\mathbf{n}$	index vector ($c\mathbf{k} / \omega$)
ν	frequency
ω	angular frequency ($2\pi\nu$)
$\mathbf{r}$	point of space
ρ	density (of the ether)
$\mathbf{s}$	ray vector
σ_{ij}	stress tensor
t	time
θ	angle
$\mathbf{u}$	displacement of a medium
U	velocity of the earth
υ	velocity of a particle
V	wave velocity, potential, or Hamilton's characteristic function
x, y, z	Cartesian spatial coordinates
x_i	ith Cartesian coordinate, with $i = 1, 2, 3$.
∂_i	partial derivative with respect to the coordinate x_i
∇	gradient operator

$\nabla\times$	curl of a vector field
$\nabla\cdot$	divergence of a vector field

- For the sake of brevity, Cartesian-component equations are often rendered as vector equations, even though none of the authors discussed in this book used the vector notation.
- Citations are in the author–date format and refer to the appended bibliography. Square brackets around the date indicate manuscript sources. Page numbers refer to the last mentioned source in the bibliographical item. Abbreviations are listed at the head of the primary bibliography. For Aristotle's texts, the Latin title is given, followed by the Bekker number.
- Translations from French, German, and Latin are mine, unless the source given in reference is itself a translation. In the latter case, I have freely modified the translation whenever I judged it necessary.

1

FROM THE GREEKS TO KEPLER

Shadows, reflections, rainbows, and other optical wonders could be seen and interpreted from the beginning of mankind. Optical artifacts such as mirrors and burning lenses, optical techniques for astronomical observations, and medical knowledge about ocular diseases antedate the first millenary B.C. The first documented speculations on the nature of light and vision occurred toward the middle of that millenary in oriental and Greek schools of philosophy. Chinese Mohism dealt with the ray propagation, reflection, and refraction of light. In India, the Samkhya, Nyaya, and Vaisheshika made light or fire (*tejas*) one of the five elements of the world; the two latter schools distinguished between the fire from luminaries and the fire from the eye, and conceived both as diverging streams of atoms. The Greek case is the only one of interest here, because it is the one that led to a corpus of specialized literature on which later European and Mediterranean science much depended. The ancient Greeks held diverse theories of visual perception that all differed from modern optics in essential manner. Two of their explanatory frameworks, the atomist and the mediumnist, nevertheless remained influential until the seventeenth century. Geometrical aspects of their optics crossed many centuries as well.[1]

This chapter is devoted to this multifarious Greek optics, to its Arabic reinterpretation at the turn of the first millennium, and to Kepler's optics, much of which remains valid today. As we will see, the Arabic contribution was essential in making light the primary object of vision, whereas for the Greeks light had only been a secondary circumstance of vision. The Arabs nonetheless saved the basic structure of Ptolemean ray optics through an ad hoc theory of vision. In contrast, Kepler's theory of vision introduced the concept of optical images that is the basis of modern geometrical optics. As one may suspect, this evolution strongly depended on cultural conditions that changed enormously in the huge time span of this chapter. Let us begin with the dawn of Greek philosophy.

1.1 Greek theories of vision

In the sixth century B.C., a few bold Greek thinkers rejected the traditional appeal to supernatural forces in the explanation of natural phenomena and replaced these forces with causes to be found in nature itself. They trusted reason and observation more than myths and religious authority. They held diverse views and were eager to debate their differences. Whereas the Milesians sought to reduce phenomena to a single material principle (water for Thales, air for Anaximenes, or the "boundless" for Anaximander),

[1] On mohist optics, cf. Needham 1954–2004, vol. 4:1, pp. 85–6. On ancient Indian optics, cf. Subbarayappa 1971, pp. 478–81; Preisendanz 1989. I thank Agathe Keller for information on the latter optics.

the Pythagoreans believed that numbers were the principle of all things. In the fifth century, Parmenides of Elea formulated his paradox of change, according to which nothing can come to be from not being, and concluded that the true world of being was one, unchanging, and uniform whereas the world of experience was mere illusion. Whether or not Parmenides meant to criticize Milesian monism, his paradox made it difficult to conceive change in a world made of one substance only.[2]

Whether or not later philosophers meant to answer Eleatic paradoxes, they offered concepts of change that partially circumvented them. Later in the fifth century, Empedocles of Acragas developed the idea of four stable "roots" (earth, water, air, and fire) whose variable sympathies and antipathies accounted for apparent change; Leucippus and Democritus reduced natural phenomena to the variable configuration of stable atoms of various sizes and shapes in an infinite vacuum. In the fourth century, Plato adopted a doctrine of matter that compromised between Empedocles's, Pythagoras's, and the atomists': he analyzed Empedocles's elements into atoms of Pythagorean polyhedral shape. Aristotle retained the Empedoclean elements, in a less rationalist guise in which sensible qualities played a central role. He rejected the atomists' endeavor to reduce all qualities to quantity. Besides locomotion, he envisioned two other kinds of change: variation in the intensity or degree of a quality, and actualization of a potentiality. As we will see in a moment, the latter mode of change played an important role in his theory of perception.

During these first three centuries of Greek philosophy, the early Milesians accepted the testimony of senses, the Eleatics belittled it as mere illusion, and all other philosophers recommended a critical approach in which senses should be used under the control of reason (with minimal trust in Plato's case, and maximal trust in Aristotle's). Pythagoras, Empedocles, Leucippus, Plato, Aristotle, and their successors all gave causal accounts of perception through which they defined the perceived properties of objects and the means by which we become aware of them. These accounts varied considerably, despite some interconnections. They depended on the broader philosophical outlook of the author, on casual observation, and on popular beliefs about the nature of the different sensations. They usually treated the five senses in parallel, although sight and hearing received by far the most attention.[3]

The visual fire

In the Greek popular understanding of the visual process, the eye emits a fire whose rays probe the surface of the observed object. Traces of this view are found in ancient Greek poetry and theater, well before the beginning of philosophy. For instance, in Homer's *Iliad* one may read: "[Agamemnon's] heart was black with rage, and his eyes flashed fire as he scowled on Calchas"; or "Achilles was roused to still greater fury, and his eyes gleamed with a fierce light, for he was glad when he handled the splendid present [armour] which the god had made him." The visual purpose of the fire from the eyes is evident in another extract:

[2]Cf. Lloyd 1970. [3]Cf. Beare 1906.

> And Jove answered: 'Juno, you need not be afraid that either god or man will see you, for I will enshroud both of us in such a dense golden cloud, that the very sun for all his bright piercing beams shall not see through it.'

Homer's personified sun saw through his rays, just as a Greek eye saw through visual rays.[4]

This concept of vision evidently derives from analogy with touch: as the astronomer Hipparchus put it around 200 B.C., the fire from the eye acts like a "visual hand." According to Theon of Alexandria (fourth century A.D.), it easily accounts for the fact that attention is needed to see a small object like a needle on the floor. According to Pythagoras's disciple Alcmaeon, the existence of the ocular fire is corroborated by the fact that a blow on the eye excites visual sensations: "And the eye obviously has fire within, for when one is struck [this fire] flashes out."[5]

Among pre-Socratic philosophers, the Pythagoreans and Empedocles shared the popular belief in the visual fire. In a poem written toward the middle of the fifth century B.C., Empedocles compared the eye to a lantern:

> As when a man, thinking to go out through the wintry night, makes ready a light, a flame of blazing fire, putting round it a lantern to keep away all manner of winds; it divides the blasts of the rushing winds, but the light, the finer substance, passes through and shines on the threshold with unyielding beams; so at that time [when Aphrodite created eyes] primeval fire, enclosed in membranes, gave birth to the round pupil in its delicate garments which are pierced through with wondrous channels. These keep out the water which surrounds the pupil, but let through the fire, the finer part.

Empedocles believed vision to depend on two kinds of fire, from flames (including the sun) and from the eye, according to the general principle of action of the like on the like: "By earth in us we perceive earth; by water in us water; by air in us, the gods' air; and consuming fire by fire in us."[6]

Atomist effigies

The only dissenters of this view were the atomists, who reduced every sensation to the impact of atoms from the observed object on the organ of observation. If we believe Theophrastus (Aristotle's successor at the Lyceum), Empedocles and Democritus may in fact have adopted a compromise-theory, in which a material efflux met the visual fire somewhere between the object and the eye. Democritus's most influential heirs, Epicurus and Lucretius, completely eliminated the visual fire and imagined thin layers of atoms traveling all the way from the object to the eye. In a letter he wrote to Herodotus circa 300 B.C. to summarize his philosophy, Epicurus wrote:[7]

[4]Homer, *Iliad*, book 1, v. 101; book 19, v. 12; book 14, v. 341. Cf. Mugler 1964, pp. 7–13 (Introduction), ἀκτίς and ὄψις entries.

[5]Hipparchus cited by Aetius of Antioch, in Diels 1879, p. 404; Theon, in Ver Eecke 1959, p. 54; Alcmeon, fragment A5, in Diels & Kranz 1951–52, vol. 1, p. 212. Cf. Lindberg 1976, 3–4; Simon 1988, pp. 22–8.

[6]Empedocles, fragments B84, B109, in Diels & Kranz 1951–52, vol. 1.

[7]Theophrastus, *De sensibus*, 49–83, in Diels 1879; Epicurus, letter to Herodotus, in Diogenes Laertius 1925, vol. 2, pp. 577–9. Cf. Lindberg 1976, pp. 2–3; Simon 1988, pp. 36–41.

> There are outlines or films, which are of the same shape as solid bodies, but of a thinness far exceeding that of any object that we see. For it is not impossible that there should be found in the surrounding air combinations of this kind, materials adapted for expressing the hollowness and thinness of surfaces, and effluxes preserving the same relative position and motion which they had in the solid objects from which they come. To these films we give the name of 'images' or 'idols' ... Besides this, remember that the production of the images is as quick as thought. For particles are continually streaming off from the surface of bodies, though no diminution of the bodies is observed, because other particles take their place. And those given off for a long time retain the position and arrangement which their atoms had when they formed part of the solid bodies, although occasionally they are thrown into confusion ... We must also consider that it is by the entrance of something coming from external objects that we see their shapes and think of them. For external things would not stamp on us their own nature of color and form through the medium of the air which is between them and us or by means of rays or currents of any sort going from us to them, so well as by the entrance into our eyes or minds, to whichever their size is suitable, of certain films coming from the things themselves, these films or outlines being of the same color and shape as the external things themselves.

Lucrecius' *De rerum natura*, written circa 50 B.C., gives a detailed and poetical expression of this view:

> And thus I say that effigies of things,
> And tenuous shapes from off the things are sent,
> From off the utmost outside of the things,
> Which are like films or may be named a rind,
> Because the image bears like look and form
> With whatso body has shed it fluttering forth.–
> A fact thou mayst, however dull thy wits,
> Well learn from this: mainly, because we see
> Even 'mongst visible objects many be
> That send forth bodies, loosely some diffused–
> Like smoke from oaken logs and heat from fires–
> And some more interwoven and condensed–
> As when the locusts in the summertime
> Put off their glossy tunics, or when calves
> At birth drop membranes from their body's surface,
> Or when, again, the slippery serpent doffs
> Its vestments 'mongst the thorns–for oft we see
> The briers augmented with their flying spoils.

These effigies (*simulacra*) strike the eye and convey to it all the information needed to recognize the shape and colors of bodies. Figure corresponds to the arrangement of the atoms of the effigy, and color to their shape.[8]

Lucretius addressed two evident difficulties of this picture. First, it seems to exclude that the eye could perceive the entirety of the effigy of a large object. Lucretius suggested that a

[8]Lucretius 2004, p. 105 (book 4, verses 50–60).

small part of the effigy was enough to convey all its properties, just as the hardness of a big stone can be inferred by touching only a small part of it. Secondly, this picture does not explain why objects are only seen in the presence of a source of light, namely, the sun or a flame. Lucretius solved this difficulty by admitting the continuous emission of subtler atoms of light from the luminaries. These atoms, being able to enlarge the pores between the atoms of air, permitted the free traveling of the effigies from the object to the eyes. Consequently, for the (later) atomists vision required two entities, the light from the sun, and the material effigies from the observed bodies.[9]

Plato's synthesis

In one of his Socratic dialogues, the *Timaeus* written in the early fourth century B.C., Plato included an analysis of visual perception that combined Pythagorean, Empedoclean, and Democritean elements. In addition to the popular fire from the eye, he assumed a fire from the sun or flames in order to explain the necessity of (day)light for vision. As appears from their poetry and from their indifferent use of the word ἀχτίς (ray) both for the visual rays and for sunrays, the ancient Greeks assumed a deep analogy between the visual fire and sunlight. In harmony with Empedocles's principle of the interaction of the like with the like, Plato assumed that the fire from the sun coalesced with the fire from the eye to form a coherent, homogenous, and percipient body between the eye and the sighted object:[10]

> The fire within us, which is akin to the daylight, [the gods] made to flow pure smooth and dense through the eyes ... Whenever this visual current is surrounded by daylight, then it issues forth as like into like, and coalesces with the light to form one uniform body in the direct line of vision, wherever it strikes upon some external object that falls in its way. So the whole from its uniformity becomes sympathetic; and whenever it comes in contact with anything else, it passes on the motions thereof over the whole body until they reach the soul, and thus causes that sensation which we call seeing.

What comes in contact with this percipient union of light and visual fire is not the seen object itself, but a third fire emanating from the surface of this object. Here Plato probably followed Democritus, although he preferred to explain the perception of images by extended sensitivity rather than by traveling effigies. As was mentioned, Plato also shared the Democritean idea of an atomic constitution of the elements, with some Pythagorean additions such as the regular polyhedral shape of atoms. In order to explain contrast and brightness, he assumed different sizes and velocities for the particles of the fire emanating from bodies. When these particles are smaller than those of the visual current, they dilate this current and cause the sensation of whiteness. In the contrary case, they cause the sensation of blackness:

> The particles which issue from outward objects and meet the visual stream are some of them smaller, some larger, and some equal in size to the particles of that stream. Those of equal size cause no sensation, and these we call transparent; but the larger

[9]Ibid., pp. 114–15.

[10]Plato 1888, p. 157. Cf. Lindberg 1976, pp. 3–6; Simon 1988, pp. 29–30.

> and smaller, in the one case by contracting, in the other by dilating it, produce effects akin to the action of heat and cold on the flesh, and to the action on the tongue of astringent tastes and the heating sensations which we termed pungent. These are white and black, affections identical with those just mentioned, but occurring in a different class.

The swifter particles are able to travel against the visual current and reach the eye to cause the sensation of brightness. Colors (other than black and white) result from the mixture of the two fires (from the eye and from the object) within the eye in various proportions and speeds.[11]

Aristotle's insensitive medium

Plato's most influential student, Aristotle, criticized every earlier theory of vision. He rejected Democritus's idea of effigies as erroneous inference from the fact that a small picture of an object can be seen in the eye of another person (he correctly interpreted this fact as a case of mere reflection). He equally rejected Plato's visual fire, for he judged the coalescence of this fire with daylight or its quenching by darkness to be absurd notions. He questioned the ability of the visual rays to reach so far as the remote stars, and brushed away the mending assumption that the light from the stars only met the visual rays in the vicinity of the eye.[12]

In his own theory, Aristotle retained Plato's idea that a homogenous medium was required between the eye and the object of sight:

> Seeing is due to an affection or change of what has the perceptive faculty, and it cannot be affected by the seen color itself; it remains that it must be affected by what comes between. Hence it is indispensable that there be *something* in between—if there were nothing, so far from seeing with greater distinctness [as the atomists would think], we should see nothing at all.

Aristotle's medium no longer involved the visual fire and no longer pertained to an extended sensitivity. It just was the celestial "ether" contained in air and other transparent bodies, and its sole function was to permit the transmission of some intrinsic qualities of the objects of sight, named colors (including black and white). Just as Plato's theory required sunlight for visibility, the transparency of Aristotle's medium needed to be actualized by fire in the sun or on burning bodies. By Aristotle's definition, light was the actualized transparency (whereas for Plato and the atomists, light was a fire from the sun):

> Every color has in it the power to set in movement what is actually transparent; that power constitutes its very nature. That is why it is not visible except with the help of light … Light is the activity [of the ether in transparent bodies]. Light … exists whenever the potentially transparent is excited to actuality by the influence of fire or something resembling the uppermost body.

Aristotle regarded both the actualization of transparency and the transmission of colors as instantaneous, global processes involving no displacement of the parts of the medium:

[11] Plato 1888, p. 248.

[12] Aristotle, *De sensu*, 436a-439a. Cf. Lindberg 1976, pp. 6–9; Simon 1988, pp. 42–5.

"Light has its *raison d'être* in the being of something, but it is not a movement." He did refer the transmission of colors to a "movement" through the lit transparent medium; but by movement he only meant qualitative change, which can "conceivably take place in a thing all at once ... e.g. it is conceivable that water should be frozen simultaneously in every part."[13]

In order to account for black and white, Aristotle assumed that every body—not only the reputedly transparent ones—included some ether and was therefore able to be activated by daylight. The surface of the body appeared white when this activation took place, and black in the contrary case:

> Now, that which when present in air produces light may be present also in the translucent which pervades determinate bodies; or again, it may not be present, but there may be a privation of it. Accordingly, as in the case of air the one condition is light, the other darkness, in the same way the colors white and black are generated in determinate bodies.

Somewhat like Plato, Aristotle reduced the other colors to a mixture of black and white so fine as to elude separate perceptions of its components:

> It is conceivable that the White and the Black should be juxtaposed in quantities so minute that [a particle of] either separately would be invisible, though the joint product [of two particles, a black and a white] would be visible; and that they should thus have the other colors for resultants. Their product could, at all events, appear neither white nor black; and, as it must have some color, and can have neither of these, this color must be of a mixed character—in fact, a species of color different from either.

This idea plausibly derived from the common observation of colors in the sky when the light from the sun is dimmed or reflected by clouds (including the rainbow). With sunset and sunrise in mind, a peripatetic scholar obtained crimson by mixing dusky black with sunlight and purple by mixing feeble sunlight with thin dusky white.[14]

The stoic pneuma

Chrysippus of Soli (third century B.C.) and other stoic philosophers shared with Plato and Aristotle the idea of a medium relating the seen object to the eye. Plato generated this medium from daylight and visual fire. Aristotle took it to be the ether activated by daylight. The Stoics identified it with air properly modified by the joint action of daylight and of a visual *pneuma* (combination of air and fire) emitted by the eye. They followed Plato in retaining the popular idea of an emission from the eye and in imagining a percipient body between the eyes and the object. As Cicero put it, "the air itself sees together with us." The Stoics however rejected Plato's idea of an emanation from the

[13]Aristotle, *De anima*, 419a, 418b, 446b, 447a. Aristotle introduced the ether as a fifth element in which natural motion was circular and unimpeded, as was required for celestial bodies.

[14]Aristotle, *De sensu*, 439b; Pseudo-Aristotle, *De coloribus*, 791a–799b.

surface of bodies and imagined a direct transmission of the pattern of the object through stressed air. So reported Alexander of Aphrodisias:[15]

> [The Stoics] explain vision by the stress of air. The air adjoining the pupil is excited by vision and formed into a cone which is stamped on its base by an impression of the object of vision, and thus perception is created similar to the touch of a stick.

In an influential variant of the stoic view, the Greco-Roman physician Galen (second century A.D.) rejected the walking-stick analogy because a stick cannot convey color, size, and position. He rather compared the illuminated air to an innervated body part, somewhat like the tentacles of an octopus. He held the same principle, the visual pneuma, responsible for the sensory power of the nerves and for that of illuminated air:

> Whenever we see, the ambient air seems to be an instrument similar to the nerves which exist permanently in our body. Indeed the ambient air is affected by the [visual] pneuma, when it is emitted, just as it is affected by sunlight. For sunlight, when it touches the upper limit of the air, transmits its power to the whole. Similarly, vision through the visual nerves involves a pneuma-like substance, and, when it strikes the ambient air, it produces by its first impact an alteration that is transmitted to the furthest distance …
>
> The eye is built to make use of the surrounding air as an instrument. This instrument serves to the proper distinction of sensible elements, just as the nerves do with respect to the brain. Consequently, the eye has the same relation to the air as the brain does to the nerves.

Drawing on anatomic observations, Galen identified the crystalline humor as the part of the eye through which the pneuma was emitted after traveling from the brain through the optical nerve. Unlike the visual fire of earlier authors, Galen's pneuma did not extend to remote objects: it only served to induce a tension of the intermediate air. This conception answered Aristotle's main objection against the visual fire, that is, its incapacity to fill the enormous spaces separating us from the stars.[16]

Euclid's geometry of rays

The atomists and the medium theorists (Plato, Aristotle, and the Stoics) had a holistic understanding of the perception of images, either based on the traveling of effigies or based on global transmission through the medium. Their theories were essentially qualitative and they did not provide any precise understanding of visual appearances and deceptions. There was, however, a Greek geometrical optics according to which the perception of images depended on the incidence of visual rays on the various points of the objects. In this theory, any observer is aware of the directions in which its eye emits visual rays. Consequently, he or she can appreciate the angular distribution of the rays' contact points with the object. This distribution is the main information on which the perception of images is based.

[15]Cicero, quoted in Sambursky 1959, p. 28; Alexander, quoted ibid. p. 124; Octopus analogy from Aëtius of Antioch, cited ibid. p. 24. Cf. Lindberg 1976, pp. 9–10; Simon 1988, pp. 31–2; Hülser 1987–88. The stoic pneuma was responsible for the coherence of material objects, plants, animals, and even of the whole universe.

[16]Galen, *De placitis*, pp. 619, 625.

The Greek concept of visual fire, singular though it may be to modern eyes, is likely to have favored the emergence of a geometrical optics. In the opposite view in which rays emanate from the object, it is not natural to assume that the eye should be able to appreciate the direction of the rays. Moreover, the finite aperture of the eye implies that a whole conic beam of rays enters it. The modern answer to these difficulties involves a theory of the eye that goes far beyond the possibilities of Greek optics. In contrast, the idea of geometrizing vision through the angular distribution of visual rays was a natural one for believers in the visual fire.[17]

The earliest known text based on this simple idea is Euclid's *Optics*, written circa 300 B.C. It is mainly a treatise on angular perspective regarded as an application of Euclidean geometry. Its typical problems concern the apparent ratios of lengths situated at various distances from the eye or at various inclinations with respect to the visual axis. It does not include other aspects of vision such as color and the appreciation of distances, and it tells very little about physical foundations. Euclid's text nonetheless suggests that the rays issue from the eye, except in the case of shadows for which rays from the sun are blocked by opaque bodies (e.g. in Prop. XVIII). This double recourse to visual and solar rays suggests a vague Empedoclean or Platonic framework. Geometry nonetheless dominated Euclid's optics.[18]

Euclid's treatise begins with the following set of assumptions:

1. The straight lines drawn from the eye diverge to embrace the magnitudes seen.[19]
2. The figure contained by a set of visual rays is a cone of which the apex is in the eye and the base at the limits of the magnitudes seen.
3. Those magnitudes are seen upon which visual rays fall, and those magnitudes are not seen upon which visual rays do not fall.
4. Magnitudes seen under a larger angle appear large, those under a smaller angle appear smaller, and those under equal angles appear equal.
5. Magnitudes seen by higher visual rays appear higher, and magnitudes seen by lower visual rays appear lower.
6. Similarly, magnitudes seen by rays further to the right appear further to the right, and magnitudes seen by rays further to the left appear further to the left.
7. Magnitudes seen under more angles are seen more distinctly.

Through the first six assumptions, Euclid defines an angular perspective in which the visual appearance of objects is governed by the angles under which their limits are seen, these angles being reckoned with respect to the frontal direction of the (implicitly standing) observer.

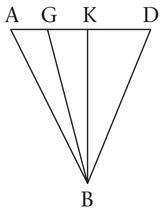
[17]In the second century A.D., Galen (*De placitis*, 471) emphasized that Aristotle's and Epicurus's theories did not account for the perception of size and place, whereas his own version of the stoic theory did so through sensitive visual rays.

[18]Euclid, *Optics*, in Ver Eecke 1959, pp. 1–52. Cf. Lejeune 1948; Lindberg 1976, pp. 12–14; Simon 1988, pp. 63–72. I use the version that Johan Heiberg and Paul ver Eecke regarded as most authentic, although Wilbur Knorr has recently argued for the precedence of Theon's recension (Knorr 1994).

[19]A more literal rendering of the Greek (from the Heiberg manuscript) reads: "The straight lines drawn from the eye carry with them the intervals of large magnitudes."

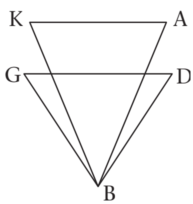
Assumption 7 concerns the distinctness of vision. Its meaning is clarified through Propositions I and II:

Prop. I: No observed magnitude is seen simultaneously as a whole.



Call AD the observed magnitude, and B the eye from which the visual rays BA, BG, BK, BD fall. Since the visual rays diverge, they do not fall on the magnitude AD in a contiguous manner; so that there are intervals of this magnitude on which the visual rays do not fall. Consequently, the entire magnitude is not seen simultaneously. However, as the visual rays move rapidly, it is as if we saw [the entire magnitude] simultaneously.

Prop. II: Among distant magnitudes, the closer ones are seen in more distinct manner.



Call B the eye. GD and KA denote the observed magnitudes, which are taken to be equal and parallel. Suppose that GD is the closer magnitude, and let the visual rays BG, BD, BK, BA fall [from the eye]. I do not mean that visual rays that fall from the eye on the magnitude KA reach it by passing through the points G, D; for [in this case] the segment KA of the triangle BDAKGB would be larger than the segment GD, which is equal by assumption. Hence the magnitude GD is seen under more visual rays than the magnitude KA. Consequently, the magnitude GD is seen more distinctly than the magnitude KA; for the magnitudes seen under more numerous angles are seen more distinctly.

Clearly, Euclid assumed the discreteness of visual rays, and measured the distinctness of vision by the number of rays reaching the object. He conciliated this picture with the apparent continuity of vision by imagining a fast scanning of the objects by the visual rays.[20]

Except for this concept of distinctness, Euclid was mostly concerned with the apparent dimensions and situation of objects as judged through the angular distribution of visual rays. Typical examples are Proposition VII, which states that "equal magnitudes situated on the same straight line, not next to each other, and at a different distance from the eye appear unequal" (see Fig. 1.3), and Proposition VIII, which states that "equal and parallel magnitudes situated at different distances from the eye are not seen proportionally to the distances" (see Fig. 1.4).

These two examples make clear that Euclid's angular perspective differs from the linear perspective of Renaissance painters. In the latter perspective, the apparent size of segments is determined by the size of their conic projection on the plane of representation. If, in the case of Prop. VII (Fig. 1.3), the trace of this plane is taken to be parallel to AΔ, the conic projections of the compared segments are equal. If, in the case of Prop. VIII (Fig. 1.4), this trace is parallel to AB and ΓΔ, the projections of the compared segments are proportional to their distances from the eye.

Euclid's optics has often been regarded as a partial anticipation of modern geometrical optics, the argument being that the orientation of the rays (from eye to object, or from

[20]Euclid, figures from Heiberg 1895; translated text from Ver Eecke 1959.

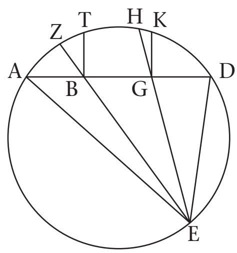


Fig. 1.3. The equal segments AB and GD appear unequal to the eye situated at E because the angles AEB and GED are unequal. From Euclid's *Optics* (Heiberg 1895), Prop. VII.

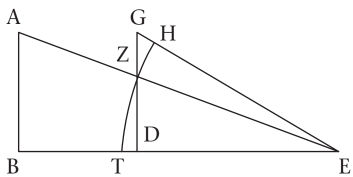


Fig. 1.4. The apparent lengths of the segments AB and GD are not proportional to their distances EB and ED from the eye E, because the angles AEB and GED are not proportional to these distances. From Euclid's *Optics* (Heiberg 1895), Prop. VIII.

object to eye) does not matter to Euclid's geometric constructions. There are good reasons to disagree with this view. First, the notion that only one ray should be considered between a luminous point and the eye is foreign to modern geometrical optics. Secondly, the latter theory deals with the way vision is modified by various artifacts including mirrors and lenses, whereas Euclid's optics deals only with direct vision. Thirdly, the modern theory involves a concept of vision in which the distance of a point-like (not too distant) object from the eye is perceptible, whereas the appreciation of distance is foreign to Euclid's optics.

Euclid's optics is more akin to the theory of linear perspective, since it shares with it the idea of a conic projection through a cone of rays joining the points of the objects to the eye regarded at a point. In Euclid's case, the projection is over a sphere centered on the eye; in the case of linear perspective, it is over a plane located between the eye and the object. In conformity with this analogy, Euclid's optics has a probable origin in the problem of giving the illusion of depth in the painted scenery of theaters. According to Vitruvius's treatise on architecture, Democritus and Anaxagoras solved this problem by means of the concept of visual rays. Although Euclid did not explicitly address it, some propositions of his optics (e.g. Prop. X and Prop. XXI) can be read as rules for the size that should be given to the paintings of familiar objects in order that they appear to be at a given distance from the spectator. In general, linear perspective can be defined as the art of representing objects on a plane so that the real objects and their representations have the same angular perspective. There remains an important difference between the two kinds of perspective: whereas angular perspective concerns direct vision through visual rays, linear perspective concerns vision mediated through depiction.[21]

[21]Cf. Brownson 1981. Euclid's motivation also included altimetry (Prop. XVIII) and surveying (Props. XIX–XXI).

Ptolemy's broader optics

Euclid's *Optics* was original and influential in its application of geometry to vision, but unique in its exclusive focus on angular perspective and direct vision. Later Greek optical geometry included a much broader variety of problems and approaches. Already in the third century B.C., Archimedes is reported to have had a geometrical theory of mirrors as well as some remarks on refraction. This included a theory of burning mirrors and a problem abundantly discussed by later Greek geometers: finding the shape of the mirror that makes all sun rays converge to the burning point. Euclid may have himself developed a now lost catoptrics, and there is a pseudo-Euclidean *Catoptrics* of unknown origin. In Alexandria, Hero produced his own *Catoptrics* in the first century A.D., and the great astronomer Ptolemy his own *Optics* in the second. Even though the latter text has only reached us through an incomplete and unreliable Latin translation of an Arabic translation, it is worth special attention because it represents the most advanced stage of Greek optics and because it played a major role in later Arabic optics.[22]

In the now lost first book of his *Optics*, Ptolemy explained vision by the interaction between three entities of the same genus: the light from luminaries, the colors of bodies, and visual rays. An extract of the second book conveys the same idea:

> The passion undergone by the visual flux is illumination and coloration. Illumination by itself comes from luminous bodies and is too abundant; so that is offends and hurts the sense. Illumination joins coloration in the bodies which receive their light from elsewhere. Light and color modify each other because they change from one species to the other, being both of the luminous genus: color, when it is lit, becomes luminous, and light, when it is colored, is manifestly altered. In contrast, their quality is not altered by the visual flux. The visual sense must indeed be pure, devoid of any anterior quality, and it must receive from them its qualities, as it belongs to the same genus. Yet it undergoes this change of state without reciprocity.

These ideas can be traced to the popular belief in an analogy between the sun and the eye, and to the old principle that the like acts on the like. Ptolemy pushed the analogy between light rays and visual rays very far, as he made both of them susceptible of the accidents of color, reflection, and refraction. In his view, colors were inherent qualities of the (surface of) bodies activated by daylight and sensed by the visual pneuma. Ptolemy thus relied on Aristotle's concept of color and on the Stoics' pneuma, without retaining their idea that daylight served to activate the medium. Instead he held light responsible for the luminosity of seen objects.[23]

Although Ptolemy shared with Euclid the definition of optics as a theory of perception through visual rays, he strongly departed from Euclid in his assessment of the nature and function of the visual rays. In his view, the rays were an abstract representation of a continuous flow of pneuma from each eye. The flow was symmetric with respect to the axis of the eye, concentrated around this axis, and centered on the center of curvature of the

[22]Cf. Lejeune 1948, 1956, 1957; Simon 1988, chap. 3; Mark Smith 1996, 1999; 1998, pp. 13–21. On Greek theories of burning mirrors, cf. Rashed 2000.

[23]Ptolemy, *Optics*, book 2, §23.

cornea (presumably to avoid refraction when leaving the eye). Vision was most distinct where the rays were densest, that is, near the axis. To this concept of a visual cone for each eye, Ptolemy added the notion that the axes of both eyes had to converge toward the object of sight in order to avoid double seeing (diplopia). Unlike modern opticians, he did not associate this process with the perception of depth. Instead he assumed our approximate awareness of the distance traveled by a visual ray between the eye and the object.[24]

Because of this increased complexity of the visual apparatus, direct vision now depended on several parameters: the angular distribution of the rays from each eye to the object, the passion of these rays under the effect of the object's luminosity or color, the orientation of each eye, and the distance traveled by the visual rays. Ptolemy conveyed to the mind the ability to judge the actual size, position, and colors of objects from this complex information. He thus departed from Euclid's narrow concept of vision, which only yielded angular appearances and did not require any psychic activity. Ptolemy's sophisticated theory enabled him to discuss various kinds of optical illusions deriving from unfavorable physical conditions, from intrinsic limitations of the visual apparatus, or from misjudgment of the data provided by this apparatus.

Ptolemy spent much time discussing visual illusions or deceptions, especially those resulting from the reflection or refraction of visual rays. In the case of reflection, he adopted Archimedes's and Hero's rule that the angle of incidence of the visual ray on the mirror should be equal to the angle of refraction, verified it through the experiments described on Fig. 1.5, and justified it through a mechanical analogy with the reflection of a ball on a wall. From Hero or (pseudo-)Euclid, he borrowed the evident idea that the (point-like) object should be seen in the direction of the ray issuing from the eye before reflection on the mirror, and the more enigmatic assumption that the apparent position of the object should lie on the perpendicular dropped from the object on the surface of the mirror (the so-called *cathetus*). In the case of a plane mirror, this construction implies that the object is seen as if it were located in a position symmetric to its real position with respect to the plane of the mirror (Fig. 1.6). Ptolemy applied similar reasoning to convex and concave mirrors, with a dubious extension of the *cathetus*.[25]

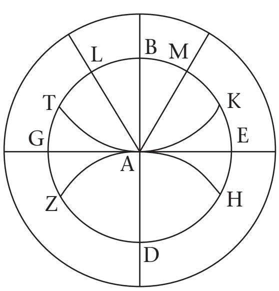


Fig. 1.5. Ptolemy's device for verifying the equality of the angles of incidence and reflection of visual rays. The larger circle is the outline of a bronze disk. GAE, TAK, and ZAH are the traces of a plane, concave, or convex mirror. LA is the trace of the visual ray passing through a collimator, M the position that a mark on the circle GBE must have in order to be seen through reflection by one of the three mirrors. From Ptolemy's *Optics*, book 3, §10. Courtesy of Mark Smith and the American Philosophical Society.

[24] Ibid. §§ 26–44.

[25] Ptolemy, *Optics*, book 3, §3 (laws of reflection), §17 (*cathetus*), §19 (ball and wall). Ptolemy implicitly assumes that the plane of the incident and reflected rays contains the perpendicular to the mirror at their intersection point.

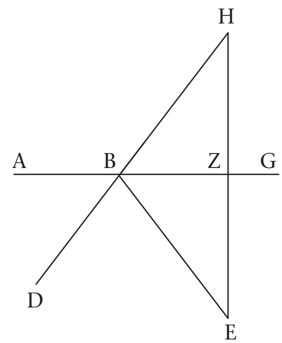


Fig. 1.6. Ptolemy's construction of the image of a point (E) through a plane mirror (AG). The ray DB issuing from the eye is broken at B, which is located so that the angles ABD and EBG are equal. The image must belong to the extension of the visual ray DB. It must also belong to the perpendicular EZ drawn from E to the mirror (*cathetus*). The equality of ZH and ZE results from this construction. From Ptolemy's *Optics*, book 3, §74. Courtesy of Mark Smith and the American Philosophical Society.

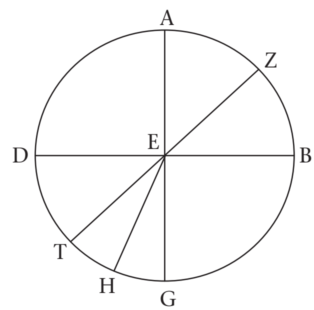


Fig. 1.7. Ptolemy's device for measuring refraction. The bronze disk ADGB is half immersed in water, DEB being the trace of the interface between air and water. The visual ray ZEH reaches the mark H on the graduated circle DGB. From Ptolemy's *Optics*, book 5, §7. Courtesy of Mark Smith and the American Philosophical Society.

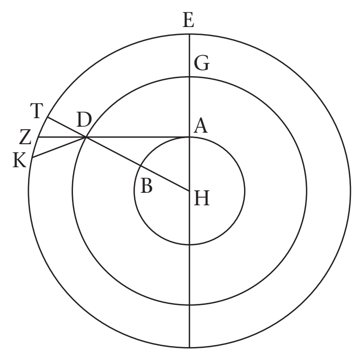


Fig. 1.8. Ptolemy's figure for the effect of atmospheric refraction on the apparent position of stars. AB is the surface of the earth, A the position of the observatory, DG the spherical boundary between the ether and the air, ADK the visual ray refracted in D, K the real position of the star, Z its apparent position. From Ptolemy's *Optics*, book 5, §25. Courtesy of Mark Smith and the American Philosophical Society.

Ptolemy's treatment of refraction was somewhat similar to his treatment of reflection. He assumed that the angle of refraction of visual rays had a well-defined and constant relation to the angle of incidence for a given pair of media, and he tabulated this relation on the basis of measurements performed with the device of Fig. 1.7. His interest in refraction largely resulted from his intention to take into account atmospheric refraction in astronomic measurements. For this purpose, he imagined an abrupt transition between a homogenous ether and a homogenous air (see Fig. 1.8). Ptolemy's activities in astronomy probably determined other characteristics of his approach to optics, such as the experimental devices and procedures used in his experiments on reflection, refraction, and binocular vision.

Conclusions

To summarize, the ancient Greeks held highly diverse views on optics, which they defined as the study of vision. The differences depended on their broader philosophical outlook and on the weight they gave to the popular belief in a visual fire. As none of them admitted action at a distance, there were three basic options: an efflux from the object to the eye, a sensitivity extending to the object, and an inert medium between the object and the eye. The atomists imagined fluxes of particles emanating from luminous bodies and retaining their superficial arrangement; Plato and the Stoics had the visual fire or pneuma engender a sensitive medium between the eye and the object; Aristotle believed in the instantaneous conveying of the superficial pattern of objects through a subtle but insensitive medium.

Light, defined as a fire from the sun and flames, played a role in all these theories as a necessary condition of visibility: for the atomists, it eased the traveling of the material effigies; for all medium theorists, it served to activate the medium; for Ptolemy it caused the luminosity of (not self-shining) objects. Yet light was not the proximate cause or means of vision. Material emanations from bodies were so for the atomists, and the activated medium was so for the medium theorists. Unlike modern optics, ancient optical theories cannot be defined as theories of light. Unlike early modern optical theories, they cannot be regarded as mechanical theories (despite their reliance on contact action). This is obvious for any theory involving the visual fire and extended sensitivity. Although atomist theories seem more mechanical, they require a mysterious, non-mechanical apprehension of the effigies through the eye. Aristotle's optical medium or ether is non-mechanical for it instantaneously conveys qualitative changes that are not reducible to local motion.

The role of the visual fire is a striking feature of Greek optics. It completely upsets the modern separation between physical, physiological, and psychological aspects of vision. It introduces sensitivity where we see only purely physical processes. Yet the rays of this visual fire were the means through which the Greeks were able to produce the first geometrical accounts of the visual perception of spatial properties. Sensitive though they were, these rays could be reasoned and experimented upon. Where Aristotle and the atomists assumed a global conveying of forms without further analysis, believers in visual rays could trace the perception of forms to the distribution of the visual rays that reached the objects. So did Euclid in an idealization in which vision was traced to the angular distribution of rays from a single center. So also did Ptolemy in a more complex theory in which the angle, length, coloration, reflection, and refraction of the visual rays from the two eyes became the basis of easily delusive judgments of the position, shape, and color of objects.

1.2 Medieval optics

Despite the vicissitudes of their transmission, Greek sources alimented the discussion of optical matters until the Renaissance. The diversity of Greek theories of vision was preserved, perhaps because different motivations favored different theories. Astronomers and geometers naturally focused on Euclid's and Ptolemy's theories; philosophers and theologians on Plato's or Aristotle's; physicians on Galen's stoic theory. In the Christian world, the general decline of natural philosophy long prevented any creative departure from received ideas; the most widely accepted theory of vision remained Plato's until the

twelfth century (it still played a central role in Robert Grosseteste's metaphysics). In the Arabic and Persian worlds, each of the main Greek theories of vision found clever supporters and critics when the systematic study of Greek texts began, in the ninth century.[26]

Arabic learning in the ninth, tenth, and eleventh century

The main reason for the Arabs' lead in preserving and adapting Greek science was the Islamic interest in assimilating Greek and oriental knowledge after the conquests of the seventh century. The scholars of the new empire translated a large amount of Greek sources, and subjected their contents to a meticulous criticism often informed by careful experiments. Whereas Ptolemy's and Galen's emphasis on observation and experimentation was exceptional in the ancient Greek world, it became common practice in Arabic science. Religious and cultural reasons, such as the Koran's emphasis on observation-based knowledge of God or the techno-practical needs of the new empire, can be found for this development. Or it may just be the logical consequence of any critical assessment of an imported science that comes in the form of several contradictory theories.[27]

In optics, Euclid was the first source translated and discussed by the Arabs. In ninth-century Baghdad, ibn Ishak al-Kindi wrote an influential treatise in which he consolidated Euclid's optics. Whereas Euclid treated the straightness of rays as an axiom, al-Kindi proved it by simple experiments on the shadows projected by opaque screens (owing to the Greek analogy between sunrays and visual rays, the proof concerned both). Whereas Euclid took extramission for granted, al-Kindi justified it by a variety of arguments including the mobility of the eye (meant to orient the beam of visual rays) and the appearance of a circle whose plane contains the visual axis: in Euclidean perspective the circle is seen as a segment, as it ought to be; in the intromissionist theories known to al-Kindi (the atomists' and Aristotle's), the form of the circle should be globally transmitted to the eye, irrespective of its orientation. Al-Kindi nonetheless rejected the discreteness of Euclid's rays, and replaced it with a concept of visual field similar to Ptolemy's.[28]

In the tenth century, Arab scholars such as al-Razi and al-Farabi assimilated Aristotle's works and consequently favored the intromissionist concept of vision. Remember that, according to Aristotle, the true object of vision is colors transmitted from the object to the eye through the medium. Around the year 1000, the great Abdullah ibn Sina (Latinized as Avicenna) produced a wealth of arguments against every variety of extramission. For instance, the visual fire cannot reach remote objects because this fire would have to fill enormous portions of space; Euclid's discrete visual rays imply a spotted vision of remote objects; the Stoic idea of air stressed by the visual pneuma makes vision impossible in windy weather. None of these objections was truly inescapable. The visual fire could be saved by arguing its complete immateriality; Euclid's discrete rays could be complemented

[26]Cf. Lindberg 1978a, pp. 52–5; 1978b.

[27]Cf. Lindberg 1978a, pp. 55–8.

[28]Cf. Lindberg 1976, chap. 2; Rashed 1992, 1996, 1997, 2005. Theon of Alexandria anticipated Al Kindi's proof of rectilinear propagation (cf. Ver Eecke 1959, pp. 53–4).

with Euclid's notion of scanning; and the Stoics' stressed air could be replaced with a subtler stressed medium. Their accumulation nevertheless had rhetoric force.[29]

The most efficient way to reject a theory is to provide a more compelling alternative. Avicenna's somewhat younger contemporary Ibn al-Haytham (Alhazen or Alhacen in Latin), a prolific natural philosopher who spent most of his life in Cairo, did this. Alhazen's monumental treatise on vision, the *Kitab al-Manazir* (or *De aspectibus* in a twelfth-century Latin translation), begins with the assumption that light from the sun or flames reaches the eye whenever vision occurs. This light affects the eye in a violent and hurtful manner if we direct our gaze to its source. It does the same when we look at the source through a mirror, because it is reflected by the mirror. And some of it also reaches the eye when we look at an illuminated body, because this body reflects the incoming light in a diffuse manner. There is little in here that could not be found in Greek geometrical optics. Since Archimedes at least, every Greek writer in this field assumed the reflection of light on mirrors. Alhazen's main inspirer, Ptolemy, made the illumination of a body a necessary condition for its visibility and therefore implicitly admitted that light reflected by this body would reach the eye. Moreover, everyone knew that an illuminated body, such as the moon, could act as a secondary source of light.[30]

Alhazen was more original in his discussion of the role of light in vision. In his opinion, the blinding effect of intense light, the dependence of the distinctness of vision on the intensity of light, and the existence of after-images proved that light "performed some operation in vision." He meant that the direct or indirect illumination of the eye played an active role in the internal functioning of the eye:

> Light emanates in every direction from any luminous body, however it is illuminated. Thus, when the eye faces any visible object that shines with some sort of illumination, light from that visible object will shine on the eye's surface. And it was shown that it is a property of light to affect sight, while it is in the nature of sight to be affected by light. It is therefore fitting to say that sight senses the luminosity of a visible object only through the light that shines from it upon the eye.

In this citation, Alhazen clearly states that light is sufficient to explain vision. As he later explains, visual rays are redundant because they require a transmission from the object to the eye that is already performed by light itself. He thus comes out as an intromissionist of a new kind. For the atomists, the eye received particles from the surface of bodies and light served to increase the permeability of the intermediate air. For Aristotle, the eye received pure forms of color from the surface of bodies, and light served to activate the transparency of the medium. For Alhazen, the air is transparent in itself and the diffusely reflected light is the direct cause of vision.[31]

Although Alhazen's new concept of light and vision flatly contradicted Aristotle's, it had some Aristotelian elements. Alhazen regarded the propagation of light as the transmission of "luminous forms" through a medium, in probable analogy with Aristotle's understanding of the transmission of colors. Moreover, he shared Aristotle's idea of color as

[29]Cf. Lindberg 1976, chap. 3; Rashed 1997.

[30]Alhazen, *Optics*, book 1, §§4.1, 4.27. Cf. Sabra 1972, 1989; Lindberg 1976, chap. 4; Crombie 1990; Rashed 1997; Mark Smith 1998, pp. 21–30; 2001.

[31]Alhazen, *Optics*, book 1, §§4.5, 6.1 (citation), 6.58, 6.59.

an intrinsic property of bodies transmitted to the eye through the transparent medium. He only departed from this Aristotelian concept by requiring colors to be accompanied by light in their transmission to the eye:[32]

> The form of the color of any tinted body that shines with any sort of illumination is always mingled with the light shining in every direction from that body, and light and the form of color will always correspond with one another. Therefore, since the form of the color of the visible object will always coexist with the light shining from the visible object to the eye, and since light and color will reach the surface of the eye together, and since sight senses the color that is in the visible object by means of the light shining upon it from the visible object, it is quite fitting to say that sight senses the color of the visible object only from the form of color reaching the eye along with the light.

By renouncing the visual rays, Alhazen lost the basis of Greek geometrical optics. His next task was to recreate this science through the concept of light rays. He inherited this concept from the Archimedean treatment of burning mirrors, the Euclidean explanation of shadows, and Ptolemy's thorough analogy between the accidents of light rays and those of visual rays. From Theon's analysis of the shadows cast by an extended source, he may have derived the idea of analyzing luminous bodies in points from which rays emanate in every outward direction:

> The light shining from the self-luminous body into the transparent air therefore radiates from every part of the luminous body facing that air; and the light in the illuminated air is continuous and coherent; and it issues from every point on the luminous body in every straight line that can be imagined to extend in the air from that point.

When applied to illuminated bodies, the same idea seems to ruin the prospect of understanding the perception of images. Indeed each point of the surface of the eye receives rays from every point of the illuminated bodies. Alhazen avoided this confusion by bringing into play a new theory of the eye based on Galenian anatomy.[33]

According to Alhazen's idealization of Galen's representation, the eye is made of various humors separated by spherical tunicae (see Fig. 1.9). The *spaera glacialis* (which corresponds to our crystalline lens), *glacialis* in short, contains the *glacialis humor* and the *vitreus humor*. It is the sensitive component of the organ, where light meets the visual spirit traveling from the brain through the hollow optical nerve. The anterior limit of this humor and the *cornea* belong to concentric spheres defining the center of the eye. Alhazen focused on the pattern of light and color within the *glacialis*. If every ray falling on the cornea from a given luminous point would reach the crystalline with equal intensity, he reasoned, then a large portion of the crystalline would be lit by these rays and the distinction of the various points of a luminous object would be impossible. In order to avoid this paradox, Alhazen selected from among these rays the one perpendicular to both the cornea and the front of the *glacialis* (see Fig. 1.10). This ray, he argued, is the only one that is not refracted, and it is the one that makes the strongest effect on the crystalline:

[32]Ibid. §6.2. Unlike Aristotle, Alhazen assumed a finite propagation time for the propagation of (luminous) forms (*Optics*, book 2, §§3.60–3.62).

[33]Ibid. §3.19. For Theon's analysis, cf. Ver Eecke 1959, pp. 53–4.

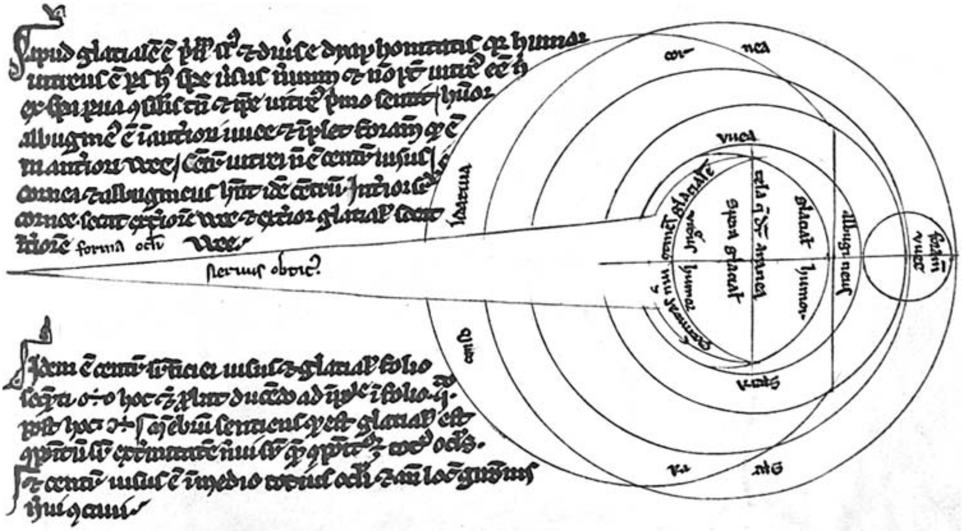


Fig. 1.9. Inner geometry of the eye according to Alhazen. The two largest circles delimit the external tunica of the eye, called the *spaera cornea*; the two smaller circles delimit the *spaera uvea* (iris and choroid); the smallest circle on the right represents the pupil, which is an opening in the uvea (*foramen uveae*); the uvea contains the albugineous humor (*albugineus*) and the lens-shaped *spaera glacialis*, which encloses the *glacialis humor* (frontward) and the *vitreus humor* (rearward); the conic-shaped optical nerve (*nervus obticus*) is attached to the rear of the *spaera glacialis*. Most importantly for Alhazen, the cornea and the front of the *glacialis* are concentric spheres. From a late thirteenth-century manuscript of the *De aspectibus*, in Mark Smith 2001, vol. 2, p. 399. Courtesy of Bibliothèque Nationale de France.

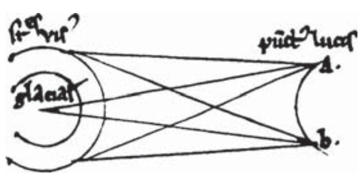


Fig. 1.10. Illustration of Alhazen's selection of efficacious rays. Although the light from the luminous points A (*punctum lucis*) reaches a large portion of the surface of the eye (*sr^{ies} vis' = superficies visus*), only one ray reaches the *glacialis*. From a late thirteenth-century manuscript of the *De aspectibus*, in Mark Smith 2001, vol. 2, 404. Courtesy of Mark Smith and the American Philosophical Society.

The effect of light arriving along perpendiculars is stronger than the effect of light arriving along oblique lines. Therefore, it is quite fitting to say that at any given point the *glacialis* senses only the form reaching it straight along the perpendicular and does not sense any form that strikes it at that point along refracted lines.

For an extended luminous source, these rays draw a conic projection of the surface of this source on the *glacialis.*[34]

Alhazen's selection of the perpendicular rays was clearly ad hoc. He later justified it by comparison with a sword, which better penetrates the enemy when the blow is perpendicular. He also argued that the refracted rays, for some obscure reason, ended up having the same effect as the perpendicular ones. At any rate, conic projection within the eye has an undesired consequence: the rays from the object cross at the center of the eye, which is situated before the hollow of the optical nerve; consequently, the distribution of rays at the entrance of the nerve corresponds to an inverted image of the object. In his second book, Alhazen solved this difficulty by imagining a refraction of the rays when passing from the *glacialis humor* to the *vitreus humor* before entering the optical nerve. He did not give up the sensitivity of the *glacialis*, for he traced the needed refraction to a discontinuity of sensitivity and transparency at the interface between the two humors. His purpose was to preserve the angular ordering of luminous points and thus to save the geometrical core of Euclidean perspective.[35]

Contrary to Ptolemy's visual rays and more like Euclid's, Alhazen's perpendicular rays carried no information about the distance of the eye from the object. Alhazen nonetheless shared Ptolemy's ambition to explain how we judge the distance, size, form, and color of objects. His solution involves the mind's ability to learn from previous perceptions and to memorize patterns through which sensorial data are unconsciously interpreted. For example, memory helps us to identify objects of a priori known size (a flower, a friend). As vision directly informs us of the angle within which we see this object, we can evaluate its distance; moreover, we can evaluate the size of neighboring objects by comparison. Although Alhazen thus pioneered important ideas of modern cognitive psychology, this aspect of his doctrine had little effect on the later history of optics.[36]

Like Ptolemy, Alhazen devoted a whole book to illusions of various kinds, and several books to reflection and refraction. Despite the replacing of visual rays with rays of light and the unprecedented thoroughness of analysis, Alhazen's approach to the latter phenomena is similar to Ptolemy's in scope and methods. One reason for this similarity is that both theories share the same angular perspective (and lack the modern concept of image). Another is that Alhazen inherits Ptolemy's taste for controlled experiments: he verifies for light rays the laws of reflection and refraction that Ptolemy had already verified for visual rays. Still another reason is the shared fondness for mechanical analogies: like Ptolemy, Alhazen offered mechanical analogies for reflection and refraction. To the already mentioned analogy of the sword, he added that of an iron ball crossing a thin board. He explained the inclination of the refracted ray toward the perpendicular (when penetrating a denser medium) by a diminution of the parallel component of the velocity of the sword or ball. Such reasoning may surprise, coming from someone who defined the propagation

[34]Ibid. §§6.24 (citation), 6.68 (sensitive *glacialis*).

[35]Alhazen, *Optics*, book 2, §§2.6–2.18; book 7, §2.8 (sword).

[36]Alhazen, *Optics*, book 2. Cf. Mark Smith 2001, vol. 1, pp. xlii–xlvi.

of light in terms of an Aristotelian transmission of immaterial forms. Alhazen probably did not mean his analogies to be taken too literally.[37]

To summarize, Alhazen preserved the general organization of Ptolemy's *Optics* as well as the geometrical core of Euclid's optics, which is the cone of rays joining points of the object to the center of the eye. Yet Alhazen transcended his Greek models in four essential manners: he eliminated the visual rays; he made light and its accidents the principal object of optical theory; he traced visual perception to the projection of a pattern of light on a sensitive receptor within the eye; he understood the role of learning and memory in the interpretation of visual sensations. Although he espoused the Aristotelian concept of forms transmitted through a medium, he did not dwell on the deeper nature of light and colors. Ray propagation and the various accidents of light in its travel from sources to the ocular receptor were the only assumptions he needed to develop his theory of visual perception.

Late medieval and Renaissance Europe

Twelfth- and thirteenth-century Europe experienced a deep cultural transformation through the numerous translations of Greek and Arabic sources undertaken in the linguistic and cultural borderlands of Spain and Sicily. Whereas in earlier times Christian theology had compromised the diffusion and development of Greek philosophy, the new scholars of the twelfth century were eager to learn from nature and from ancient and foreign sources. In the domain of optics, Albert the Great promoted and expanded Aristotle's doctrines. Roger Bacon's treatises offered a syncretic theory of vision, adding Platonic and Aristotelian elements to Alhazen's *De aspectibus*. In particular, Bacon subsumed Alhazen's idea of medium-transmitted forms of light and color under the broader Aristotelian concept of the *multiplicatio specierum*, which implied the ordered replication of any (spatially or temporally) distributed quality of the object (the *species*) from one layer of the medium to the next. The Polish friar Witelo offered the most detailed and faithful account of Alhazen's theory, although his encyclopedic *Perspectiva* also presented earlier theories and the Baconian synthesis. The spirit of this reception was more conciliatory than critical. The visual rays survived thanks to the popularity of Euclid's and Ptolemy's newly accessible optics. The most arbitrary parts of Alhazen's system, such as the perpendicularity of efficient rays or the sensitivity of the *glacialis*, remained unchallenged.[38]

In the fourteenth and fifteenth centuries, scholasticism seems to have damped interest in geometrical optics or *perspectiva*, as Alhazen's European followers called it. Bacon's multiplication of species invaded natural philosophy, with little geometric benefit, but with strong support for intromission. *Perspectiva* resurfaced in various Renaissance contexts, including artistic perspective, ocular anatomy, astronomy, and popular entertainment.

[37]Alhazen, *Optics*, book 7, §2.8 (iron ball). Cf. Lindberg 1968a; Sabra 1972; Mark Smith 2006, 2008. Alhazen adopted the Euclidean rule of the *cathetus*, which he justified through the (empirical) equality of object and image: cf. Mark Smith 2006, vol. 1, p. xxv. In order to complete the determination of the image through this rule, he solved the (then) daunting problem of finding the locus of reflection of a ray on a spherical or cylindrical mirror when the incident ray and the reflected ray pass through two given points: cf. ibid., pp. xlvi–lxvi.

[38]Witelo 1535 [1270s]. Cf. Lindberg 1976, chaps. 6–8; Lindberg 1978a, pp. 58–67. For a broad study of the relation between optics and arts, cf. Kemp 1990.

As a consequence of the naturalist movement that accompanied the twelfth-century revival of learning, painters sought realistic representation of perspective, light, and shade. In his beautiful frescoes for the Arena Chapel in Padova (c. 1300), Giotto di Bondone pioneered linear perspective without giving any theory. A century later, the Florentine architect Filippo Brunelleschi is reported to have had such a theory, although the credit for its first systematic exposition goes to Leon Battista Alberti. In his *Della pittura* (1435), Alberti explained how the artist obtains a correct perspective by conic projection of the scene onto the plane of the representation. Albrecht Dürer later offered startling representations of the concrete devices that Alberti imagined for this purpose (see Fig. 1.11). This theory plausibly derived from the angular perspective shared by Euclid's, Ptolemy's, and Alhazen's optics, although contemporary surveying or scenographic techniques may also have played a role. It did not offer any means to decide between alternative concepts of vision, as Alberti himself noted:

> Among the ancients there was no little dispute about whether the rays come from the eye or the plane. This dispute is very difficult and is quite useless for us. It will not be considered … Nor is this the place to discuss whether vision … resides at the juncture of the inner nerve [as Alhazen thought] or whether images are formed on the surface of the eye as on a living mirror [as Ptolemy and Avicenna thought]. The function of the eyes in vision need not be considered in this place.

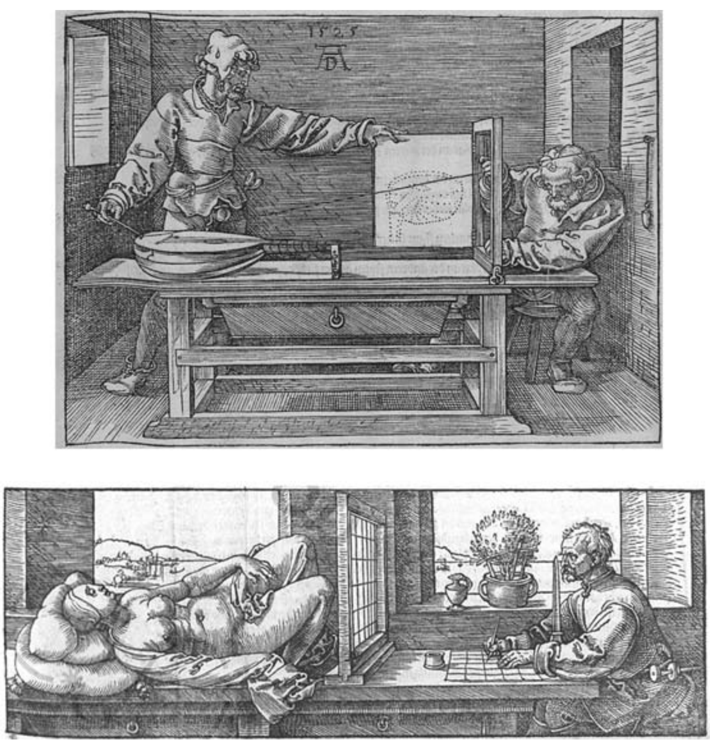


Fig. 1.11. Dürer's representation of devices for perspective drawing. From Dürer 1525, book 4.

Not every artist shared Alberti's lack of interest in the theory of vision. Lorenzo Ghiberti and Leonardo da Vinci discussed received theories and proposed their own speculations. In scholarly circles, the growing importance of linear perspective may have boosted interest in *perspectiva.*[39]

Perspectiva was also relevant to anatomical studies of the eye, although anatomists were often concerned with details that were irrelevant to visual perception. The number and quality of these studies sharply increased in the late sixteenth century. Most investigators retained Galen's basic organization of the eye, which made the crystalline humor the primary organ of vision; and they accepted Mondino dei Luzzi's early scholastic rephrasing of Alhazen's notion of a form of light and color entering the eye and channeled and sensed by the crystalline lens.[40]

A significant exception was Felix Platter, a member of the medical faculty at Basel (see Fig. 1.12). Without proof but with great clarity, Platter's anatomic treatise of 1583 made the retina the sensor of the visual *species*, and the crystalline humor a magnifying glass (*perspicillum*):

> The primary organ of vision, namely the optic nerve, expands when it enters the eye into a hollow retiform hemisphere. It receives and judges the species and colors of

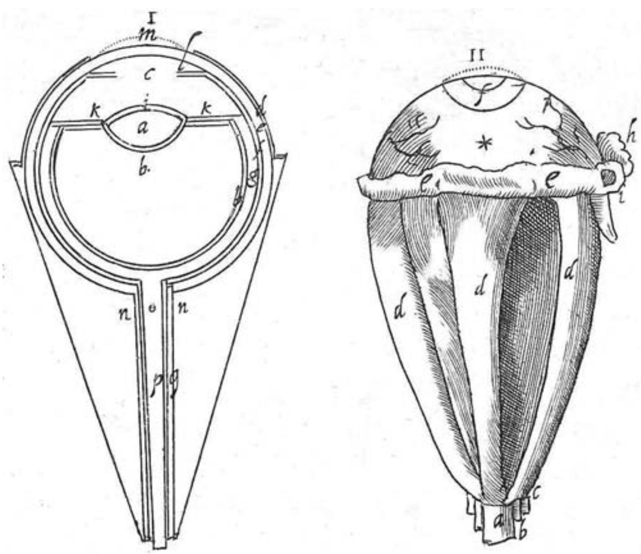


Fig. 1.12. The human eye according to Platter. In the cross-section on the left side, *a* denotes the crystalline humor, *b* the vitreous humor, *c* the aqueous humor, *f* the uvea, *g* the retina, *o* the optical nerve. From Platter 1583.

[39]Alberti [1435] *De pictura*. MS first published as *De Pictura praestantissima et numquam satis laudata arte libri tres absolutissimi Leonis Baptistae Albertis* in 1540 (Basel: Westheimer); Alberti 1966, pp. 46–7; Dürer 1525. Cf. Lindberg 1976, pp. 147–54. Perspectivist elements can be found in the optics that Galileo used in his astronomical observations: cf. Dupré 2001.

[40]Cf. Lindberg 1976, pp. 168–75.

> external objects, which, along with brightness, fall into the eye through the pupil and are manifest to it through its *perspicillum* as will be described.
>
> The crystalline humor is the *perspicillum* of the optic nerve; and, placed between the nerve and the pupil, it collects the species passing into the eye as rays and, spreading them over the whole of the retiform nerve, presents them enlarged in the manner of an internal *perspicillum*, so that the nerve can more easily perceive them.

Although the *retina* had long been known as a cup-shape, net-like expansion of the optical nerve, other anatomists followed Alhazen in making it (or the *aranea*, which prolongs the *retina* and covers the crystalline humor) the receptor of forms already sensed in the crystalline humor. In contrast, Platter compared this humor to a *perspicillum* that received a downsized *species* of the object and sent a magnified version of this *species* over the *retina. Perspicillum* was the Latin name then given to a growing popular device: the eyeglass. As Platter retained the scholastic interpretation of this device in terms of modified *species*, his insight into the working of the eye should not be confused with the modern understanding of the crystalline lens.[41]

The several treatises on *perspectiva* written in the second half of the sixteenth century did not benefit from Platter's insights. They adhered to the Galenian notion of the sensitivity of the crystalline humor, and they usually accepted Alhazen's idea of conic projection of the object into the eye. Those who ventured to question the latter idea did it at the price of some confusion. In his *Photismi* posthumously published in Naples in 1611, the Benedictine monk Francesco Maurolico compared the crystalline humor to a double-convex lens and explained shortsightedness and farsightedness by an excessive or insufficient convergence of this lens that could be corrected with the lenses of spectacles: "Pupils [he meant the crystalline lens] are the spectacles of nature, and spectacles are the pupils of the art." Although Maurolico thus introduced non-perpendicular refracted rays in the mechanism of vision, he did not address the resulting confusion of forms, and he did not give up the sensitivity of the crystalline humor.[42]

Maurolico was more rigorous in his analysis of the *camera obscura* (a dark chamber with a small opening through which an image is projected onto an opposing surface). In the fourth century B.C. the peripatetic author of the *Problemata* had already noted the crescent shape of the eclipsed sun where light rays shone through dark foliage onto the dark ground. In the Middle Ages, the *camera obscura* became a means to observe solar eclipses without being blinded by the excessive light of the sun. In the ninth and tenth centuries, al-Kindi and Alhazen used it to prove the rectilinear propagation of light and the non-interference of intersecting rays. In his treatise *On the shape of the eclipse*, Alhazen also explained why the shape of the hole of the camera did not affect the shape of the image on the opposing surface if this surface was distant enough from the source. The principle of this explanation, which is the superposition of the images produced by the individual

[41]Platter 1583, p. 187. Cf. Lindberg 1976, pp. 175–7. Convex lenses started to be used in the thirteenth century to correct presbyopia, and concave lenses in the mid-fifteenth century to correct myopia: Cf. Helden 1977, p. 10; Ilardi 2007.

[42]Pseudo-Aristotle, *Problemata*, 911b, 912b; Alhazen, *Optics*, book 1, §6.85; *On the shape of the eclipse*, in Wiedemann 1914; Maurolico 1611, p. 80. Cf. Lindberg 1968b; 1976, pp. 178–82; 1983.

luminous points of the source, eluded late-medieval perspectivists who had no access to Alhazen's untranslated treatise. Maurolico rediscovered it in his *Photismi*.[43]

The Napolitan aristocrat Giovanni Battista della Porta described the wonders of the *camera obscura* in his widely read *Magia naturalis* of 1558 and 1589. On this occasion, he showed how to improve the image by placing a glass lens at the opening of the camera:

> Now will I declare what I ever concealed till now, and thought to conceal continually. If you put a small lenticular crystal glass [*crystallina lens*] to the hole, you shall presently see all things clearer, the countenances of men walking, the colors, garments, and all things as if you stood hard by. You shall see them with so much pleasure, that those that see it can never enough admire it.

In an otherwise conservative discussion of the functioning of the eye, Porta compared this organ to a *camera obscura*:

> Hence Philosophers and opticians clearly see how vision takes place, and the question of intromission, which has been so much debated since antiquity, is thus solved; and no other artifice will ever demonstrate it. The representation [*idolum*] is introduced through the pupil, which plays the role of the hole in the window, and the part of the crystalline sphere situated in the middle of the eye plays the role of the screen: I know this will very much please ingenious minds.

Porta still regarded the crystalline humor as the sensitive part of the eye, and did not conceive any analogy with the lens of his improved pinhole camera. Yet he seems to have understood that lenses could produce a distinct picture of an object on a sheet of paper:

> If you put the thing to be seen behind the lenticular [*lens*], that it may pass through the center [axis], and set your eyes in the opposite part, you shall see the image [*spectrum*] between the glass and your eyes, and if you interpose a paper [*papyrum*], you shall see it clearly. So that a lighted candle will seem to burn upon the paper.

Porta produced a similar phenomenon with crystal balls. Although he was primarily interested in images floating in the air, he also knew how to materialize these images. In modern terms, he anticipated the notion of real image.[44]

To summarize, late-medieval translations of Arabic sources permitted the European integration of Alhazen's intromissionist theory of vision, based on the idea of a conic projection of luminous objects over a sensitive crystalline humor. As this theory could easily be rephrased in the Aristotelian language of *species*, it became part and parcel of scholastic learning under the name of *perspectiva*. Euclidean and Ptolemean optics were nonetheless taught in parallel, and there was still talk of visual rays at the end of the Middle Ages. In the Renaissance, the flourishing of arts and techniques intensified the interest in *perspectiva*, and suggested analogies of the eye with spectacles or with the *camera obscura*. With Flatter's exception, the authors of these analogies retained the sensitivity of the crystalline lens. None of them understood this lens's ability to form an image of the object.

[43]Maurolico 1611, theorems 20–22. On Alhazen's and earlier anticipations, cf. Lindberg 1968b, 1970.

[44]Porta 1589, 266 (*camera*), 267 (eye), 269 (lens), 270–1 (crystal ball); 1658, book 17, chaps. 6, 10, 13 (*Pila* is there mistranslated as pillar). Cf. Lindberg 1976, pp. 182–5; 1983.

1.3 Kepler's optics

From astronomy to vision

On 10 July 1600, the German astronomer Johannes Kepler observed a solar eclipse by means of a *camera obscura*. On this occasion, he confirmed a then well-known anomaly: the angular diameter of the moon, as seen through the *camera*, was larger than the value given by direct observation. Suspecting that the finite diameter of the opening on the *camera* played a role, he rediscovered Alhazen's and Maurolico's principle that the image on the wall of the camera should be built by superposition of the conic beams issuing from the luminous points of the object and passing through the hole of the camera. This construction precisely explained the moon-diameter anomaly. Kepler soon speculated that it might also play a role in the functioning of the eye. Like Porta, he compared the front part of the eye to a *camera obscura*:

> The pupil plays the role of a window and the crystalline behind plays the role of a screen, except that the intersection [of beams] is not yet complete because of the proximity of the crystalline, so that everything is still confused.

To Porta's "I know this [analogy] will very much please ingenious minds," he replied:

> Surely, we are very pleased, oh excellent initiate of the mysteries of nature! Over is the quarrel about whether vision proceeds by reception or emission! ... In me thou hast an admirer and a man who gratefully publishes thy name and I hope others will do the same.

Yet for Kepler the projection on the crystalline humor could only be the first step of a more complex process. The finiteness of the pupil implied a blurring of the image, as was the case for the eclipsed sun observed in the *camera obscura*.[45]

As every astronomer, Kepler was also concerned with atmospheric refraction and its effect on the apparent position of the stars in the sky. While he was struggling to establish the correct law of refraction, he thought of using the position of the image of a point-like object seen through refracted rays as a means to measure the angle of refraction. This brought him to the ancient notion of *cathetus*, according to which the image should be at the intersection of the prolongation of the visual ray (that is, the ray reaching the center of the eye) and the perpendicular drawn from the object to the surface. First considering the simpler case of reflection through a mirror, Kepler dismissed all received justifications of the *cathetus* from pseudo-Euclid to Witelo. In any theory in which only one ray from a point-object is used for vision, Kepler reasoned, the distance of the object or its image is undetermined: the image of the object could be anywhere on the prolongation of the visual ray.[46]

[45]Kepler 1604, pp. 173, 209. In a letter to his mentor Michael Maestlin of 10/20 December 1601, Kepler wrote: "Why would it not take place in the eye what I have demonstrated in the case of apertures, namely, that luminous [objects] appear dilated while obscure [objects] appear contracted? For in the eye too there is an aperture." He thus expressed his intention to exploit the analogy between the pupil and the aperture of the *camera obscura* in studying errors of vision. The theory of monocular distance appreciation in Kepler 1604, pp. 65–6 also suggests an early stage of Kepler's understanding of vision in which he used the analogy of the *camera obscura* without the stigmatic property of the crystalline. On Kepler's optics in general, cf. Lindberg 1976, pp. 185–208; Chevalley 1980; Straker 1981; Crombie 1991.

[46]Kepler 1604, pp. 56–9.

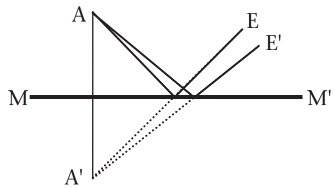


Fig. 1.13. Kepler's construction of a binocular mirror image. The two rays entering the pupils of the eyes E and E′ after reflection on the mirror MM′ intersect at point A′, which is the symmetric of the luminous point A with respect to the mirror. For a plane mirror, this remains true when the line EE' does not lie in the reflection plane.

Kepler found the key to the perception of depth in binocular vision. Like Ptolemy, he assumed that the visual rays from the two eyes should converge to the luminous point when it is observed directly. Unlike Ptolemy, he traced the perception of depth to our awareness of the orientation of the two eyes: the basis and the angles of the (equilateral) triangle formed by the two eyes and the luminous point together determine the distance of the object. When a refracting or reflecting surface intervenes between the eye and the object, "the true location of the image is the point of intersection of the continuation of the visual rays issuing from the two eyes and passing through their own points of refraction or reflection." Since, in the case of a plane mirror, the prolongation of every reflected ray from the luminous point passes through the symmetric of this point with respect to the plane of the mirror, this symmetric is the image (see Fig. 1.13). Kepler thus justified the rule of the *cathetus*. He then showed that in the case of reflection through a convex or concave mirror and in the case of refraction through a plane surface, this rule only held in the (most common) case in which the line joining the two eyes was perpendicular to the (average) plane of reflection or refraction (in this case, the two reflected or refracted rays intersect on the intersection of the two incidence planes, which coincides with the *cathetus*). He thus corrected an error that had lasted nearly two millennia, "a shameful stain over the most beautiful science." At the same time, he introduced what we may call the *principle of intersection*, namely the idea that image formation results from the intersection of rays.[47]

Pursuing his reflections on vision, Kepler convinced himself that the theory of Alhazen and Witelo was unacceptable. His strongest objection concerned the selection of rays perpendicular to the anterior surface of the crystalline humor. There was no reason to believe, Kepler argued, that rays slightly deviating from the perpendicular would be much less intense than perpendicular rays; and these oblique rays were enough to create a confusion of forms within the crystalline and beyond. Kepler's other objections had to do with improved knowledge of the anatomy of the eye. He chanced to see Platter's plates, which disconnected the optical nerve from the crystalline humor so that the latter could not longer be regarded as the sensitive part of the eye. Platter and others also refuted the flatness of the posterior surface of the crystalline humor, which Alhazen needed to avoid the crossing of rays before the channeling of forms through the optical nerve.[48]

Most importantly, Kepler learned from Platter that the retina was the sensitive receptor of the eye and that the crystalline humor acted like an optical lens. In his attempts to

[47]Kepler 1604, pp. 59 (citation), 62 (evaluation of distance), 69 (citation), 70–5 (violations of the rule of the *cathetus*).

[48]Ibid., pp. 166–7 (crystalline disconnected from retina), 204–5 (no selection of perpendicular rays), 205–6 (inverted picture).

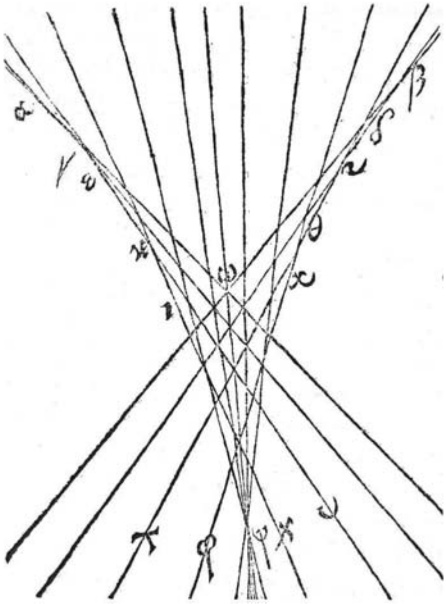


Fig. 1.14. Plane section of the caustic under a refracting globe (not represented) exposed to sunlight. The letters α, β, γ, δ, ε, ζ, η, θ, ι, κ denote the intersections of successive refracted rays (which were equidistant in the original parallel beam from the sun) and define the caustic; ω the first intersection of the refracted rays with the axis (the central ray); ψ the last of these intersections, which is also the apex of the caustic. From Kepler 1604, p. 194.

understand the role of this optical lens, Kepler drew on Porta's magics. One of Porta's tricks employed a crystal sphere to produce a "floating image" of objects seen through it. Namely, the object was seen in the air between the eye and the sphere. Porta further indicated (in the case of a lens) that a picture of the object appeared on a piece of paper placed at the locus of the illusion. Kepler sought a geometrical explanation of these strange phenomena, based on rays issuing from the luminous points of the object and twice refracted through the sphere.[49]

The ability of glass balls to concentrate light and heat had long been known as the principle of burning spheres and it had been the object of numerous theoretical studies, the best of which probably was Alhazen's. Kepler carefully studied the intersection of rays after crossing the sphere and showed that they formed what we now call a caustic, namely a surface on which the density of rays is infinitely larger than elsewhere. In the case of a glass sphere, this surface is engendered by rotating a cusp-shaped curve around the unrefracted ray passing through the center of the sphere (see Fig. 1.14). The end of the cusp is the brightest point, situated at the distance d from the center of the sphere. When a

[49]Ibid., pp. 166–7 (the retina is the receptor), 180 (Porta on crystal balls and lenses). Cf. Dupré 2007. On Porta, see above p. 25.

luminous object is placed far from the sphere, Kepler reasoned, each of its luminous points projects a quasi-parallel beam on the sphere with varying inclination. To each of those beams corresponds a bright point situated behind the glass sphere at the intersection of the unrefracted ray with a sphere of radius *d*. Kepler thus explained Porta's mysterious observation. He confirmed it by filling a spherical phial with water and interposing it between a luminous object and a piece of paper.[50]

A new theory of the eye

In Kepler's words, the phial "paints" a likeness of the object on the paper. Kepler suspected that a similar phenomenon occurred in the eye: the crystalline humor is somehow able to produce a painting of distant objects on the curved surface of the retina. The comparison is however imperfect, for at least three reasons: the incoming beams are narrowed by the pupil of the eye, the crystalline humor does not have a spherical shape, and a first refraction occurs at the cornea. Kepler understood that the first of these circumstances acted favorably in narrowing the zone of intersection of the outgoing rays (that is, in making the caustic smaller; see Fig. 1.15). He further argued that nature had adjusted the shape of the posterior surface of the crystalline humor so that this zone was nearly reduced to a point (thus causing what we now call stigmatism). As for the refraction at the cornea, he argued that is was so strong that the issuing rays were close to the perpendicular and therefore little refracted by the anterior surface of the crystalline humor (which he assumed to be spherical and concentric with the cornea). This implied that the main refractions occurred at the surface of a complex of roughly spherical shape made of the aqueous and crystalline humor.[51]

The essential point as Kepler himself saw it was that the eye acted as an optical instrument that produced an accurate painting (*pictura*) of distant objects on the retina:

> Vision takes place by a painting of the visible object on the white and concave wall of the retina; the leftward objects are painted on the right side of the wall, the rightward on the left side, the upward on the upper side, the downward on the lower side; green is painted with the same green color, and in a general manner every object is painted with its original color; so that if this painting on the retina could be exposed to daylight by removing the interposed parts of the eye that serve to form it, and if there were a man with sufficient visual acuity, he could recognize the identical figure of the hemisphere [of vision] on the tiny inside of the retina. Proportions are indeed conserved: the angle under which lines drawn from a given point of the visible object would reach a certain point within the eye is about equal to the angle under which these points are depicted; even the smallest points are not omitted; the sharper is a man's vision, the subtler is this painting in his eye.

The rationale for the painting process was the principle of intersection, now transposed from virtual to real images. In modern parlance, an image is formed when all the rays issuing from a given luminous point intersect at a well-defined image point after traveling through the optical system; the image is real when this intersection belongs to the actual

[50]Kepler 1604, pp. 177–8 (experiment), 193–5 (theory).

[51]Ibid., pp. 171–2 (refraction negligible at the anterior surface of the crystalline), 196–9 (role of the pupil).

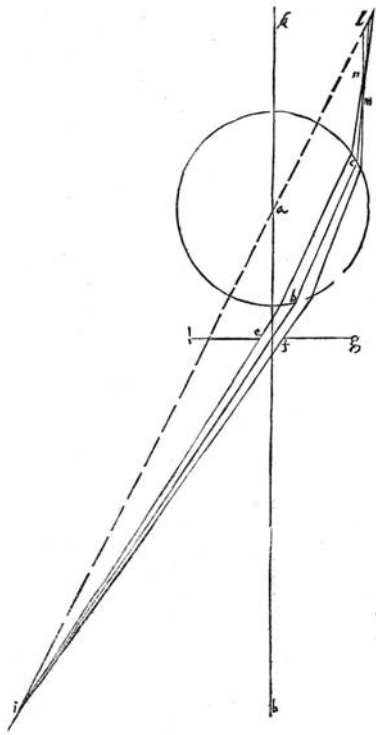


Fig. 1.15. Picture formation behind a refracting globe. The rays issuing from the distant object *i* and passing through the hole *ef* of the diaphragm *dg* are twice refracted, around *b* and around *c*. They intersect in the small region *mn*. From Kepler 1604, p. 197.

path of light; it is virtual when this intersection belongs only to an imaginary straight extension of this path. Kepler used different words in these two cases, *pictura* for the real image, and *imago* for the virtual image: "*Whereas until now the image was a being of reason* [ens rationale]*, we shall henceforth call pictures the figures of objects that really exist on a paper or on any other screen.*"[52]

Kepler did not fail to note that for an object placed close to the eye, its picture in the eye was further from the crystalline humor than it was for a distant object. This circumstance blurs the painting of the object on the retina. Kepler originally assumed that the interval between the crystalline and the retina was set to the value for which vision is most distinct at moderate distances. He explained shortsightedness and farsightedness by the inadequate setting of this interval, to be corrected by concave and convex lenses respectively. At a later stage of his optics, he assumed that the eye was able to control the distance between the crystalline humor and the retina. He did not use this mechanism to explain depth perception for a single eye. He nonetheless understood that the cone of rays entering the

[52]Ibid., pp. 170, 193 (italics in original).

pupil from a single luminous point contained all the information needed to locate this point; and that any (moderately) divergent cone of rays entering the eye produced the impression of a luminous point placed at the apex of the cone, whether or not this apex belonged to the actual path of the light. He thus extended his foreshadowing of the concept of virtual image from binocular vision to monocular vision.[53]

As a consequence of Kepler's theory, the picture on the retina is inverted. We earlier saw that Alhazen eliminated a similar inversion of forms by flattening the posterior surface of the crystalline humor. Kepler originally struggled to avoid inversion in his own theory, for instance by imaging a crossing of rays in the space between the cornea and the crystalline humor. In the end, he accepted the inversion in the name of a sharp distinction between the physical and physiological aspects of vision. For Alhazen and Witelo, the travel of a form or *species* in the crystalline humor, in the vitreous humor, and in the optical nerve had both inanimate and animate aspects: it implied the visual spirit from the brain but it also obeyed the rules of transparency and refraction. This partially inanimate character of the *species* presented to the brain seemed to require a faithful, non-inverted depiction of the object. Kepler avoided this dilemma by making the retina the end of the inanimate process of vision and the beginning of a purely spiritual process which he left to the ingeniousness of "physicists" (that is, physiologists). In his view, the retina was an opaque surface in which no light penetrated; and it also was an impenetrable barrier for the spirits from the brain:[54]

> How the representation or picture is connected to the visual spirits which reside in the retina and in the nerve ... , this I leave to be disputed by the Physicists. For the armament of the Opticians does not take them beyond this first opaque wall encountered within the eye [the retina].

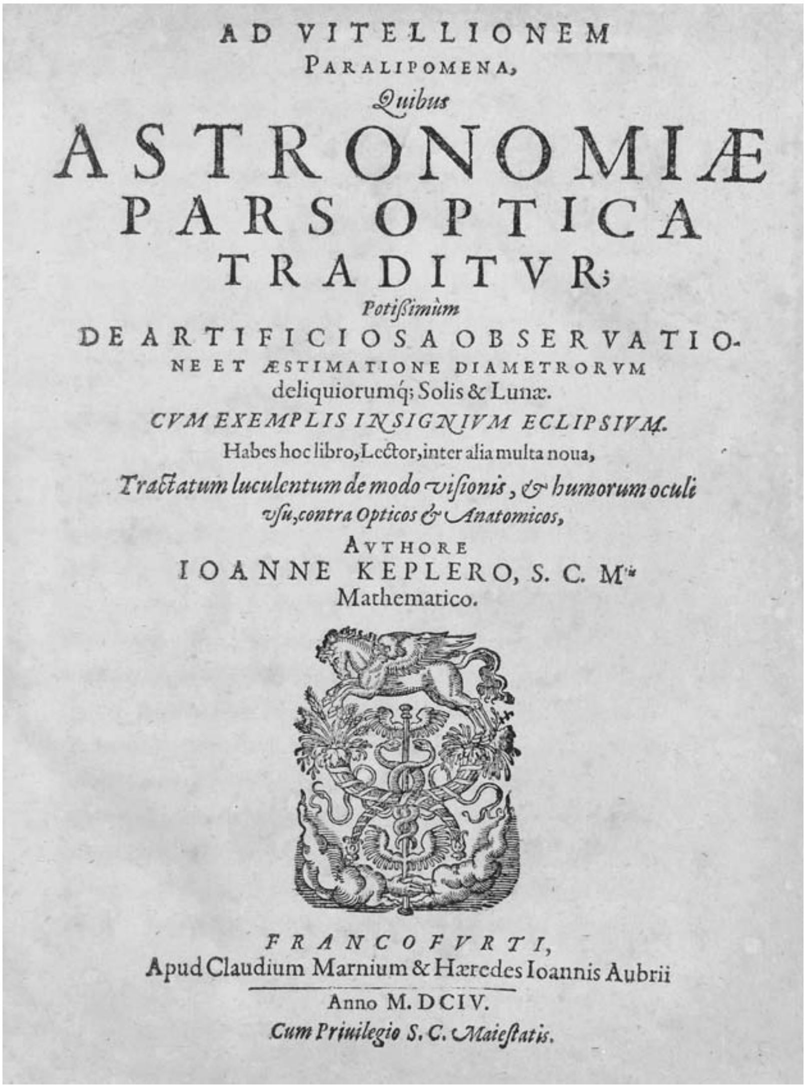
The Paralipomena

Kepler published his theory of vision in 1604, under the biblical title *Ad Vitellionem paralipomena* (the Chronicles of the Old Testament, which complete the Kings, are called *Paralipomena*, which is the Greek for "omitted things"). Witelo's three-century old exposition of Alhazen's optics was Kepler's main point of departure. A series of subtitles indicated that this volume was intended as the optical part of astronomy, that it contained a solution to the paradox of the apparent diameter of the moon as observed in solar eclipses, and that it offered a new theory of vision (see Fig. 1.16). The above-given reconstruction of Kepler's itinerary is based on the historical remarks included in the *Paralipomena*. This itinerary and the resulting theory of vision did not imply any precise concept of light besides Alhazen's notion of rays issuing from every point of an illuminated body.

Kepler nonetheless included a first chapter in which he defined light as an "emanation" from the luminous points of the source. This emanation is a hollow spherical amplification of the source point, in analogy with the holy trinity (center, sphere, and radii). Rays are

[53]Ibid., pp. 63–6 (monocular depth perception), 199–200 (perfectly distinct vision only at a certain distance), 200–3 (defects of vision); Kepler 1611, pp. 26–7 (accommodation).

[54]Kepler 1604, pp. 206 (attempts at eliminating inversion), 168–9 (citation).



AD VITELLIONEM
PARALIPOMENA,
Quibus
ASTRONOMIÆ
PARS OPTICA
TRADITVR;
Potissimùm
DE ARTIFICIOSA OBSERVATIO-
NE ET ÆSTIMATIONE DIAMETRORVM
deliquiorumq; Solis & Lunæ.
CVM EXEMPLIS INSIGNIVM ECLIPSIVM.
Habes hoc libro, Lector, inter alia multa noua,
Tractatum luculentum de modo visionis, & humorum oculi
vsu, contra Opticos & Anatomicos,
AVTHORE
IOANNE KEPLERO, S. C. M^tis
Mathematico.

FRANCOFVRTI,
Apud Claudium Marnium & Hæredes Ioannis Aubrii
Anno M. DCIV.
Cum Priuilegio S. C. Maiestatis.

Fig. 1.16. Title page of Kepler's *Paralipomena* (1604).

the lines of flow of this emanation. As their density decreases with the inverse square of the distance from the center, so does the intensity of light. Owing to its divine character, the emanation is massless and therefore instantaneous. Color is a disposition of bodies that produces a specific "attenuation" of light as well as an injection of some of their matter. Reflection and refraction result from the old Greek principle of the interaction of the like with the like: the spheres of light, being surfaces, must interact with the surface of bodies.

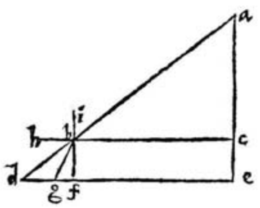


Fig. 1.17. Kepler's justification of refraction. If *bc* denotes the refracting surface and *a* a point source, the intensity of the light delimited by the rays *ai* and *ac* decreases at a rate depending on the angle *bac* when traveling from *a* to *bc*. When the light enters the denser medium, its rate of decrease is smaller, so that the angle between the refracted ray *bg* and the perpendicular ray *ce* must be smaller than the angle *bac*. From Kepler 1604, p. 15.

Kepler struggled to derive the law of refraction from this conception. For instance, he argued that the surface of a dense body implied a smaller divergence of a light beam after crossing the surface because divergence and density belonged to the same genus (see Fig. 1.17). As reasoning of this kind failed to yield a definite law, he turned to the empirical method that led to his understanding of image formation.[55]

Kepler's concept of light differed from any anterior one. He had some sympathy for the atomists and for Empedocles, who at least admitted an emanation from the sun and flames. His *bête noire* was the ambient scholasticism, to the demolishment of which he devoted a long appendix:

> The strange passions of men have become so vain that in order to gain renown through one's work, one must either build or burn Diana's temple: I mean that one must either use Aristotle's authority as a wall or ostensibly wage a war against him … In this day, Aristotle reigns everywhere and the Opticians are in hiding, forced to content themselves with a private liberty. I have therefore decided to exploit their passion for contradiction and erudition in order to force them into the School of the Opticians by publicly denouncing Aristotle's fancies about vision.

The Opticians to which Kepler here alluded probably were Witelo and other perspectivists, who had a higher respect for Aristotle than suggested in this citation. These authors nonetheless shared the main target of Kepler's criticism: Aristotle's concept of activated transparency. In their eyes, it was the colors of bodies, not the transparency of the medium, that light activated; vision required a light *species* traveling from the illuminated body to the eye besides the color *specie*s. To this idea Kepler added that a transparent body, when lit, could only impede vision because absorption and the activation of its own colors interfered with the travel of light from the illuminated body to the eye. He therefore agreed with the atomists that light could only be an emanation that best traveled through a vacuum.[56]

The Dioprice

Antischolasticism was on the rise in the early seventeenth century. In 1610, Galileo Galilei published his *Sidereus nuncius* ("Starry messenger") in which he challenged Aristotelian

[55]Ibid., pp. 6–7 (spherical emanation); props. 2 and 4 (rays), 5 (instantaneous), 9 (inverse squared law), 12 (reflection and refraction), 15 (colors); chap. 4 ("On the measure of refractions").

[56]Ibid., appendix to chap. 1 (citation on p. 29).

cosmology with his amazing telescopic discoveries. Kepler, whom Galileo informed through private letters, promptly understood that his optics contained the true principles of Galileo's instrument. He expounded this theory in his *Dioptrice* of 1611.[57]

The considerations of this treatise are limited to weakly inclined rays (with respect to the common axis of a system of lenses), for which the refraction law is approximately linear. By means of the concept of stigmatism of the *Paralepimonena*, Kepler first showed that a convex lens produced a distinct picture of a small object (placed on the axis of the length), the picture being further from the lens when the object is closer (see Fig. 1.18). After summarizing his theory of vision, he judged the effect of a lens placed between the object and the eye from the convergence and mutual orientation of the beams that issued from points of the object, crossed the lens, and entered the pupil (see Fig. 1.19). For instance, when a convex lens is placed in front of the eye at a distance inferior to the focal distance, a distant object is seen upright, larger, and fuzzy (because the beams intercepted by the pupil do not converge on the retina). Kepler then obtained magnification without fuzziness by the proper combination of two convex lenses. He also explained how Galileo's combination of a convex and a concave lens served the same purpose.[58]

To a modern reader, Kepler's analysis of lenses and telescopes seems strange, with its constant appeal to the beams penetrating the eye and forming a not-necessarily focused picture of the object on the retina. Today, we would reason by means of virtual images at the virtual intersection of the rays of these beams. Kepler did not do so, even though he had used the similar concept of *imago* in the case of a single refraction. It was left to later opticians (including Descartes, Gregory, Barrow, Newton, and Huygens) to systematize Kepler's theory and to eliminate the eye from the analysis of optical instruments. Kepler can nonetheless be credited with having introduced the concept of stigmatism from which the functioning of any optical instrument, including the eye, derives. His *Paralipomena* and his *Dioptrice* contain the germs of modern geometrical optics, except for the chromatic dispersion later discovered by Newton.[59]

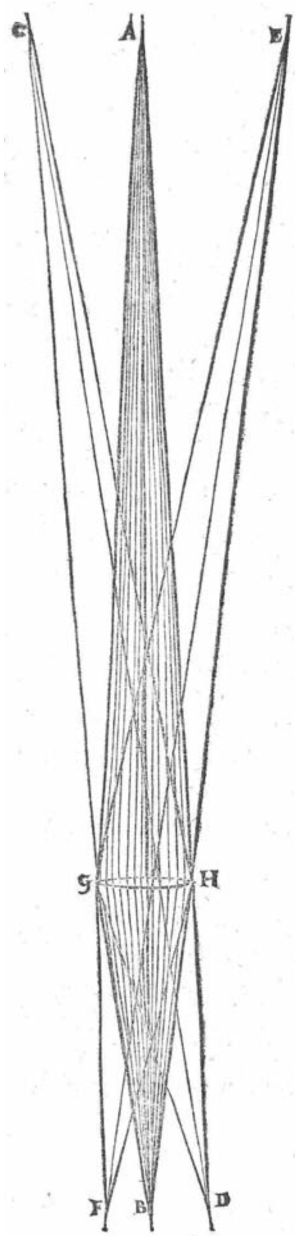
1.4 Conclusions

We may now reflect on the slow transformation of optics from its Greek beginnings to Kepler's accomplishment. During this whole period, optics remained essentially a theory of vision (with the exception of burning mirrors and burning lenses). In Greek antiquity, light was only a circumstance of vision: it facilitated or permitted the process through which an image of the object was transferred to the eye. This process did not imply the

[57]Galilei 1610; Kepler 1611. On the invention of the telescope (around 1608, probably in the Netherlands), cf. Helden 1977; Willach 2008. On Galileo's optics, cf. Dupré 2001.

[58]Kepler 1611, pp. 16–17 (picture by convex lens), 21–8 (theory of vision), 28–42 (effect of single convex lens on vision); 42–5 (two convex lenses), 45–53 (concave lenses); 53–66 (Galilean telescope). The Keplerian telescope (with two convex lenses) was yet unknown. In the *Sidereus nuncius* (pp. 6–7), Galileo described the contraction of the visual field by refraction of the visual rays, and announced a full theory—which he never published.

[59]On Kepler's ignoring virtual images and later optical geometry by James Gregory and Isaac Barrow, cf. Shapiro 1990; Malet 1990, 1997. For Kepler, the image of an object was not always located where we now locate the virtual image (at the common intersection of the rays entering the eye) because the eye did not have to accommodate to the distance of the latter image and could favor an unfocused image at a different distance.



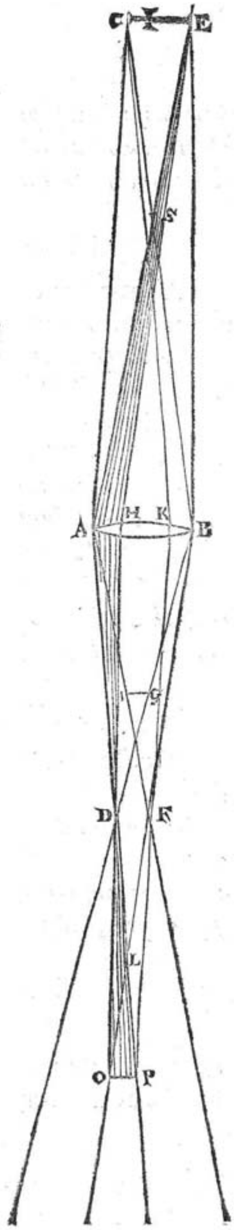


Fig. 1.18. Picture produced by a convex lens. The rays issuing from the points C, A, E of the object converge to the D, B, F of the picture after crossing the lens GH. From Kepler 1611, p. 17.

Fig. 1.19. Images produced by a convex lens according to Kepler. When the pupil of the eye is located at OP (beyond the focal plane DF), an inverted, downsized image of the object is seen. When the pupil is located at IG a blurred, magnified image of the object is seen. Kepler reasons from the inclination of the beams entering the eye (DOP and FOP in the former case), not from the intersection points D and F (as we would now do). From Kepler 1611, p. 32.

traveling of light from the object to the eye; the atomists traced it to the emanation of films from the surface of the object; other philosophers had it depend on a form-propagating medium; the medium was inert for Aristotle, and sensitive for Plato, Euclid, the Stoics, and Ptolemy. Geometrical optics was anchored on the notion of rays issuing from the eye at conscious angles. At the beginning of the second millennium, Alhazen had vision depend on light entering the eye after diffuse reflection on the surface of things. He thus made light the central object of optics. He nevertheless preserved Euclid's angular perspective, thanks to the ad hoc selection of rays passing through the center of the eye. Kepler rejected this selection and drew on Porta's magics and Platter's anatomy to argue the formation of a picture of the object on the retina. He understood that this picturing involved the common intersection of all the rays from a given luminous point, and that it gave a measure of the distance of the object. In the same breath, he made the eye an optical instrument and explained how instruments could correct or enhance vision. Gone were the sensing visual rays of the Greeks and the sensing crystalline humor of the Arabs. The narrower physics of the inanimate now ruled optics from the sun to the surface of the retina.

2

MECHANICAL MEDIUM THEORIES OF THE SEVENTEENTH CENTURY

Most writers on optics in the seventeenth century adopted Kepler's concept of vision and its consequences on the theory of optical instruments. They ignored his concept of light as a divine immaterial emanation, presumably because it contradicted both the declining scholasticism and the rising mechanism that were the main competing natural philosophies of the time. As is well known, the most innovative natural philosophers of the seventeenth century sought to replace the scholastic multiplication of species with mechanical explanations inspired from macroscopic devices and geometry. Many reasons have been evoked for this trend, including the rise of techniques (which brought new mechanical contrivances), the Copernican revolution (which erased the distinction between sublunar and supralunar physics), neo-Platonism (which favored mathematical explanation), the improved social status of mathematicians (whose utility was better recognized), and the success of Galileo's terrestrial mechanics. As far as optics was concerned, there were essentially two kinds of mechanical explanation: one based on analogy with fluid pressure or motion, the other based on analogy with the impact of projectiles. This chapter is devoted to the first kind.[1]

The mechanical character of the new optical theories of the seventeenth century made light more similar to sound, which had been regarded as a mechanical process since Greek antiquity (with the exception of the scholastic doctrine of immaterial sound *species*). This change partly explains why fruitful analogies between light and sound only began to occur in the seventeenth century. There are other reasons, to be found in the evolving relations between music, physics, mathematics, and optics. Some preliminary remarks on this evolution will be useful.[2]

Although the ancient Greeks perceived analogies between different senses, they did not use these analogies to model one sense after another. Nor did they see any relevance of music to optics, save for shared Pythagorean harmonies. Until the Renaissance, music was closely related to arithmetic and had a lot less connection with physics than optics had. There was not much of a physics of sound that could be transferred to optics. This state of affairs changed at the turn of the sixteenth and seventeenth centuries: the rise of mechanical and experimental philosophy as well as the need to justify newer musical practice now favored acoustic inquiries. The multiplication of new temperaments called for a physical

[1]Cf. E. J. Dijksterhuis 1961; Hall 1962; Westfall 1971; Cohen 1994; Dear 2001.

[2]Cf. Darrigol 2010a; and Mancosu 2006, who offers a valuable summary of the history of acoustics and optics in the early modern period.

justification of consonance, which was found in the frequent coincidence of the successive pulses of the two superposed tones. This interpretation required the correspondence between pitch and frequency, which thus became commonly accepted.[3]

In any other respect the physical nature of sound remained controversial, until the 1670s at least. Opinions varied on whether a medium was needed, on whether this medium was air or some other substance, on whether sound occurred in pulses or in smooth oscillations. The most common concept of sound was the ancient Greek concept according to which the strokes of the sounding body displace air all the way to the ear, as a missile or as a breath. In a popular variant of this concept, which the Minim friar Marin Mersenne adapted from Aristotle and defended in his authoritative *Harmonie universelle* of 1636, the air between the source and the ear moves globally like a pestle. To this view the Cartesians added that some time was needed to compact the particles of air before the strokes were transmitted.[4]

In the name of Divine harmony, Mersenne imagined a deep analogy between light and sound, but rather applied it from light to sound, for instance in subjecting echoes to the optical law of reflection. The vagueness of Mersenne's concept of sound, and the competition of other concepts did not encourage its use as a template for optical theories. His contemporaries either ignored the analogy; or they used it discreetly, even when it was responsible for an essential feature of their optics. This state of affairs lasted until Robert Boyle's air-pump experiments of the 1660s provided the basis for a more precise physics of sound.[5]

Boyle's experiment with a ticking watch in an evacuated vessel showed the plausibility of air as a medium for sound. His experiments on the spring of air opened the possibility of explaining the propagation of sound by the compression of successive layers of air, although Boyle himself shied from explanatory theories of this kind. As we will see, the pioneers of this approach, Ignace Gaston Pardies, Christiaan Huygens, and Isaac Newton were also the most ardent advocates of the acoustic analogy in constructing optical theories. Newton's concomitant derivation of the velocity of sound as a function of the density and elasticity of the air played an important role in promoting the compression-wave theory of sound.[6]

Since Greek antiquity, the propagation of sound had been compared to the waves formed on calm water when a stone is thrown into it. This analogy failed to compensate the insufficiencies of acoustic analogies, because until the beginning of the nineteenth century very little was known on the propagation of water waves. Before Pardies and Newton, these waves were usually understood as the horizontal projection of an excess of water, not as the pendulous oscillation that we now imagine to be transferred from one wave to the next (for periodic waves of small amplitude). The wave metaphor therefore supported the popular concept of sound as a breath. Moreover, it wrongly

[3]Cf. Wardhaugh 2006, 2008.

[4]Mersenne 1636–37, vol. 1, p. 10.

[5]Mersenne 1627, pp. 77, 309–310; 1636–37, vol. 1, props. 25, 47, 48.

[6]Cf. Darrigol 2010a and further reference there.

suggested—even to Newton—that the length of sound waves produced by impact on an elastic solid depended on the size of the impacting body.[7]

To summarize, the seventeenth-century rise of mechanical philosophy made it possible to conceive light as a perturbation transmitted through a mechanical medium. Acoustic or water-wave analogies were not necessarily a good guide in this approach, because sound and water-wave propagation long remained ill-understood. The first proponent of a mechanical medium theory of light, René Descartes, needed neither sound nor waves. The first section of this chapter is devoted to his optics, the second section to various theories by Hobbes, Hooke, Grimaldi and others that made a discreet use of analogy with sound in the breath or pestle view, the third to Pardies's and Huygens's theories assertively based on analogy with sound as a compression wave. The fourth section, on optical imaging, goes a little beyond the scope of this chapter, as it deals not only with issues raised by the medium theorists but also with their development in the hands of other seventeenth and eighteenth century opticians.

2.1 Descartes's optics

The nature of light

Of all sciences, geometry was the one that best resisted the systematic doubt on which the young Descartes famously based his method. This explains why his new natural philosophy rested on the doctrine that matter was nothing but spatial extension. He regarded space as completely filled with perfectly rigid particles of various sizes and shapes. Those of the "third element," or ordinary matter, are the grossest and have an arbitrary shape. Those of the "second element," or "subtle matter," are round and they fill as much as they can of the space between the former particles. Those of the "first element" are arbitrarily small and they fill the remaining interstices; they are the scrapings (*raclure*) generated during the production of the balls of the second element by mutual attrition of rotating particles; in this process they acquired an intense agitation. The sun and stars are spherical accumulations of the first element. They are immersed in the subtle matter of the second element. Light is nothing but the pressure (*inclination au mouvement* or *conatus*) that the sun and stars exert on the balls of the second element. This pressure is instantaneously and rectilinearly transmitted to the eye, owing to the contiguity of the balls and to their perfect rigidity.[8]

Descartes illustrated this process by a suitably modification of the old Stoic analogy with the walking stick:

> It has sometimes doubtless happened to you, while walking in the night without a torch through places which are a little difficult, that it became necessary to use a stick in order to guide yourself; and you may then have been able to notice that you felt,

[7]Cf. Darrigol 2010a, pp. 22–3.

[8]Descartes [1633], chaps. 5, 8; Descartes 1644, part 3, §§46–52. Light does not imply any actual motion of the subtle matter because the first element that makes the luminaries is neither compressible nor expandable. Descartes imagined a vortex motion of the subtle matter around each celestial body in order to explain gravitation. On Descartes's optics, cf. Sabra 1967, chaps. 1–4; Shapiro 1973; Mark Smith 1987. On Descartes's life and natural philosophy, cf. Garber 1992; Gaukroger 1995.

> through the medium of this stick, the diverse objects placed around you, and that you were even able to tell whether they were trees, or stone, or sand, or water, or grass, or mud, or any other such thing. True, this sort of sensation is rather confused and obscure in those who do not have much practice with it; but consider it in those who, being born blind, have made use of it all their lives, and you will find it so perfect and so exact that one might almost say that they see with their hands, or that their stick is the organ of some sixth sense given to them in the place of sight. And in order to draw a comparison from this, I would have you consider light as nothing else, in bodies that we call luminous, than a certain movement or action, very rapid and very lively, which passes toward our eyes through the medium of the air or other transparent bodies, in the same manner that the movement or resistance of the bodies that this blind man encounters is transmitted to his hand through the medium of his stick.

As a better analogy for transparent bodies, Descartes offered a pierced vat of half-pressed grapes. The wine plays the role of the subtle matter, the grapes that of gross matter. According to Descartes, a portion of the wine located near the free surface tends to move in the direction of the holes on the bottom of the vat (see Fig. 2.1). These endeavors are rectilinear, although the actual motion may not be. In the model of contiguous balls, Descartes argued that rectilinear propagation corresponded to the least number of displaced balls to transfer a lacuna from the source to the eye. He also explained that the necessarily imperfect alignment of the balls did not interfere with the rectilinear transmission of pressure, by analogy with the pressure transmitted by a curved stick. And he justified the independence of crossing rays by analogy with air blown in intersecting straight pipes.[9]

The multiplicity of Descartes's analogies suggests his awareness of weaknesses in his deduction of rectilinear propagation. Yet he did not doubt the central tenet of his model of light: light is a pressure instantaneously propagated through contiguous chains of rigid balls. There can be no delay in the transmission of the pressure because the matter of the

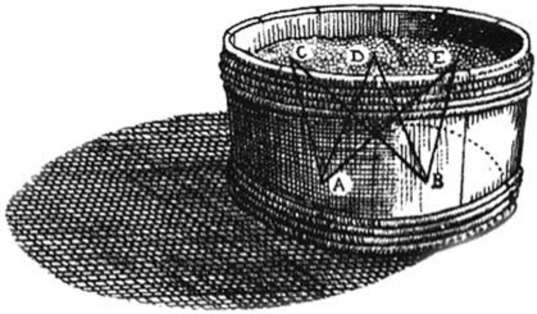


Fig. 2.1. Descartes's vat of half-pressed grapes. The wine tends to flow from the points C, D, E, of the surface to the hole A and B on the bottom. From Descartes 1637, p. 86.

[9]Descartes 1637, pp. 83–4 (citation), 86–7; [1633], pp. 47–53 (rectilinear propagation).

balls, being pure extension, is necessary incompressible. As Descartes wrote to his Dutch mentor Isaac Beeckman: "The instantaneous propagation of light is to me so certain that if its falsity could be shown, I would be ready to admit my complete ignorance of Philosophy." This concept of light can be regarded as intermediate between the Aristotelian concept and later wave-theoretical concepts. Of the former concept, Descartes retained instantaneous propagation through a medium; of the latter he anticipated the notion of a mechanical medium but excluded the compressibility of this medium.[10]

The law of refraction

Descartes first expounded the doctrine of the three elements and the resulting concept of light in the *Traité de la lumiere*, which he completed in 1733 but renounced publishing after he heard of Galileo's troubles with the Church. In 1737, Descartes published his *Dioptrique* as an illustration of the directives expressed in the *Discours de la méthode*. Although the walking stick and wine-vat analogies already appear in this context, Descartes kept his full theory of matter for his *Principia philosophiae*, published in 1644 as an ambitious replacement of Aristotelian philosophy.[11]

The main purpose of the *Dioptrique* was the improvement of optical instruments. To this end, Descartes derived the sine law of refraction by analogy with the inflection of the motion of a tennis ball upon entering water. Like Alhazen, he decomposed the motion in to a component parallel to the surface of the water and a component normal to this surface. Unlike Alhazen, he regarded the latter component as the one that is modified when the ball penetrates the water. Moreover, he assumed that the velocity (modulus) of the ball was changed by a constant fraction when the ball entered the water. The sine law immediately results from these assumptions (see Fig. 2.2). As early as 984, the Arabic geometer Ibn Sahl had implicitly used this law as a rule to derive the properties of burning lenses; the English astronomer Thomas Harriot had established it secretly in 1601; and Beeckman's teacher Willebrørd Snel van Royen had privately circulated it in 1621. It is not clear whether Descartes knew of the latter discovery or whether he performed relevant experiments. There is no doubt, however, that he gave the first theoretical derivation as well as the first application to a theory of optical instruments and to a theory of the rainbow.[12]

Descartes's derivation of the law of refraction has a few apparent weaknesses. First, it relies on the actual motion of a ball, whereas Descartes defines light as a mere tendency to motion. Descartes deflected this objection by making clear that he only meant a partial analogy between the two cases: "For it is very easy to believe that the action or inclination to move which I have said must be taken for light, must follow in this the same laws as does motion." Accordingly, the velocity of the ball should not be identified with the velocity of light in each medium. This velocity is always infinite for Descartes. A second difficulty in Descartes's reasoning is the assumption that the velocity of the ball changes by a constant fraction when passing from one medium to the other. There is nothing, in Descartes's

[10]Descartes to Beeckman, 22 August 1634, in Tannery 1908, pp. 307–12.

[11]Descartes [1633], 1637, 1644.

[12]Descartes 1637, pp. 93–105. On earlier occurrences of the sine law, cf. Rashed 1990; Lohne 1959; Shirley 1983; Sabra 1967, pp. 100–2. On Descartes's derivation, cf. Mark Smith 1987; Heeffer 2006.

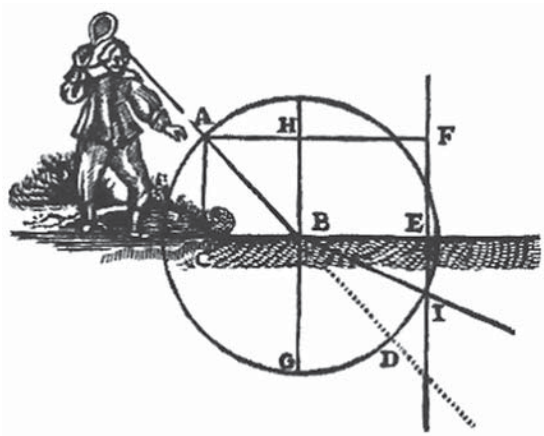


Fig. 2.2. Descartes's derivation of the law of refraction. AB represents the displacement of the tennis ball in the air in a unit time, AH the horizontal component of this displacement, and HF the horizontal displacement of the ball in the water in the time for which its total displacement (BI) equals AB. This time being the same for any angle of incidence, the conservation of the horizontal velocity implies the proportionality of AH and HF. Hence the sines AH/AB and BE/BI are also proportional. From Descartes 1637, p. 97.

primitive mechanics, to justify this assumption. Lastly, Descartes's reasoning requires the strange assumption that the velocity of the ball is higher in the denser medium (in order that the refracted ray should be closer to the perpendicular). Descartes gave the following excuse: the grosser particles of the third element being freer to move in air than in glass (or water), the particles of the second element (tend to) move faster in glass than in air just as a ball rolls faster on a hard floor than on a soft carpet.[13]

Natural and aided vision

Descartes went on with a clear and persuasive exposition of Kepler's theory of vision, without caring to name Kepler. Through a striking illustration (Fig. 2.3), he explained how the formation of a painting of the object at the bottom of the eye could be proved by removing the skins that cover the bottom of the eye of some big animal and observing this bottom. To Kepler's binocular mechanism for the estimation of depth, he added our awareness of the deformation that the eye underwent in order to produce a focused painting of the object on the retina. Most originally, he traced the brain's reception of images to the movement of fluid or "spirit" circulating in the hollow fibers of the optical nerve. In terms of the famous Cartesian divide, nervous influx belongs to the *res extensa* and does not relate to the *res cogitans* before entering the brain. Descartes drew the following corollary on visual perception:[14]

[13]Descartes 1637, pp. 89 (citation), 103.

[14]Ibid., pp. 112 (citation), 137 (depth). In his influential *Oculus* (1619), the Jesuit astronomer Christoph Scheiner had provided much anatomic evidence in favor of Kepler's theory of vision; in his *Rosa ursina* (1630, p. 110), he had reported observations of inverted images on the bottom of animal and human eyes. Cf. Daxecker 2004.

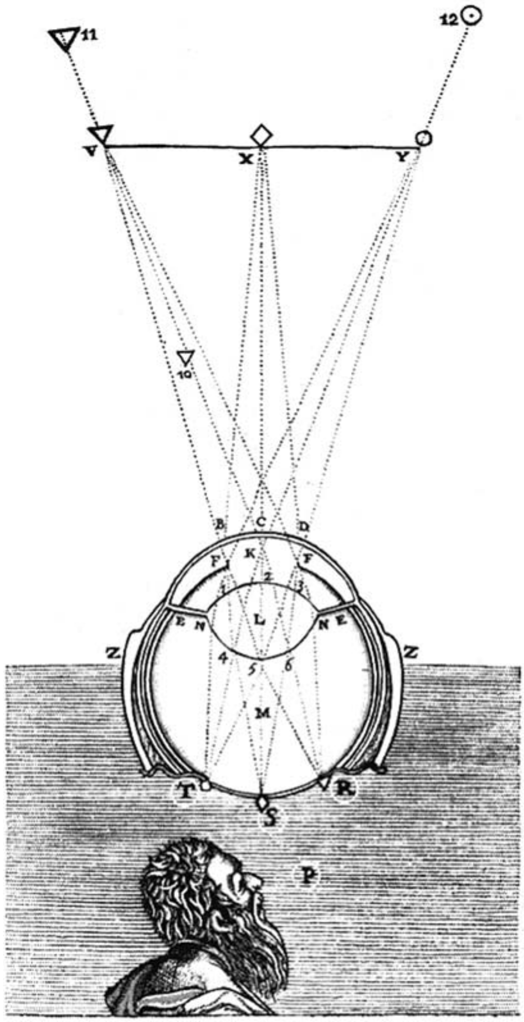


Fig. 2.3. Descartes's illustration of Kepler's theory of vision. The gentleman at P sees the images R, S, T of the luminous points V, X, Y at the bottom of a big eye, whose inferior skins have been removed. From Descartes 1637, p. 122.

We must beware of assuming that in order to sense, the mind needs to perceive certain images transmitted by the objects to the brain, as our philosophers commonly suppose; or, at least, the nature of these images must be conceived quite otherwise than as they do … They have had no other reason for positing these images except that, observing that a picture can easily stimulate our minds to conceive the object painted there, it seemed to them that in the same way, the mind should be stimulated by little pictures which form in our head to conceive of those objects that touch our senses; instead we should consider that there are many things besides pictures which

> can stimulate our thought, such as, for example, signs and words, which do not in any way resemble the things which they signify … It is only a question of knowing how the images can enable the mind to perceive all the different qualities of the objects to which they refer; not how they hold their resemblance.

Descartes, unlike Kepler, had the notion of virtual images formed by the common intersection of the prolongation of the rays deflected by an optical device and originating in the same point of the object. This can be judged from the figures he drew for the effects of prisms, lenses, and mirrors on vision (Fig. 2.4). He did not, however, exploit

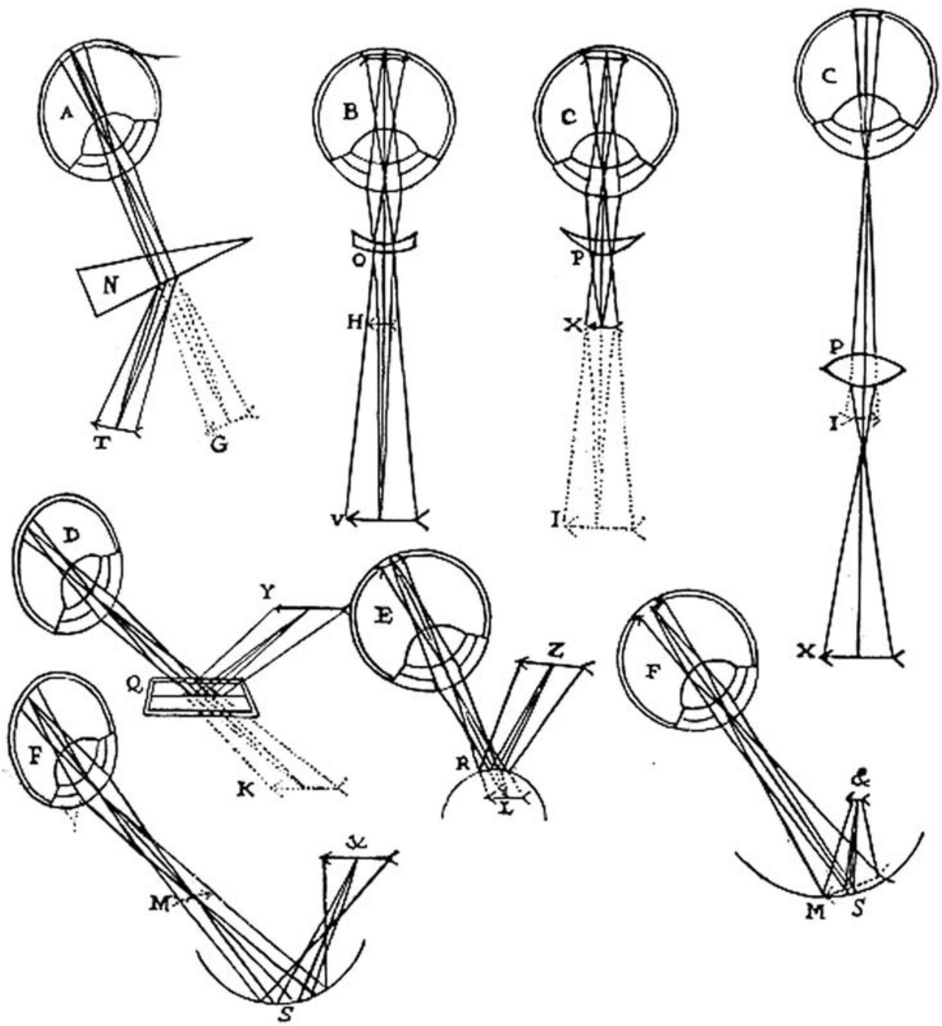


Fig. 2.4. The alteration of vision by a prism, lenses, and mirrors of various shapes according to Descartes 1637, p. 143. The dotted arrows represent what we would now call the virtual image of the object.

this notion in his analysis of the magnifying power of telescopes. Rather, he traced (angular) magnification to “making the rays that come from the various points of the object cross each other as far as possible from the bottom of the eye.” In this view, the role of the telescope was to transfer the zone of crossing rays from the pupil of the eye to the object lens of the telescope. Descartes’s main innovation in the theory of optical instruments was the determination of the refracting surfaces that cause the refracted rays from a parallel beam to converge to a “burning point.” He found these surfaces to be those engendered by the rotation of ellipses or hyperboles. More generally, he showed that the surfaces engendered by what we now call “Descartes’s ovals” bring the rays from a given point to converge to another point. His new geometry was here essential. In the last chapter of his treatise, he imagined machines that could grind glasses to the desired hyperbolic shape.[15]

The rainbow and colors

As another illustration of his philosophical method, Descartes included a treatise on meteorology (*Les météores*) in his volume of 1837. There he gave his theory of the rainbow, based on reflections and refractions occurring at the surface of the quasi-spherical droplets of a cloud. The idea that the rainbow results from the reflection of the light of the sun by clouds is an ancient notion, which Aristotle related to the fact that the parts of a given rainbow are seen under a constant angle with the direction of the sun. Aristotle easily but vaguely accounted for the colors by evoking the dimming effect of the clouds, in conformity with his general notion of color as a mixture of brightness and darkness. Around 1300, the Teutonic theologian Theodoric von Freiberg and the Persian astronomer Kamal al-Din al-Farisi experimented with spherical glass balls and understood that the primary and secondary rainbows resulted from two refractions and one or two internal reflections at the surface of water droplets. Their insights were lost. Closer to Descartes’s times, Giovanni Battista della Porta and Marco Antonio de Dominis emphasized the role of refraction in generating the colors of the rainbow in analogy with prismatic colors, and the latter rediscovered Theodoric’s explanation of the primary rainbow.[16]

Descartes did not mention any of these earlier investigations of the rainbow. From his experiments with a spherical phial, he measured the limiting angles of the two rainbows to be 42° and 52°. As he verified by blocking the path of light in his phial, the former case corresponds to one reflection within the droplets, the second to two reflections (see Fig. 2.5). Applying his law of refraction to parallel rays entering the droplet at various distances from its rim and experiencing one or two reflections within it, he found that the angle of emergence had a maximum of 41° 30′ in the first case and a minimum of 51° 54′ in the second case. He observed that the density of emerging rays was much higher near these extrema, and thus explained the observed angular radii of the two rainbows.[17]

[15]Descartes 1637, pp. 142–3, 155 (citation). Ibn Sahl anticipated the refracting hyperbole: cf. Rashed 1990. On Descartes’s lens making machines, cf. Burnett 2007.

[16]Descartes 1637, eighth discourse of *Les météores*. On rainbow history, cf. Boyer 1959, 1987; Blay 1983, chaps. 1–3; Blay 1995. On al-Farisi’s contribution, cf. Wiedemann 1910.

[17]Descartes 1637, pp. 325–9, 336–41. Descartes says that he computed the refracted rays in the course of his investigation of the analogy between rainbow and prismatic colors. Cf. Buchwald 2008.

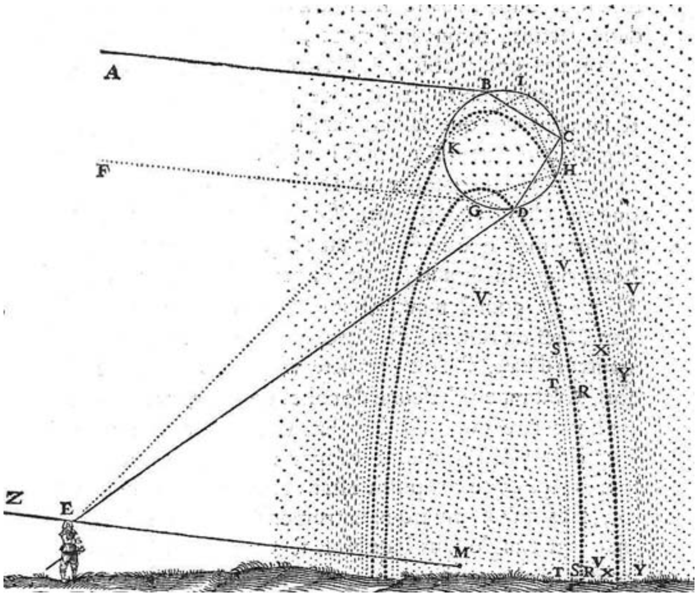


Fig. 2.5. Descartes's explanation of the rainbow. The circle represents an exaggerated droplet. The twice refracted and once reflected sunray ABCDE contributes to the primary rainbow (smaller dotted semi-circle). The twice refracted and twice reflected ray FGHIKE contributes to the secondary rainbow (smaller dotted semi-circle). From Descartes 1637.

In order to understand the rainbow colors, Descartes experimented with a prism mounted in the manner of Fig. 2.6. The light from the sun is refracted when exiting the prism through the hole DE. When this hole is small enough the rainbow colors are projected over the white surface PF. For a larger hole, the luminous spot is mostly white and colors only appear on the upper and lower sides of it. Descartes concluded that the production of colors depended both on refraction and on the sideways limitation of the refracted light, and that the order of colors depended on which side (of the perpendicular to the refracting surface) the refraction was done. These rules enabled him to explain the colors of the two rainbows, since the limiting angles of emergence provided the desired limitation of the refracted light.[18]

Descartes reached or consolidated his rules of refractive coloring by appealing to his globular model of light propagation. Besides the translational tendency of the balls of the second element, he imagined a rotational tendency whenever the balls interacted with the grosser particles of the third element (ordinary matter). He had in mind the rolling of a tennis ball over a hard floor, in which case "the rotation is about equal to the rectilinear translation." He further imagined that refraction made the balls spin in a

[18]Descartes 1637, pp. 329–31. The divergence of the beam DFEH in Fig. 2.6 is only a consequence of the exaggerated diameter of the sun.

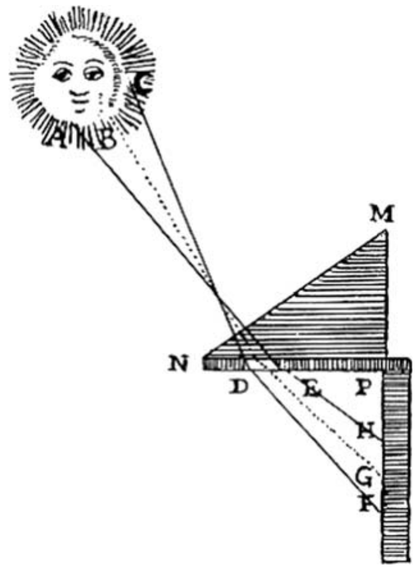


Fig. 2.6. Descartes's experiment with the prism. The rays from the sun enter the prism perpendicularly, and experience refraction before exiting through the hole DE on the supporting board. The resulting beam DEHF projects the rainbow colors on the part HF of the vertical screen.

sense depending on the sign of the angle of incidence, by analogy with a tennis ball entering water obliquely (see Fig. 2.7). After refraction, a ball at the limit of the refracted beam DEFH (Fig. 2.6) touches balls moving faster than itself on the inward side, and slower than itself on the outward side. Consequently, balls situated near the lower side of the beam rotate faster than those situated near the higher side. Descartes identified color with this differential spin:

> All of this shows that the nature of the colors appearing near F consists just in the parts of the subtle matter that transmit the action of light having a much greater tendency to rotate than to travel in a straight line; so that those that have a much stronger tendency to rotate cause the color red, and those that have only a slightly stronger tendency cause yellow. The nature of the colors that are seen near H consists just in the fact that these small parts do not rotate as quickly as they would if there were no hindering cause; so that green appears where they rotate just a little more slowly, and blue where they rotate very much more slowly.

In his *Dioptrique*, Descartes similarly explained the colors of bodies through the spin communicated to the balls of subtle matter during their impact with surface irregularities.[19]

Descartes thus completed his mechanization of optics. Light and colors no longer were the irreducible, multiplied species of scholastic philosophers. They were tendencies to motion for the ultimate particles of a subtle matter, transmitted from one particle to the

[19]Ibid., pp. 333–4.

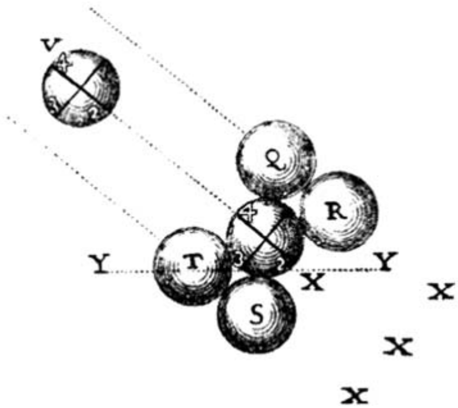


Fig. 2.7. Descartes's spinning balls. The impact of the ball V on the water surface YY prompts it to rotate clockwise (1234) because it enters this surface sideways (through its part 3). It may be touched by two faster balls Q and R on one side, and two slower ones T and S on the other side. This contact increases the rotation.

next. In the elaboration of this simple scheme, Descartes relied on vague analogies with macroscopic processes. He could not do better in the absence of any adequate theory of mechanics. As we will see, his contemporaries easily spotted weaknesses in his reasoning, and not always because they misinterpreted his intentions.

Fermat's principle

Not even the sine law of refraction eluded criticism, even though its empirical content could easily be divorced from its mechanical foundation. For instance, in a letter to Mersenne of December 1637 the French mathematician Pierre de Fermat denied every step of Descartes's reasoning, from the distinction between endeavors and motions to the conservation of the parallel component of motion. Twenty years later, Fermat received from the physician Cureau de la Chambre an optical treatise in which the principle that "Nature always acts by the shortest courses" played an important role. La Chambre recalled that Hero of Alexandria, in his *Catoptrics*, had shown that the law of reflection on a plane mirror followed from the condition that the total length of the ray between the eye and the sighted point should be a minimum (Fig. 2.8). La Chambre deplored that the same principle, when applied to light traveling from one medium to another, led to a straight line instead of the observed broken line.[20]

To which Fermat replied that the difficulty could be solved by assuming that different media offered different "resistances" to the passage of light. In mathematical terms, this means that the sum $\alpha\mathrm{AM} + \beta\mathrm{BM}$ has to be a minimum, A and B being two fixed points,

[20]Fermat to Mersenne, December 1637, in Fermat 1891–1912, vol. 2, pp. 117–20; La Chambre 1657, p. 354; Hero, 1st century A.D., in Hero 1900, pp. 325–7 or in Cohen and Drabkin 1948, pp. 263–4. Cf. Montucla 1758, vol. 2, pp. 188–91; Sabra 1967, chaps. 3–5.

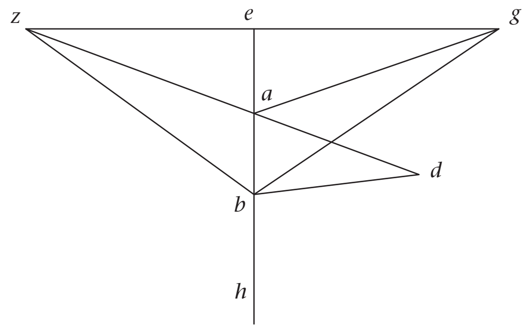


Fig. 2.8. Hero's theorem of least path. From Hero 1900 [1st century A.D.], p. 326. The ray from the eye g is reflected at point a of the mirror eh and reaches the observed point d. The produced ray ad intersects the perpendicular eg to the mirror at the mirror image z of point g. The alternative ray path gbd is longer than the true path gad because $gb + bd = zb + bd > za + ad = ga + ad$.

one on each side of a refracting plane, M a variable point on this plane, and α and β measures of the resistance of the two media. It took Fermat five more years to solve this problem by his method of minima and maxima, which is formally equivalent to the modern requirement of vanishing differential. To his surprise, the result exactly complied with Descartes's law. Fermat then meant the resistance to be inversely proportional to the velocity of light, in which case the minimized quantity is the time that light takes to travel between two points. He did not fail to notice that his reasoning made light travel faster in the rarer medium, against Descartes's contrary and unnatural supposition.[21]

By the time Fermat's derivation became known, the law of refraction had been repeatedly corroborated by experiments. Its consequences for the rainbow and for optical instruments were widely accepted. Descartes was no longer alone in basing optics on a mechanical medium. Although his heirs or competitors did not necessarily follow him in assuming the infinite speed of light or the relation between color and rotation, the Cartesian dream of reducing optics and other domains of physics to a geometry of matter had begun to influence natural philosophy.

2.2 From Hobbes to Hooke

Hobbes's lines of light

A contemporary of Descartes, the English philosopher Thomas Hobbes, pursued the more radical aim of reducing physics, mind, and God himself to pure motion. In 1644, he published an optical treatise as part of his friend Marin Mersenne's *Synopsis* of mixed mathematics. Hobbes defined light as the motion of a medium induced by global dilations

[21]Descartes to La Chambre, August 1657, in Fermat 1891–1912, vol. 2, pp. 354–9; 1 January 1662, ibid., pp. 460–2; "Analysis ad refractiones" [1662], in Fermat 1891–1912, vol. 1, pp. 170–2. In "Synthesis ad refractiones" [1662], ibid., pp. 173–9, Fermat offered a synthetic proof that light takes less time to travel on the Snel-Descartes path than on any other broken path. Huygens 1690, pp. 39–41 has a simpler proof of the same kind. Cf. Sabra 1967, chap. 5.

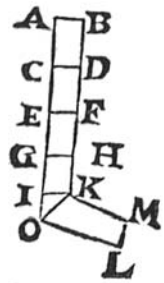


Fig. 2.9. Hobbes's drawing of a ray, with the successive lines of light AB, CD … ML (refraction occurs at KO). From Hobbes 1644, p. 570.

and contractions of the source, which he compared to a beating heart. He cited the scintillation of stars and the simultaneous emission of light in every direction as evidence for this conception. Implicitly assuming the incompressibility of the medium, he showed that successive concentric shells of the medium had a velocity decreasing with their radius, which explained their decreasing effect on the eye.[22]

Most originally, Hobbes defined a ray as "the path through which the motion is propagated." He represented this path by an evenly sliced column (or pyramid) such that the medium in a given slice occupied the next slice after an expansion of the source (see Fig. 2.9). He called the limits of (a plane section of) these slices "propagated lines of light" and derived their perpendicularity to the sides of the rays from the uniformity of the motion of the medium.[23]

Hobbes exploited this perpendicularity in a new derivation of the sine law of refraction. When one end of a line of light reaches the frontier between two media of different densities, this end travels at a speed different from the speed of the other end as long as the latter remains in the first medium.[24] In order to preserve the length of the line of light (which Hobbes identified with its intensity) and its perpendicularity to the sides of the rays, Hobbes had it rotate during its crossing of the frontier (see Fig. 2.10). The geometry of the resulting figure implies that the sines of the angles of incidence and refraction should be proportional to the velocities in the two media. As Hobbes assumes the velocity of the denser medium to be smaller, this implies that the refracted ray should be closer to the normal in the denser medium. At the end of his treatise, Hobbes vaguely suggested that the rotation of the lines of light during refraction might explain the prismatic colors: if this rotation somehow persisted until the ray reached the eye, it could enhance the impact of one end of a line of light and diminish that of the other end, thus causing the sensations perceived as red and violet.[25]

[22]Hobbes 1644, pp. 567–8. Cf. Brandt 1928, pp. 105–10; Shapiro 1973, pp. 145–72; Bernhardt 1990; Médina 1997, pp. 33–48. Hobbes's theory resembles an emission theory (he earlier held one), except that it requires a continuous, space-filling medium and implies no net flux of this medium because of the alternating contraction and expansion of the source.

[23]Hobbes 1644, pp. 570–2. Alhazen and Witelo already had finite-width rays, though not in relation with wave propagation: cf. Shapiro 1973, p. 150n. Hobbes truly meant his rays to be conical, but thin enough and far enough from the source to be approximated by cylinders: cf. Shapiro 1973, p. 161.

[24]Hobbes may have had in mind the conservation of flux, which requires the product of velocity and density to be the same in the two media.

[25]Hobbes 1644, pp. 572–89. On Hobbes and colors, cf. Blay 1990, p. 153–68.

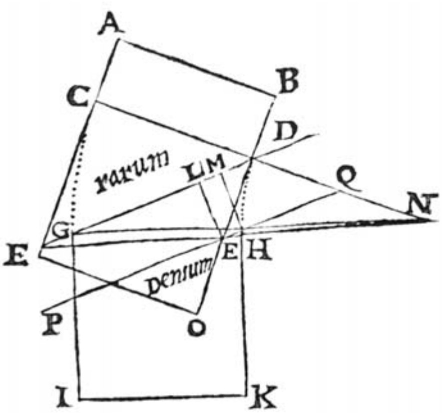


Fig. 2.10. Refraction according to Hobbes 1644, p. 572. The line GD separates the rare medium from the dense medium. After the incoming line of light AB reaches the position CD, the extremity C travels on the arc CG at the end of which it enters the dense medium and assumes the rectilinear path GI. The extremity D of the line of light CD travels on the arc DH while C travels on CG. As CG belongs to the rare medium and DH to the dense medium, the ratio of their lengths (or of the radii NC and ND) must be equal to the ratio of the velocities of the two media. Consequently, the sine law of refraction holds.

As we will see in a moment, Hobbes's original derivation of the law of refraction inspired later wave-theoretical derivations, just as Descartes's derivation inspired later corpuscular derivations. It must be noted, however, that both Descartes and Hobbes believed the propagation of light to be instantaneous. Consequently, the velocity ratio, which they equated to the ratio or to the inverse ratio of the sines of incidence and refraction, did not refer to the propagation velocity. For Descartes, the implied velocity belonged to an analogy with a process involving instantaneously transmitted endeavors. For Hobbes, it referred to the instantaneously transmitted shift of an incompressible fluid. In both cases, this velocity had more to do with the intensity of light than with its speed.

Although the only analogy found in Hobbes's optical treatise is that of heart beats, he is likely to have been inspired by the analogy with sound propagation. The global dilations and contractions of Hobbes's shining body resemble the global vibrations of a sounding body. The flow of the optical medium resembles the concept of sound adopted by his friend Mersenne in the monumental *Harmonie universelle*. As the ancient Greeks already did, Mersenne assumed sound to consist of air being pushed like a pestle all the way from the source to the ear. Hobbes's discussion of sound in his later *De corpore* (1755) confirms his drawing on Mersenne: there he adopted the pestle concept with circular waves, explained hearing trumpets by the confinement of the flow, and echoes by the reflection of sound rays. Hobbes still did not mention the analogy between light and sound. One reason for this silence may have been that no theory of sound then had sufficient authority to serve as a template for other theories.[26]

[26]Hobbes 1655, chap. 29.

Another reason is that Hobbes, by that time, had renounced his original theory of light, probably because he now judged the contractions and expansions of the source to imply a vacuum incompatible with his philosophy of motion. He replaced his ingenious theory of refraction with an erroneous modification of Descartes's and joined the French philosopher in asserting that light only implied an endeavor to motion:

> As vision, so hearing is generated by the motion of the medium, but not in the same manner. For sight is from pressure [*pressio*], that is, from an endeavour [*conatus*]; in which there is no perceptible progression of any of the parts of the medium; but one part urging or thrusting on another propagateth that action successively to any distance whatsoever; whereas the motion of the medium, by which sound is made, is a stroke [*percussio*]. For when we hear, the drum of the ear, which is the first organ of hearing, is stricken.

Not much was left of the original analogy with sound, except for the general idea of a mechanical medium.[27]

Hobbes's publication of his theory of light in a friend's compilation, his later abandonment of it, Descartes's misunderstanding and rejection of it, the scandal of Hobbes's *Leviathan* of 1651, John Wallis's questioning of his geometrical competence in 1655, and his difficult character all contributed to this theory almost never being cited by later writers on optics. Yet a few of them arguably benefited from it. In his *Perspectiva horaria* of 1648, Mersenne's close friend Emmanuel Maignan reinterpreted Hobbes's derivation of the law of refraction in an emissionist framework in which the lines of light became aggregates of light corpuscles whose speed depended on the transparent medium (see Fig. 2.11). In his optical lectures of 1669, the Lucasian Professor Isaac Barrow used a similar figure and reasoning in a mixed pseudo-Cartesian view of light in which the rays implied a flux of the subtlest element (akin to Descartes's first element) as well as accompanying impulses of the particles of a grosser element (akin to Descartes's second element) (see Fig. 2.12). The success of these transpositions is easily explained by noting that Hobbes's derivation requires only two ingredients: the perpendicularity of the rays to the lines of light, and different velocities for the parts of a line of light when this line crosses the interface between two different media. These two assumptions can be justified both in an emission theory (with resisting medium) and in a continuous-medium theory not necessarily of Hobbes's kind. The meaning of the relevant velocities of course depends on the selected framework. In Maignan's and Barrow's theories, these velocities are identical to the propagation velocities. They are not in Hobbes's theory.[28]

[27]Hobbes 1655, p. 280 (Latin), p. 486 (English). On pp. 264–5 (Latin), Hobbes gave an obscure interpretation of prismatic colors as an inclination of the lines of the propagated endeavor, inclination caused by the transverse reaction of the refracting medium; this might have inspired Hooke's later theory of colors.

[28]Maignan 1648, pp. 576–8; Barrow 1669. Cf. Brandt 1928, pp. 211–16; Shapiro 1973, pp. 172–9 (Maignan), 179–81 (Barrow). As Shapiro reports, Maignan and Barrow also had derivations of the law of reflection. As can be judged from their correspondence (via Mersenne), Descartes and Hobbes did not understand each other's concepts of ray nor the relations they assumed between velocity and medium: one theory was by contact of hard balls, the other by incompressible fluid; there was no well-established mechanics at the time, so that frictional and inertial effects were not clearly separated; there were no uniform concepts of rigidity and continuity. Cf. Brandt 1928, pp. 111–19; Shapiro 1973, pp. 155–9.

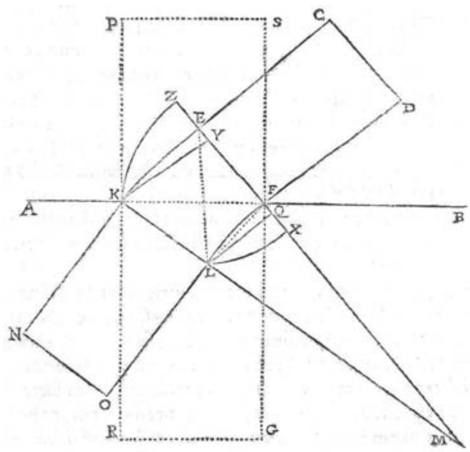


Fig. 2.11. Refraction according to Maignan 1648, p. 632.

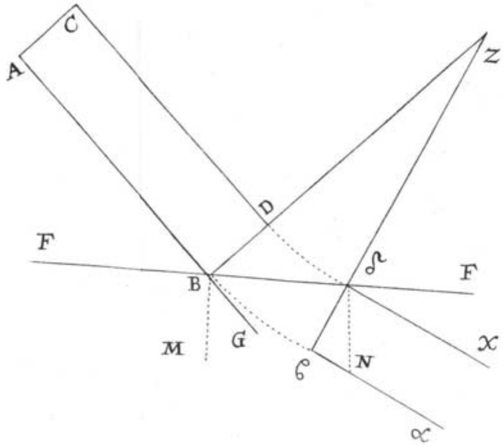


Fig. 2.12. Refraction according to Barrow 1669, plate.

Hooke's pulses

In his celebrated *Micrographia* of 1665, the Royal Society's demonstrator and chief mechanical philosopher Robert Hooke appealed to Hobbes's notion of physical ray (without naming Hobbes) and modified his theory of refraction in a strange manner. His motivation was to explain colors and their production by thin transparent plates, as studied by his mentor Boyle and produced by foliated bodies under Hooke's microscope. Hooke defined light as a periodic succession of pulses caused by very quick and tiny vibrations of the luminous source, and traveling at a very high but finite and well-defined speed in a given homogenous medium. To support the vibrational character of light he appealed to the following phenomenon and analogy:

> A *Diamond* being *rub'd, struck*, or *heated* in the dark, shines for a pretty while after, so long as that motion … remains (in the same manner as a Glass, rubb'd, struck, or … heated, yields a sound which lasts as long as the *vibrating* motion of that *sonorous* body).

Hooke believed sound to be the succession of pulses produced by periodic beating of the air, as he later demonstrated with a rotating toothed wheel. His concept of light was a faster version of the same process.[29]

Hooke explained spherical propagation by analogy with a stone thrown into water, and represented "physical rays" by a periodic succession of parallel segments delimited by two parallel "mathematical rays" as Hobbes had done with a different interpretation. The

[29]Hooke 1665, p. 54.

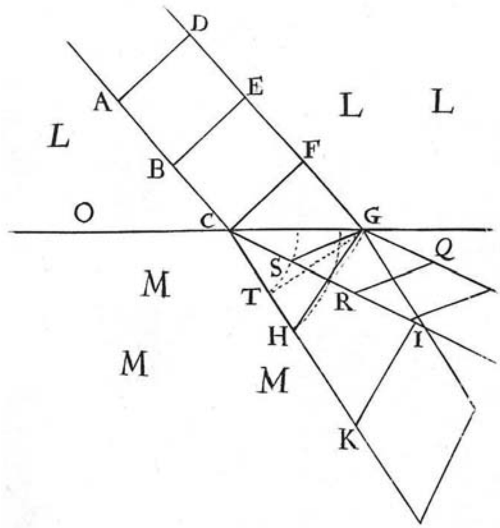


Fig. 2.13. A refracted ray according to Hooke 1665, plate. The pulses GH and KI (after refraction into a denser medium) or SG and RQ (after refraction into a rarer medium), are no longer perpendicular to the rays because the distance CH traveled in the denser medium is larger than the distance FG traveled in the rarer medium.

segments now represented the position of the successive pulses created by the source, and their velocity corresponded to the propagation velocity of light.[30] Hooke further departed from Hobbes by making the velocity of these segments increase with the density of the medium, as assumed by "the most acute and excellent philosopher Des Cartes." By reasoning otherwise similar to Hobbes, this implies that after crossing the border between two media of different densities, the pulses are inclined with respect to the rays (see Fig. 2.13). Hooke next argued that the propagation of an inclined pulse in the resisting medium implied a weaker intensity near its ends. Consequently, the front section of the pulse causes a gradually increasing sensation when entering the retina whereas the rear section causes a gradually decreasing sensation. Hooke interpreted the first kind of sensation as blue, and the second as red. He thus explained the extreme colors of a refracted beam and pretended to obtain the intermediate ones by combination.[31]

Hooke thus lost the Hobbesian explanation of refraction. In compensation, he succeeded in explaining the colors of the light reflected by thin glass plates. For this purpose, he assumed that the pulses of the incoming light were partially reflected by the first surface of the plate, and partially transmitted to the second surface and reflected there. The combination of the two kinds of reflected rays forms a periodic succession of pairs of pulses of unequal strength (see Fig. 2.14). The distance and order of these pairs depends

[30]Hooke is not quite explicit on this point. His analysis of the colors of thin plates implicitly requires it.

[31]Hooke 1665, pp. 55–64 (citation on p. 57). Cf. Shapiro 1973, pp. 189–201.

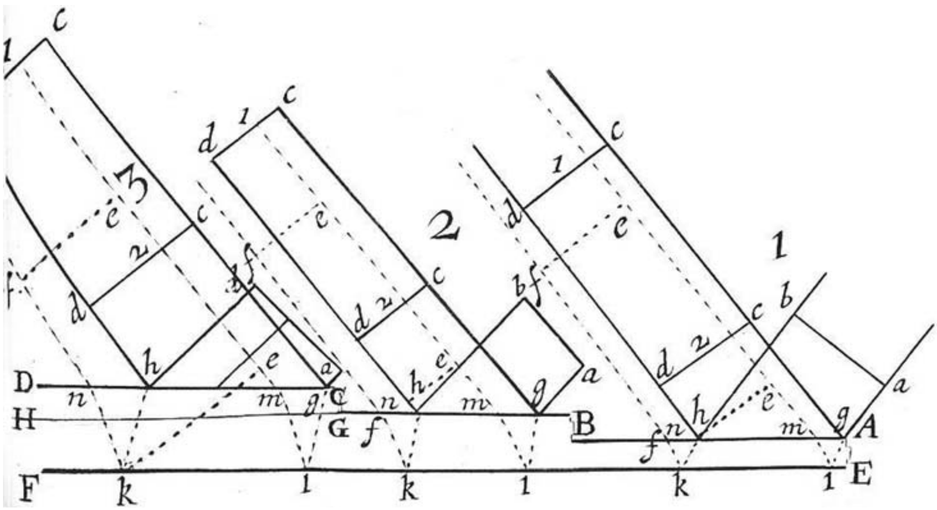


Fig. 2.14. Hooke's drawing for the reflection of three rays of light (1, 2, 3) by films of different thicknesses. From Hooke 1665, plate. Reflection on the first interface (AB, BG, CD) produces the pulses *dc*; reflection on the second interface (EF) produces the weaker pulses *ef*, with a delay depending on the thickness of the film.

periodically on the distance traveled by the weaker pulses within the glass of the plate, and the latter distance depends on the thickness of the plate and on the inclination of the rays. In conformity with his earlier definition of blue and red, Hooke interpreted a weaker pulse followed by a stronger one as blue, and the opposite pattern as red. He thus accounted for his and Boyle's main observations. He also explained the colors of bodies by the inclusion of transparent particles acting somehow like thin plates.[32]

The analogy between sound and light played an important role in Hooke's theory, since the implied periodicity and finite speed played an essential role in his explanation of color phenomena. At a Royal Society meeting in 1675, Hooke compared the harmony of colors to the harmony of tones:

> Light is a vibrating or tremulous motion in the medium (which is thence called pellucid) produced from a like motion in the luminous body, after the same manner as sound was then generally explained by a tremulous motion of the medium conveying sound, produced therein by a tremulous motion of the sounding body: and that, as there are produced in sound several harmonies by proportionate vibrations, so there are produced in light several curious and pleasant colours, by the proportionate and harmonious motions of vibrations intermingled; and as those of the one are sensated to the ear, so those of the other are by the eye.

[32]Hooke 1665, pp. 65–7 (thin plates), 68–9 (colors of bodies). Hooke's explanation of the colors of thin plates was closer to the modern explanation than Newton's, as it implied the superposition of the rays reflected by the first and second surfaces of the plate whereas Newton's implied only one ray with fits of easy reflection or transmission at the second surface of the plate.

This statement is the probable origin of Newton's subsequent claim that Hooke had adopted the correspondence between color and frequency. This claim is contradicted by Hooke's contemporary assertion that "colours begin to appear, when two pulses of light are blended so very well, and near together, that the sense takes them for one." This idea is the same as that expressed in his theory of thin plates, in which a colored impression results from the succession of two pulses of different intensity; and white light corresponds to a periodic succession of equal pulses. In Hooke's view, color and pitch are only analogous in that they both correspond to a time-ordering of pulses that the perceiving organ is unable to separate. A diary entry of January 1676 confirms this interpretation: "Compard sound and light and shewed how light produced colours in the same way by confounding pulses."[33]

In 1680–82, Hooke gave "lectures on light," posthumously published in 1705. There he defended a concept of light that implicitly contradicted his earlier views. He first formulated a few objections against atomist or emissionist theories, including the exhaustion of the sun and the impossible porosity of transparent bodies. He then pleaded for a mechanical reinterpretation of Aristotle's medium theory. Following Descartes and Hobbes, he identified the medium or "aether" with a perfectly fluid plenum, by which he meant an incompressible (and inviscid) fluid. He excluded compressibility because it would require lacunae incompatible with the Cartesian identification of matter and extension. He exploited incompressibility and the resulting conservation of flux to derive the quadratic law for the decrease of intensity (fluid velocity) with distance from the source, both for light and for gravitational forces. He regarded this result as a sufficient proof that his hypothesis represented the true nature of light: "So that thence it is evident, that Light does act according to the Proportion of the Body moved, observing the same Proportions, and therefore can be nothing else but that; for what thing soever hath all the same Properties with another, must be the same."[34]

Like Hobbes in his theory of 1644, Hooke imagined that a pulse of a point source implied an instantaneous shift of the ether at any distance from the source:

> Light then is nothing else but a peculiar motion of the parts of the luminous body, which does affect a fluid body that incompasses the luminous body, which is perfectly fluid, and perfectly dense, so as not to admit of any further condensation; but that the parts next the luminous body being moved, the whole expansum of that fluid is moved likewise.

Like many of his contemporaries, he denied that Rømer's observations of 1676 proved the finite speed of light. In the case of sound, he imagined a similar shift of the air, except that the "spongy, rarefied, or yielding" character of air as a medium implied that sound, unlike light, could only be heard "to a certain distance" (and with a certain delay).[35]

[33]Hooke, 11 and 18 March 1675, in Birch 1756–1757, vol. 3, pp. 193–5; 15 January 1676, in Hooke 1935, p. 211.

[34]Hooke, "Lectures on light," in Hooke 1705, pp. 71–148, on pp. 71–5 (emission rejected), 75–6 (Aristotle modified), 77–8 (discussing Rømer), 92–3 ($1/r^2$ law), 114 (quote), 136 ("aether"); "On comets and gravitation," in Hooke 1705, pp. 184–6 ($1/r^2$ law for gravitation).

[35]Hooke 1705, pp. 113, 116–17. Ibid. on p. 130, Hooke rejected the endeavors (*conatus*) imagined by Descartes and the later Hobbes.

Hooke was aware of an obvious difficulty of his concept of light: it seemed to imply that the ether shifts caused by the various luminous particles at the surface of an extended source should interfere with each other. As Mersenne had done in the case of sound, he justified non-interference by analogy with the distinctness of the waves created by the simultaneous impact of several drops on a calm surface of water. More fundamentally, he propounded that each sensible element of fluid had enough distinct particles to convey the pulses from different sources, or else that each sensible element of time had enough parts to permit the successive passage of the various pulses.[36]

These optical lectures contained very little on colors. Hooke only mentioned that the refraction of a white ray of light gave rise to a divergent beam of rays and vaguely related the "appearance of colour" to the refracted rays being "oblicated." Possibly, he realized that his earlier theory of the colors of refracted rays and thin plates was incompatible with the infinite velocity of light that he now assumed. Or else he did not wish to return to a topic on which he had nothing new to add and on which Newton had taken the lead.[37]

Hooke's inability to shake the Cartesian dogma of the incompressible ether was by no means unique. In the fourth volume of his *Essais de physique* (1688), the French physician and architect Claude Perrault included a theory of light that similarly conciliated the acoustic analogy with Descartes's incompressible ether. Like Hooke, Perrault argued that the emissionist theory of light would imply a quick exhaustion of the sun and exclude transparency. He compared the propagation of light to the communication of motion by a contiguous series of billiard balls as he had already done for sound:

> The essence of light consists … in the vivacity of the motion of the particles by which the particles of the luminous bodies are agitated, which is such that they have the power to shake the neighboring bodies and to sustain this shaking and to transfer it from one particle to the next over a very large distance; and this happens in the same manner as the shaking of colliding bodies shakes the air over a large distance and travels from one particle to the next until it reaches the ear.

Perrault explained the observed differences in the propagation of light and sound by the different properties of the particles of the relevant medium. Thus, he related the finite/infinite speed of propagation to the compressibility/incompressibility of the particles of the medium, the specificity of the receiving organ to the promptitude, amplitude, and kind (circular/translational) of the motion of the particles, the rectilinear/diffuse character of the propagation to the impossibility/possibility of a "dodging" motion of some of the particles on a line joining the source and the receptor. On the basis of his fine anatomy of the eye and ear, he found much difference between the two senses and used it to argue against the classical idea of universal proportions in music and architecture.[38]

[36]Ibid., p. 133. Hooke's suggestion is somewhat similar to Mairan's later idea that different particles of the air are responsible for the transmission of different sounds: see below, chapter 4, p. 137.

[37]Hooke 1705, p. 81.

[38]Perrault 1680–88, vol. 2, pp. 44–5 (incompressibility), 51–2 (circular motion); vol. 4 (1688), pp. 231 (against emission), 238 (billiard balls), 242–3 (dodging), 248 (citation). This Perrault was the brother of Charles, the author of *Cendrillon*. Originally a physician, he translated Vitruvius, and designed the neoclassical colonnade of the Louvre. Cf. Picon 1988, pp. 75, 90, 152. Perrault may have borrowed the billiard-ball analogy from his friend Huygens, although he did not share the latter's concept of light as a compression wave. He explained the colors of

To sum up, Hobbes inaugurated a kind of reasoning in which the motion of the medium around a (point) source of light was simultaneously described by rays and by (traces of portions of) surfaces orthogonal to the rays. He thus gave an influential proof of the sine law of refraction as well as a tentative explanation of colors. Maignan, Barrow, Hooke, and others similarly combined rays and intersecting surfaces in their theories of light. The meaning of the surfaces varied considerably: shifts of an incompressible fluid for Hobbes, aggregates of light corpuscles for Maignan, traveling pulses for Hooke. Perhaps this notion was in part suggested by the water-wave metaphor, which had frequently been used for sound since the Stoics. It would be wrong, however, to assimilate these theories with wave theories in the modern sense because their authors did not understand water and sound waves as we now do.

In their analogies with sound, Hobbes, Hooke, and Perrault relied on the pestle view, namely: they conceived the propagation of sound and light as the wholesale motion of the medium and not as compression waves. They followed Descartes in assuming the incompressibility of the optical medium and therefore the instantaneous propagation of light, except for Hooke, who temporarily assumed a finite velocity of propagation in his theory of colors. Hobbes and Hooke assumed the periodicity of the ethereal motion, and Hooke used this property in his explanation of the colors of thin plates. Although these authors undoubtedly relied on acoustic analogy, they were discreet about it and rather pretended to be basing their optics on observed facts and rational mechanism.

Grimaldi's undulations

Unrelated to the former theories were the singular insights of the Jesuit father Francesco Maria Grimaldi, posthumously published in his turgid *Physico-mathesis de lumine* of 1665. Grimaldi's concept of light probably originated in his discovery of the new optical phenomenon that he called *diffractio*. In a series of delicate experiments, he studied the shadow projected on a distant screen by a narrow object when illuminated by sunlight passing through a small hole in a shutter. He observed that the penumbra was larger than implied by the size of the hole and exhibited colored fringes both within and without the geometric shadow (see Fig. 2.15). He carefully determined that this phenomenon depended on a new mode of propagation of light, differing from reflection (by the edges of the object), refraction (by some heterogeneous medium), or diffusion (by the air).[39]

Grimaldi explained the observed fringes by analogy with the undulation of the surface of a stream beyond an obstacle, judging that "light seem[ed] to be some very fast fluid, sometimes also undulating, and pouring through diaphanous media." A visible, macroscopic undulation occurred in the case of diffraction fringes. A much finer, invisible undulation occurred in the light reflected or transmitted by colored bodies, as a

bodies by specificity of the motion induced in their particles, and the color of the issuing light by excitation of the same specificity in some of the particles of the ether (vol. 4, pp. 250–2). He fancifully explained refraction by multidirectional reemission of rays from the interface, followed by attraction between oblique rays and the privileged normal ray (vol. 4, p. 258); a similar explanation is found in Fabri 1667, pp. 107–10.

[39]Grimaldi 1665, prop. 1. Cf. Ronchi 1956, pp. 112–49. On Hooke's rediscovery of diffraction, see below, chap. 3, p. 93, note 29.

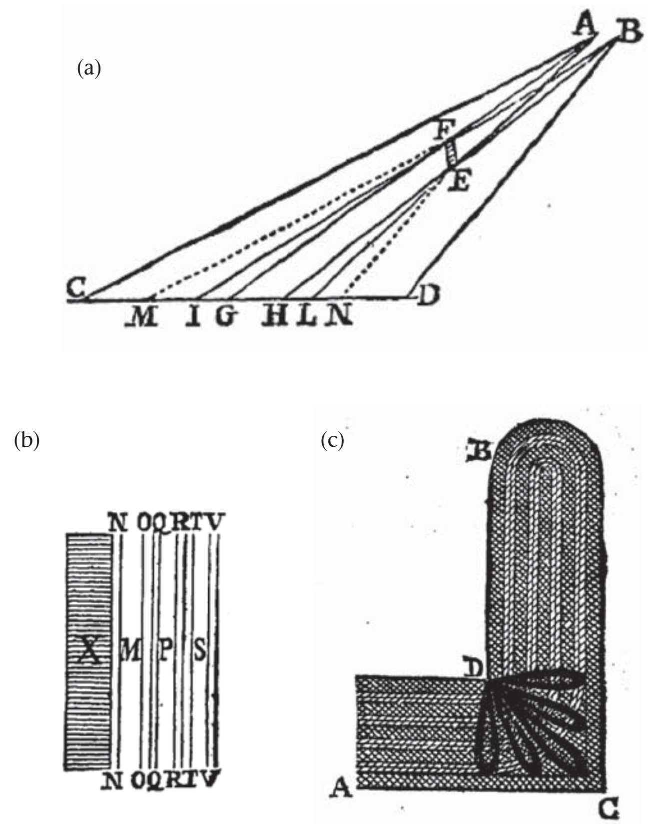


Fig. 2.15. Grimaldi's first diffraction experiment. (a) Experimental setup (AB = hole in a shutter, FE = object; CD = screen); (b) Colored fringes (N, … V) next to the geometrical shadow of the object; (c) Internal fringes in the shadow of an L-shaped object. From Grimaldi 1665, plate.

consequence of the disturbance of the luminous flow by the minute porous structure of the body:[40]

> The modification of light through which it is colored … can probably be said to be determined by its exceedingly fine undulation [*ipsius undulatione minutissime crispata*], as in some trembling flow [*tremor diffusionis*], with the most subtle rolling [*certa fluitatione subtilissima*] through which it affects the sense of vision by proper and determined application.

Although Grimaldi seems to have reached this correspondence between color and undulation through the water-stream analogy, he devoted a full section of his treatise to another analogy between the variety of sounds and the variety of colors in nature. In a

[40]Grimaldi 1665, props. 2, 43. Cf. Wilde 1838, p. 327.

lengthy preamble, he argued against the scholastic interpretation of sound as the multiplication of intentional species and he defended the idea of a mechanical vibration transmitted from the sonorous body to the ear through successive elements of the intermediate air. The implied undulations of the air, he went on, reflected the complexity of the vibrations of the source and were thus able to produce the whole range of musical or phonetic sounds. Thanks to the bridging concept of undulation, he compared this diversity of sounds with the diversity of colors acquired by light. Be it out of ignorance of contemporary music theory or out of interest in music, he did not introduce the correspondence of frequency with pitch, and he did not enunciate the analogy between pitch and color. His only point was that both for the colors of light and for the qualities of sounds (pitch, timbre, phonetics), the properties of some undulation of the medium were determinant.[41]

2.3 Pardies's and Huygens's wave theories

Pardies's acoustic analogy

The first author who explicitly and systematically constructed optics on the basis of analogy with sound and water waves was the Jesuit father Ignace Gaston Pardies, in a now lost manuscript written around 1670 and entitled "On the motion of undulations." His treatise on statics, published in 1673, contains a short description of the intent of this work: first to describe the undulations of water, "a matter of game and entertainment for children, which can be the subject of a very deep meditation for the most skilled philosophers"; and then to treat sound by analogy with these undulations, and light by analogy with sound:[42]

> In a conjecture on the propagation of light, we examine whether one could not also suppose that the vehicle of light is some similar motion in a subtler air; and we show indeed that under this hypothesis one would explain in a very natural manner all the properties of light and colors, which are otherwise very difficult to explain; and I hope my reader will enjoy the manner in which the measure of refraction is demonstrated.

A younger Jesuit, Pierre Ango, inherited Pardies's manuscript and exploited it in the first book of the optical treatise he published in 1682 in Paris. In an introductory letter to this treatise, Ango emphasized the analogy with sound and Pardies's importance in this respect:

> You will see that [this first book] contains every beautiful and solid thing that can be said on the propagation and the properties of light, on colors, and even on sounds, about which I write at first in order to ease the conception of what I subsequently say on light.—There I have used some of the thoughts of the late Father Pardies which

[41] Grimaldi 1665, prop. 44. In 1668, The Oratorian father Nicholas Joseph Poisson suggested that the distance between two colors could be measured by the number of intermediate colors as the distance between musical notes could be measured by the number of intermediate notes: "Si l'oreille juge, de combien de degrez le *Sol* est plus élevé que l'*Ut*, l'oeil s'aperçoit aussi de la difference, qu'il y a entre le *Cramoisy* et le *Jaune,* comptant les degrez de difference par les couleurs comprises entre les deux, scav. la couleur de feu, le rouge, l'incarnat etc.," cited in Oldenburg to Boyle, 17 March 1668, in Hunter, Clericuzio, and Principe 2001, vol. 4.

[42] Pardies 1673, "Préface."

> you believe to be still novel and which are in a better shape than this Father left them in the memoirs which you know I long had in my hands.

As Ango seems to have closely followed Pardies in this book, I will call its author Ango/Pardies.[43]

The two Jesuits presented their project as a defense of Aristotle's views against the neo-atomism of Gassendi and Maignan. In their opinion, Aristotle's *De anima* was the main source of the medium-based theory of sound and light. They quoted this text to justify their main assumptions, even the spring of air and the analogy of sound and light with water waves! For the optical medium, they used Aristotle's name, aether, a rare occurrence at that time.[44]

Ango/Pardies introduced the notion of undulation by means of the picture of a vibrating body that induces alternating compressions and dilations of the contiguous layer of air, followed by similar periodic deformations of the next layer, and so forth. They believed the spring of air to be a long-known property, recently confirmed by Boyle's experiments and to be explained in a Cartesian manner: by assimilating the air with a sponge of a grosser element impregnated with the incompressible ether. They insisted that the spring-based mechanism of propagation excluded any transport of air from the source, thus clearly rejecting the breath conception that had long been the most popular. They denounced the common belief that water waves implied a transport of water, and pointed to the analogy with the undulation of a tense string periodically shaken at one of its ends. This chain of analogies between light, sound, water waves, and a vibrating string, compensated for the lack of a self-standing dynamical theory of sound propagation.[45]

Ango/Pardies then introduced some of the main properties of waves by means of the water-wave illustration. They gave an essentially correct explanation of the formation of waves by the impact of a stone. They identified the direction of propagation with the force exerted on a small solid obstacle, and explained rectilinear propagation by arguing that the forces exerted on a small portion of a circular wave by the contiguous portions cancelled each other. They described the reflection of the waves by a solid wall as resulting from the specular reflection of portions of the waves when hitting the wall (Fig. 2.16), and they made the novel comment that the reflected waves were no longer circular and yet remained perpendicular to the rays when the wall was not a plane. They described the superposition of the incoming and reflected waves, and noted like Mersenne and Hooke that the waves "crossed each other without mutual hindrance" because the ones "passed like over the top of the others" (our principle of superposition).[46]

Ango/Pardies then devoted a quarter of their book to the case of sound, which they regarded as paradigmatic for other undulations. They reproduced classical arguments in favor of sound as local motion, identified pitch with frequency, expounded the coincidence theory of consonance, and dwelled on the intervals of just intonation. More originally, they discussed sounding pipes and the propagation of sound on the basis of the spring of

[43]Ango 1682, pp. iv–v. Cf. Shapiro 1973, pp. 209–17; Ziggelaar 1971.

[44]Ango 1682, pp. 7–14.

[45]Ibid., pp. 15–16.

[46]Ibid., pp. 20–8 (citation on p. 28).

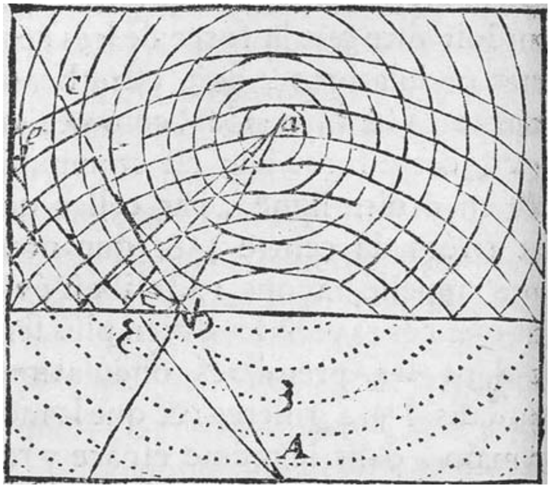


Fig. 2.16. The reflection of circular water waves by a plane wall. From Ango 1682, p. 26.

the air. The length of this section on sound no doubt resulted from their awareness that even after Mersenne's notorious efforts, no consensus existed on the nature of sound and on the mechanism of its propagation. They indeed pioneered the concept of sound as a compression wave.[47]

Ango/Pardies then turned to the refraction of undulations at the limit between two different kinds of air. They were aware of the derivation of the sine law by Maignan and by "a learned Englishman" (Barrow probably), rejected it for relying on emissionist principles, and yet adapted it to obtain the ancestor of many later wave-theoretical derivations. They replaced the lines of light with periodic undulations meant as metaphors for an alternation of compressed and expanded states. Another difference was the more general geometry: whereas Hobbes, Maignan, and Barrow considered only straight lines of light, Ango/Pardies reasoned on incoming spherical waves that became non-spherical after plane refraction. They applied a variant of Hobbes's kinematic reasoning to a small, nearly straight portion of a wave (see Fig. 2.17), then obtained the refracted rays by taking the normal to the refracted bit of wave, and the whole refracted waves by drawing a series of surfaces normal to the refracted rays and such that the distance between two consecutive surfaces was a constant (this distance being defined as the common length of the portions of rays joining them).[48]

When they finally came to the case of light, Ango/Pardies argued at length that "every luminous body breathes" and thus undergoes periodic, global vibrations that shake the surrounding ether. They also used the heart-beat analogy, without naming Hobbes. As

[47]Ibid., pp. 32–59.

[48]Ibid., pp. 60–6. Ango was aware of Hooke, whom he criticized for not proving the law of refraction (pp. 90–1). Unlike Hobbes, Maignan, and Barrow, Ango/Pardies did not assume the conservation of the lines of light. Rather, they assumed that the rays delimiting the portions of waves were broken at the interface.

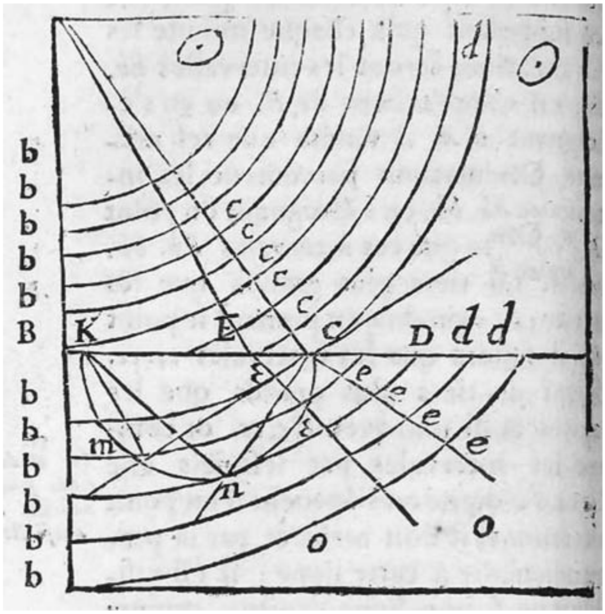


Fig. 2.17. The refraction of spherical waves according to Pardies/Ango. The refracted rays ee and εε are normal to the refracted waves eε. If cm and cn are the perpendiculars to the incoming ray cc and to the refracted ray ee, for any circle CnmK centered on the interface, the lengths Km and Kn are proportional to the velocities of the waves in the two media, as can be seen by comparing the triangles Kmc and Knc to the two small quasi-triangles formed by the interface, the ray cc, the ray εε, and the normal waves at the refraction points of these rays. This implies the sine law of refraction. From Ango 1682, p. 62.

they assumed the Cartesian incompressibility of the ether, they believed that in the absence of any matter the propagation was instantaneous (as in Hobbes's theory) but that it became truly wave-like whenever some air or other transparent matter was present. Their theory of refraction therefore implied the mingling of matter with the ether. They believed that light propagated almost instantly until it reached the atmosphere and thereafter assumed a finite velocity decreasing as the density of the air increased.[49]

On colors, Ango/Pardies adapted the theory of Aristotle's *De sensu*, according to which the colors of bodies are produced by a mixture of white and black particles in various proportions. They approved Aristotle's interpretation of the most pleasant colors as corresponding to simple ratios in analogy with musical consonance, adding that a harmonious combination of colors might also correspond to simple ratios:

> By analogy between sounds and colors … as we can give the reason why there are cords that displease and others that are most pleasant, we could just as well give the reason why there are certain colors whose assemblage pleases and others whose union is not at all agreeable to anyone.

[49]Ibid., pp. 70–3 (citation on p. 73), 80–1 (role of matter).

They further defined the proportion of black and white corresponding to various colors in a manner suggested by the lateral colors of a (broad) beam refracted by a prism. Most originally and most obscurely, they distinguished between "kinds of colors" determined by the numbers of rays, and "nuances of colors" determined by both the amplitude and the frequency of the associated waves:

> In order to explain all the variety of colors, there are in general only two things to be considered in the manner in which the rays of light are sent back by objects. Namely, the objects are only able to diminish the force or the multitude of the rays, the force by diminishing the strength and frequency of the undulations that the rays produce in the medium while pressing it against the eyes (because the objects' parts are too soft and two loose to reflect them well); the multitude by not sending the rays back in a number equal to that received from the luminous bodies (because the objects' parts are ill adjusted or cut with diverse faces that disperse the rays and deviate them in every direction without any order).

From this extract we may infer that for Ango/Pardies, the frequency of the undulations was a parameter of color, though not the only one. As they still depended on Aristotle's doctrine, they refrained from the simple and direct identification of color with frequency that the analogy with pitch naturally suggested.[50]

Huygens on the nature of light

Toward the end of his life, Pardies lent his manuscript on undulations to the great Dutch natural philosopher Christiaan Huygens, whom Louis XIV's minister Jean-Baptiste Colbert had called to Paris in order to organize the Royal Academy of Sciences. Huygens's interest in optics had developed in the 1650s, as he was conceiving and building telescopes with his brother Constantijn. With one of these excellent instruments the two siblings discovered the rings of Saturn in 1656. Christiaan worked out the theory in two manuscripts: the *Dioptrica* of 1652, which contained a general theory of systems of lenses; and the *De aberratione* of 1665, which gave the first theory of spherical aberration. The publication of Barrow's lectures in 1669 and Newton's discovery of chromatic aberration in 1672 dissuaded Huygens from publishing these treatises.[51]

Instead Huygens began working on a new treatise that would include the physical foundation of optics and explain the newly discovered birefringence of the Iceland spar. Vividly interested in sound and music, and impressed by Pardies's manuscript on light and sound, Huygens let the acoustic analogy inform his concept of light as a compression wave. In 1676, he welcomed Ole Rømer's determination of the velocity of light from observations of the eclipses of Jupiter's satellites as a confirmation of this view (see Fig. 2.18). He soon succeeded in explaining the strange properties of the Iceland spar on

[50]Ibid., pp. 155, 153–4. As we may judge from his correspondence with Newton, Pardies was aware of Grimaldi's views on colors.

[51]Cf. F. J. Dijksterhuis 2004; Shapiro 1973, pp. 218–44; 1980a.

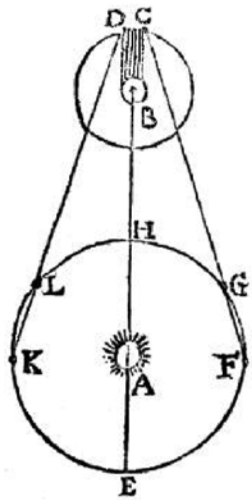


Fig. 2.18. Rømer's determination of the velocity of light. From Rømer 1676, p. 234. The larger circle represents the orbit of the earth around the sun (A). The smaller circle represents the orbit of the first satellite of Jupiter (B); it enters the shadow of Jupiter in C and emerges from it in D. L and K are the positions of the earth for two successive emersions of the satellite occurring near one quadrature (time at which Jupiter and the sun are seen in perpendicular directions). F and G are the positions of the earth for two successive immersions of the satellite occurring near the other quadrature. If the velocity of light has a finite value *c*, the time between the two emersions exceeds the period of the satellite by LK/*c*, whereas the time between the two immersions is inferior to this period by FG/*c*. Rømer found that for a large number of revolutions of the satellite, the asymmetry became measurable.

the basis of the wave theory. In 1679, he read a nearly definitive version of his *Traité de la lumiere*, which he published in 1690 with a long delay.[52]

In the first chapter of this treatise, Huygens rehearsed the then classical arguments in favor of light as a mode of motion and the controversial argument that the high speed of light and the lack of mutual perturbation of crossing rays excluded the concept of light as an emanation. He concluded: "Light therefore propagates in a different manner, and what can lead us to its understanding is the knowledge we have of the propagation of sound in air." Before dwelling on Rømer's proof of the finite velocity of light, Huygens announced the main result of the analogy with sound:[53]

[52]Cf. Blay 1992. On Huygens's interest in music and acoustics, cf. Huygens 1888–1950, vol. 19, pp. 359–77; vol. 20, pp. 1–173; Dostrovsky 1975, pp. 200–1; Cohen 1984, pp. 209–28. On Rømer's argument, cf. Rømer 1676; Pedersen 1978; Débardat 1978. Anomalies in Cassini's tables for the motion of Jupiter's satellites inspired this argument. The tables were made for the purpose of longitude measurement.

[53]Huygens 1690, pp. 3, 4.

> If light takes time to travel … , it follows that the impressed motion [of the medium] is a successive one, and consequently that it spreads out, as in the case of sound, by spherical surfaces and waves; for I call them waves by resemblance with the waves formed in water when a stone is dropped in it, which represent such a successive spreading around, though coming from another cause and occurring on a plane surface only.

Beyond these generalities, Huygens relied a lot less than Pardies on the analogy with sound. "If [sound and light] are similar in this respect [finite velocity of propagation]," he noted, "they differ in several other respects, namely: in the first production of motion that causes them, in the manner in which this motion is propagated, and in the manner in which it is communicated." Whereas Pardies assumed a global vibration of the shining body, Huygens imagined tiny fluctuations of the parts of this body in order to explain the fact that the various points of the body's surface were perceived independently of each other. Accordingly, his waves were short spherical pulses randomly emitted by the various points of a luminous surface.[54]

Regarding the medium, Huygens agreed with Pardies that it could not be the same for sound and light, since according to Torricelli's and Boyle's vacuum experiments sound required air for its propagation and light did not. Like Pardies (and Aristotle), Huygens called the light medium the aether. Whereas Pardies admitted essentially the same wave mechanism for light and sound propagation in the presence of matter, Huygens believed that the extremely high speed of light required a specific mechanism.

Huygens defined sound as a "regular trembling of the air" during which "the air acts as a spring and assumes a successive motion." In his *Traité de la lumière*, he wrote that in the explanation of the propagation of sound, "the air is considered to be of such a nature that it can be compressed and reduced to a space much smaller than that which it normally occupies; and that as it is compressed it makes an effort to expand." In the case of light, Huygens believed Descartes's model of contiguous hard spheres to be closer to the truth. Drawing on his earlier study of elastic collisions, he modified this model by allowing some elasticity of the spheres. In order to minimize the violation of Descartes's philosophy of matter, he traced this elasticity to the agitation of smaller particles confined within the spheres (as pressure is now explained in the kinetic theory of gases).[55]

Huygens believed that his model of the ether, designed to explain the high but finite propagation velocity by the elastic collisions of a succession of spheres, had another advantage over Descartes's: it allowed for the crossing of light waves without mutual hindrance (in particular two persons could see each other's eyes), because the resulting elastic deformations of the balls could be superposed. But it had a major inconvenience: it implied that a compressed ball should press every contiguous ball, whereas Descartes's perfectly rigid balls were supposed to transmit endeavors in their original direction only. Rectilinear propagation seemed impossible, and the initial pulse of the source seemed

[54]Ibid., p. 9. Owing to the lack of periodicity of Huygens's "waves," some physicists and historians refuse to call his optics a "wave theory." I disagree, because in common parlance and even in some domains of modern physics (e.g. solitons) waves need not be periodic.

[55]Huygens [c. 1674], p. 370; Huygens 1690, p. 11.

destined to be lost in diffuse motion. We will now see that Huygens turned this potentially disastrous circumstance into the greatest asset of his model.[56]

Huygens's construction

What permitted this reversal was the so-called Huygens construction, which Huygens presumably discovered while meditating on Pardies's proof of the law of refraction. This proof, with its combination of rays and non-spherical wave surfaces, must have excited Huygens's geometrical curiosity. His contemporary interest in caustics may have prompted him to examine the wave pattern in relation to the caustics produced by refraction through a curved surface: he did so in manuscripts of the 1670s and in the last section of his *Traité*.[57]

An evident defect of Pardies's derivation of the law of refraction is that it does not prove the existence of a well-defined direction of refraction for a narrow beam. It assumes this property and then derives the sine law by means of the associated waves. In the case of a broad beam, it constructs the refracted waves by drawing the surfaces normal to the rays. Yet there is a simple way to construct refracted waves by purely wave-theoretical means. At a given instant t, an incoming spherical wave (pulse) intersects the plane separating the two homogenous media. At any later time, the refracted wave must contain perturbations that have traveled from this intersection at the velocity V_2 in the second medium. According to Pardies, this velocity must be reckoned normally to the wave. Consequently, the refracted wave at the instant $t + \tau$ must be tangent to all the spheres of radius $V_2 \tau$ centered on a point of the circular intersection of the incoming wave with the separating plane. Conversely, for any positive τ the refracted wave at a given instant t must be tangent to the spheres of radius $V_2 \tau$ whose center belongs to the intersection that the incoming wave had with the separating plane at time $t - \tau$. In other words, the refracted wave must be the envelope of a well-defined family of spheres.

Huygens's Fig. 2.19 represents the traces of a single wave (pulse) in the plane of refraction at five, evenly spaced instants of time, in the simple case in which the wave is originally plane. AB is the trace of the surface separating the two media. The line AC and KL lines represent portions of the incoming wave at successive times. The circle SR centered on A has a radius AN, whose ratio to the distance CB equals the ratio V_2/V_1 of the velocities of light in the two media. The circles centered on the points K at which the wave later meets the second medium have radiuses smaller than AN by the amount HK $\cdot V_2/V_1$, so that the total time needed to travel HK in the first medium and to travel these radiuses in the second medium is a constant. The refracted wave at the last instant (at which C reaches B) is the envelope of the various circles. The OK lines represent the intermediate positions of the refracted wave. Huygens insisted on the novelty of this construction:[58]

[56]Huygens 1690, pp. 16, 21. Descartes in fact believed his model could account for the free crossing of rays: see above p. 40.

[57]Cf. F. J. Dijkterhuis 2004, chap. 4.

[58]Huygens 1690, chap. 3; p. 18 (citation). As Huygens did not fail to note, his and Pardies's derivations of the law of refraction imply the relation $\sin i_1/\sin i_2 = V_1/V_2$ between the incidence angle i_1 and the refraction angle i_2, in

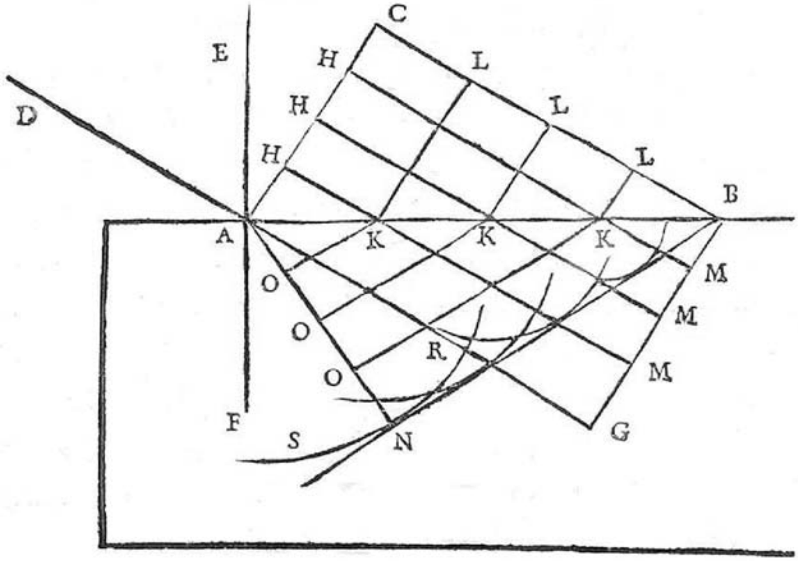


Fig. 2.19. The refraction of a plane wave according to Huygens 1690, p. 35.

> This is what has not been known to those who before me began to consider the waves of light, including Mr. Hook [sic] in his Micrography and Father Pardies, who in a treatise of which he showed me a part and which he could not complete as he died soon after that, had set out to prove through these waves the effect of reflection and refraction. But the principal foundation [Huygens's construction] was lacking in his demonstrations, and for the rest he had opinions very different from mine, as will perhaps be seen someday if his text has been preserved.

A remaining weakness of this construction is that it assumes the existence of the refracted wave as a two-dimensional pulse in the second medium. Huygens tried to remove this imperfection by reifying the spheres of his construction and treating them as real "particular waves" emitted at the interface between the two media. In this case, the refracted wave should simply be the resultant of all particular waves. Evidently, this resultant does not vanish in the space separating their envelope from the interface (the space ABN in Fig. 2.19). Probably inspired by an analogy with caustics, Huygens argued that the intensity of the perturbation in this space was "infinitely smaller" than on the envelope.[59]

Huygens justified the existence of the secondary waves through his elastic-ball model:

agreement with Fermat's principle of least time and in contrast with Descartes's assumption that the parallel component of the velocity (of a ball) is conserved during refraction.

[59]This is correct: if the circular waves are represented by δ-functions, the resultant intensity per unit area behaves as the inverse of the square root of the distance from the envelope. However, the quantity of light contained in any finite portion of the space delimited by the envelope is comparable to the quantity of light contained in any finite domain that includes the envelope.

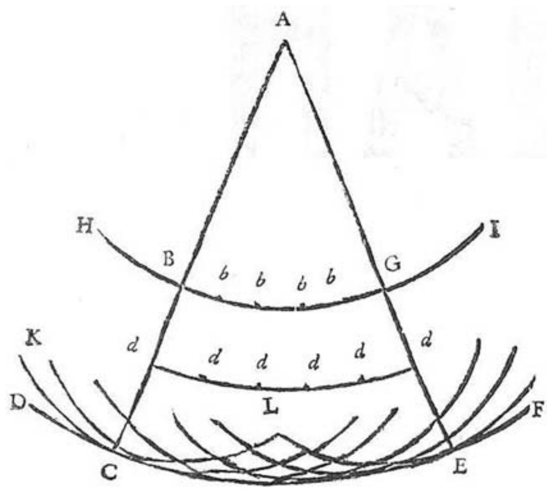


Fig. 2.20. Huygens's construction for the wave (CE) which originated in A and passed through the diaphragm BG. From Huygens 1690, p. 17.

> Every particle of the matter in which the wave spreads out communicates its motion not only to the next particle that is on the straight line drawn from the luminous point, but it also gives some motion to all other particles that touch it and that oppose its motion.

This picture implies the formation of secondary waves not only at the interface between two media, but at any point of space crossed by a wave. Huygens employed this process to explain the rectilinear propagation of light in the following manner. He considered (Fig. 2.20) a spherical wave centered on A and encountering the diaphragm made of the opaque surfaces HB and GI. The b points of the medium are excited by the wave when it passes the diaphragm. Huygens judged that at a given ulterior time the resultant of the particular waves emitted from this point had a negligible intensity everywhere except on the portion CE of the wave formed in the absence of the diaphragm. Huygens thus obtained the first wave-theoretical proof of the rectilinear propagation of light that was not too obviously insufficient. Earlier medium theorists had argued that spherical propagation implied rectilinear propagation because the radiuses of a sphere are perpendicular to its surface: this begs the question, since the concrete definition of rays requires diaphragms that truncate the spheres of light.[60]

Although Huygens introduced his theory by analogy with sound, he did not mention the common belief that sound, unlike light, propagated in curved lines. There are two possible explanations of this silence. He may have assumed, as Mersenne, Kircher, and Pardies did,

[60]Huygens 1690, p. 17. Huygens's reasoning is incorrect: even though the intensity diverges on the envelope, its integral over a finite open domain reached by wavelets has the same order of magnitude whether or not the envelope intersects this domain.

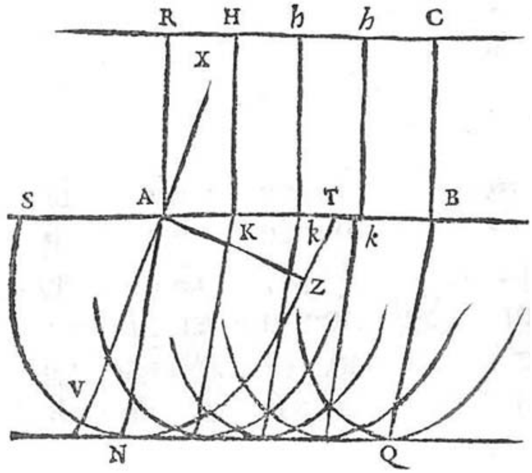


Fig. 2.21. Huygens's construction for the refraction of a plane wave (RC) when entering an anisotropic medium at normal incidence. The elliptical form of a particular waves SNT implies that the ray AN joining its center A to its contact point N with the envelope NQ of all such waves is not perpendicular to the resultant wave NQ. From Huygens 1690, p. 60.

that sound actually propagated in rays that could be reflected like rays of light and perhaps even refracted. Or he may have accepted different behaviors of light and sound in this regard and traced them to the different propagation mechanisms he imagined in each case. As we will see in a moment, he adopted a position intermediate between these two alternatives after he encountered a related objection to the wave theory of light in Newton's *Principia*. Huygens never discussed diffraction, even though he owned a copy of Grimaldi's relevant treatise.[61]

Extraordinary refraction

Huygens's greatest achievement in optics was his derivation of the laws of refraction for the Iceland spar. In 1669, the Danish physician Erasmus Berthelsen had discovered that a ray entering the spar crystal divided into two rays, one following the usual law of refraction and the other obeying a more complex rule. In the case of extraordinary refraction, Huygens assumed that the particular waves emitted at the surface of the crystal were oblate ellipsoids instead of spheres, owing to the anisotropy of the crystal (the ordinary waves are the spheres inscribed in these ellipsoids). This implies that the rays joining the centers of these waves to the enveloping wave are no longer orthogonal to the latter, so that a ray entering the crystal at normal incidence is refracted (see Fig. 2.21). With great geometrical acumen, Huygens extended this reasoning to any angle of

[61]On Huygens's neglect of Grimaldi, cf. Blay 1992, p. 25.

incidence, and confirmed the results experimentally thanks to a new technique for cutting and polishing the crystals.[62]

Huygens acknowledged two weaknesses of his system: it did not encompass the color phenomena discovered by Newton, and it did not explain a "marvelous phenomenon, discovered after writing all the above." Namely, the ordinary or extraordinary beam issuing from one spar crystal did not necessarily undergo double refraction when entering a second crystal. There were special orientations of the second crystal for which only one kind of refraction occurred. Huygens commented:[63]

> It seems that we are forced to conclude that the waves of light, for having passed the first crystal, acquire a certain form or disposition, through which when encountering the material of the second crystal in a certain position, they can stir the two different matters that serve to the two different species of refraction; and when encountering this material in another position, they can move only one of these matters. Regarding how this happens, I have not found anything that satisfies me.

Despite these shortcomings, Huygens's theory became the most respected of the medium-based theories of light of the late seventeenth century. Plausible reasons for this preference are Huygens's ability to predict new laws, his lucid and elegant style, his geometrical virtuosity, and his well established authority as one the three greatest natural philosophers of his time.[64]

2.4 Optical imaging

Stigmatism

Kepler introduced some of the basic problems and methods of modern geometrical optics, including the relation between images and crossing rays, the notion of stigmatism, the concept of rigorously stigmatic surfaces, and the relation between caustics and imaging. However, he did not provide a unified definition of virtual images. In the case of binocular vision, he placed the image of a luminous point at the intersection of the two visual axes and noted that the position of the image generally depended on the orientation of the line joining the two eyes. In the case of monocular vision, he similarly considered the intersection of rays passing through opposite sides of the pupil; and yet he judged that the image of a luminous point through a water-filled sphere was seen on the surface of the sphere because the brightness of this surface attracted the observer's attention. Another weakness of Kepler's optics is the lack of quantitative relations for the locus of images or for their (angular) size, even in the paraxial approximation. For large refracting angles, he did not know the true law of refraction and therefore could not exactly determine the stigmatic surfaces and the caustics he defined in the *Paralipomena*.[65]

[62]Berthelsen 1669; Huygens 1690, chap. 5. Cf. F. J. Dijkterhuis 2004, chap. 5. In the case of uniaxal crystals such as the Iceland spar, the ellipsoidal shape of particular waves agrees with the modern theory for the anisotropic propagation of electromagnetic waves.

[63]Huygens 1690, pp. 88, 91.

[64]Cf. F. J. Dijkterhuis 2004, chap. 6.

[65]Kepler 1604, chap. 5, props. 1–6 (water-filled sphere). Cf. Shapiro 1990, pp. 119–27.

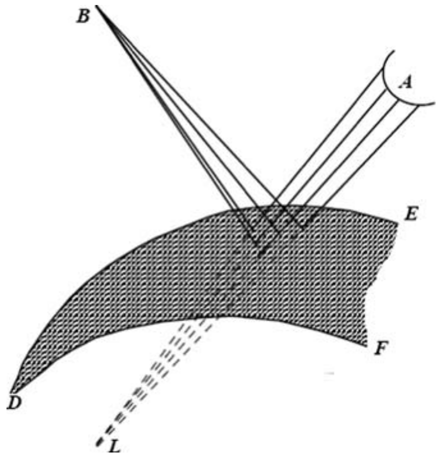


Fig. 2.22. Gregory's drawing for the definition of a virtual image. The mirror DEF reflects the rays issuing from point B. The intersection point L of the reflected rays entering the eye A is the image of B. From Gregory 1663, p. 47, redrawn in Bruce 2006. Courtesy of Ian Bruce.

In his *Dioptrique* of 1637, Descartes used the sine law of refraction to determine the rigorously stigmatic surfaces for which all the rays from a given point converge to the same point after refraction. He then concentrated his efforts on the production of hyperbolic lenses that perfectly focused a parallel beam. In his *Optica promota* of 1663, the Aberdeen astronomer James Gregory arrived at the sine law of refraction independently of Descartes, by a priori making the ellipsoid of revolution a rigorously stigmatic interface. Like Descartes he wanted perfectly focused images and therefore used parabolic and elliptic mirrors in the plans of the telescope for which he is best known. In his *Traité*, Huygens greatly simplified the determination of stigmatic surfaces by noting that they correspond to a constant traveling time along rays broken on them and joining the two foci. This property is an obvious consequence of Huygens's construction, since the waves around the foci are spherical and correspond to constant traveling times.[66]

Virtual images

Through his diagrams of modified vision (Fig. 2.4, p. 44), Descartes anticipated the modern geometrical definition of virtual images. Gregory provided the first statement of this definition, accompanied by Fig. 2.22:

> From the points of the pupil, draw through the points of reflection all the lines of reflection, in whose concourse L (provided they concur) will be the apparent place of the image of the point B. If, however, they do not concur in one point, no distinct and fixed places of the image of the visible point B will exist.

[66]Descartes 1637 (see above, pp. 44–45); J. Gregory 1663; Huygens 1690, pp. 113–18.

This definition is often attributed to Isaac Barrow, who exploited a similar one in the *Lectiones* he gave in 1668–9 from the first Lucasian chair in Cambridge. As long as the virtual image is believed to represent the perceived location of images correctly, the Gregory–Barrow principle turns instrumental optics into a purely geometrical exercise from which the subjective elements of vision disappear.[67]

The visual adequateness of this principle has often been questioned, however. Huygens, who privately introduced the real or virtual *punctum concursus vel dispersius* of a system of refracted rays as early as 1753, judged that in the case of monocular vision the appreciation of distance essentially depended on the visual context (as in Kepler's water-filled ball experiment). In his influential *Dioptrica nova* of 1692, the Irish astronomer William Molyneux similarly argued that the visual appreciation of distance depended on comparative "judgment" rather than "sense," except for close objects whose distinct vision required conscious accommodation. In his *Theory of vision* of 1709, the Irish Bishop and philosopher George Berkeley rejected any geometry of vision and related the apparent proximity of an object to the visual confusion of its image. Later in the century Robert Smith clearly distinguished the geometrical image (which he called "focus") and the visual image; and he propounded that the distance of the latter was inferred from its angular magnitude combined with prior knowledge of the size of the object. D'Alembert rejected both Smith's and Barrow's views, and found the circumstances of our visual appraisal of the location of objects to be so diverse that there might not be any general principle for the location of images.[68]

These various opinions on the visual appreciation of distance equally concern objects and images. Barrow's geometrically constructed virtual images fail to represent the apparent distance of an object seen through an optical device to the same extent as the naked eye fails to appreciate the true distance of objects. As Gregory tells in his definition, the existence of a virtual image in Barrow's sense warrants the possibility of seeing a distinct image (as long as this image is not too close to the eye). The reason is that for a point-like object the pencil of rays entering the eye should converge on a precise point of the retina after intraocular refraction; this can happen only if the produced rays of this pencil intersect at a single point (which may be infinitely remote). This state of affairs explains why Gregory's and Barrow's geometrical construction of virtual images became a basic component of the theory of optical instruments, even for authors like Smith who denied their relevance in the perception of distance.

Caustics

Strictly speaking, the Gregory–Barrow definition of virtual images is confined to rigorously stigmatic systems for which the (produced) rays entering the eye have a common point of intersection. Unlike Gregory, Barrow did not wish to confine himself to such

[67]J. Gregory 1663, pp. 46–7; Barrow 1669, p. 14. The phrase "virtual image" belongs to Claude François Milliet Dechales (1674): cf. Shapiro 2007, p. 89.

[68]Huygens [1653], p. 17 (*punctum concursus*); 1888–1950, vol. 13, pp. 775–6 [1692] (*Unde ergo locus imaginis aliquot modo percipitur? Solo apparentia magnitudine notae rei*); Molyneux 1692, 113; Berkeley 1709, §§14–27; Smith 1738, pp. 49–51; D'alembert 1761, §§5–17. Cf. Montucla 1758, vol. 2, pp. 597–602; Shapiro 1990; Malet 1997 (on Barrow); Bruce 2006 (on Gregory); Atherton 1990 (on Berkeley); Ferlin 2008, pp. 59–62 (on d'Alembert).

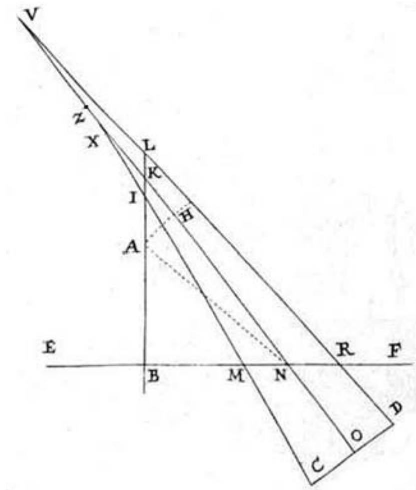


Fig. 2.23. Barrow's construction of the tangential image of a luminous point (A) through a refracting plane (BF). The refracted ray NO corresponding to the incident ray AN and passing through the center O of the pupil intersects the refracted rays MC and RD passing through the sides C and D of the pupil, at the points X and V respectively. The refracted rays closest to the central ray NO cut it at point Z, which Barrow makes the image of A. From Barrow 1669, fig. 62 of plate.

systems. He therefore needed a broader definition of images. In the special case of real images formed by a water-filled sphere (see Fig. 1.14, p. 28), Kepler had drawn the intersections of the successive refracted rays from a pencil of rays, and placed the image in the zone of intersection. Similarly, Barrow produced the rays of the pencil that enters the eye, and placed the virtual image at the intersection of the rays that are closest to the ray passing through the center of the eye (Fig. 2.23). This image is now called the "tangential image," because it is the point of tangency of the central ray with the virtual caustic generated by the optical system. Barrow only considered rays in the incidence plane of the central ray. In the case of refraction by a plane surface, Newton's subsequent consideration of the whole cone of rays entering the pupil led him to introduce a second image, now called the "sagittal image," which is the common point of intersection K of the refracted rays of all incident rays that make the angle BAN with the cathetus AB (on Fig. 2.23).[69]

Although the relation between caustics and astigmatic imagery remained unexploited until Smith's and Bouguer's relevant works, the study of caustics attracted a few seventeenth-century geometers. In his *Traité*, Huygens showed that caustics were the evolutes of the wave surfaces of his theory of light (see Fig. 2.24), and he proved that the caustic generated by the reflection of a parallel beam on a spherical mirror was an epicycloid. The

[69]Barrow 1669, lecture 5, §20; Newton 1984 [1670–72], pp. 214–21, 352–5. Cf. Shapiro 1990, pp. 133–8. As Thomas Young later realized (1801a, p. 30), the image points are in fact lines. The sagittal image satisfies the old rule of the *cathetus*, as does Kepler's binocular image when the line joining the two eyes is parallel to the refracting surface.

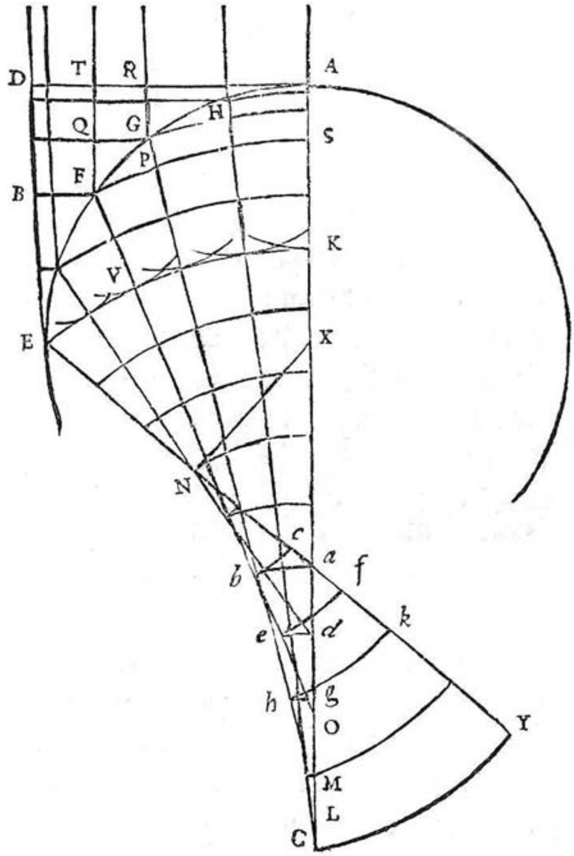


Fig. 2.24. Huygens drawing of rays, waves, and caustic after refraction on a circle (AE). The caustic is the envelope NbehC of the refracted rays. From Huygens 1690, p. 119.

German mathematician Ehrenfried Walther von Tschirnhaus published a faulty solution to this problem in 1682, and corrected it in 1690 after seeing Huygens's. In the latter memoir he inaugurated the phrase *curva caustica*, an allusion to burning mirrors. Jakob and Johann Bernoulli soon extended the study of caustics, solved the problem of the caustic of a refracting plane, and noted the connection with Barrow's determination of image points.[70]

Paraxial optics

Barrow, who treated optics as a mathematical playground, did not dwell on the paraxial (Gaussian optics) that is most useful to the theory of optical instruments. He nonetheless gave relations equivalent to the various cases of the modern formula

[70]Smith 1738, book 2, chap. 9; Bouguer 1760, pp. 98–104; Huygens 1690, pp. 118–24; Tschirnhaus 1682, 1690a, 1690b (p. 170 for *caustica*); Jakob Bernoulli 1693. Cf. Shapiro 1990, pp. 156–9.

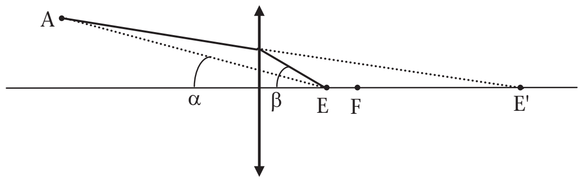


Fig. 2.25. Angular magnification of a convex lens according to Huygens. The object A is seen by the eye E under the angle α without the lens, and under the larger angle β with the lens. The point E', which Huygens uses to construct the refracted ray passing through E, is the virtual image of E.

$$\frac{n}{p'} + \frac{1}{p} = \frac{n-1}{R}$$

relating the algebraic distances p and p' (from the intersection of the axis with the refracting surface) of two conjugate points of a spherical refracting sphere with the index n and the radius R, as well as unnecessarily complicated relations of conjugation for thin lenses. In contrast, Huygens's early private treatises were aimed at improving the telescopes that he was designing with his brother. In his *Dioptrica* of 1653, he derived a variant of the now familiar

$$\frac{1}{p} + \frac{1}{p'} = \frac{1}{f} \quad \text{with} \quad \frac{1}{f} = (n-1)\left(\frac{1}{R} - \frac{1}{R'}\right)$$

for a thin lens of index n, radiuses R and R', and focal length f. This formula first appeared in print forty years later: case by case in Molyneux's treatise, and in algebraic form in a memoir by Edmond Halley also including thick lenses.[71]

Huygens computed the angular magnification of a single lens by means of a construction implying the conjugate of the position of the eye (see Fig. 2.25), and iterated this construction to obtain the magnification of any combination of lenses. He thereby treated the eye as a pin-hole camera for which accommodation and distance are irrelevant, as Kepler had done in the more qualitative reasoning of his *Dioptrice*. In order to obtain a sharp image despite the finite opening of the pupil, a small lens is placed in front of the (normal) eye so that the entering beam becomes parallel for a point source. In the *De aberratione* of 1666, Huygens computed the aberration of spherical lenses as a preliminary for the desired compensation of this defect. In a later treatise on telescopes and microscopes, he pioneered the study of two qualities of optical instruments: field and brightness. These considerations had an immediate impact on the design of his own telescopes, including the so-called "Huygens eyepiece." As we will see in the next chapter, they inspired later theorists of optics after their posthumous publication in 1703.[72]

[71]Barrow 1669, lectures 13 and 14; Huygens [1653], pp. 41–3, 99; Molyneux 1692, p. 48; Halley 1693. Cf. the excellent anonymous introduction to Huygens 1888–1950, vol. 13; Dijksterhuis 2004, chaps. 2–3; Shapiro 1990, pp. 144–51 (Barrow).

[72]Huygens [1653], pp. 175–6, 198; Huygens [1666], [1685–1692]. Descartes's *Dioptrique* included rough considerations about the field of telescopes.

2.5 Conclusions

Seventeenth-century natural philosophers inherited Kepler's concern with vision and its enhancement by optical instruments, and they significantly deepened his analysis of optical images. Their reflections on the nature of light yielded several mechanical medium theories of light, which were often seen as mechanical reinterpretations of Aristotelian optics. In Descartes's highly influential theory, light was a pressure instantaneously transmitted through the contiguous balls of the "second element." Hobbes and several of Descartes's followers agreed about instantaneous transmission, even after Rømer's contrary observations, because this property derived from the reduction of matter to spatial extension. They did not necessarily follow Descartes in neglecting the displacement of the medium. In analogy with the pestle concept of sound, Hobbes temporarily assumed finite displacements represented by surfaces orthogonal to the rays. This picture, together with the assumption of a constant ratio of the displacements in air and glass led him to an influential derivation of the law of refraction.

In the second half of the century, several authors departed more radically from Descartes by admitting a finite propagation velocity. Hooke did so temporarily, in analogy with the concept of sound as a periodic series of impulsive breaths. His main purpose was the explanation of the colors of thin plates in terms of two intercalated series of pulses. Around the same time, Pardies pioneered the explanation of sound and light as periodic compression waves. He reinterpreted Hobbes's derivation of the law of refraction as a consequence of the slower propagation of light in denser media. Huygens promoted Pardies's form of the acoustic analogy, with two liberties: he replaced the periodic waves with random series of pulses; and he introduced a specific mechanism of propagation by contact of contiguous elastic balls. He used the latter picture to justify the principle according to which each perturbed element of the medium was the center of secondary spherical waves whose envelope defined the observed wave at a given time. This principle enabled him to derive the received laws of reflection and refraction, geometric shadows, and the extraordinary refraction of Iceland spar by assuming the ellipsoidal shape of the secondary waves in this case.

Huygens believed that time was not ripe for speculating on the nature of colors. His forerunners were less reticent. Descartes linked color to the altered spin of his globules at the border between light and shadow, Hobbes to some rotation of his lines of light, Hooke to inseparable pairs of unequal pulses, Grimaldi to the fine-scale undulation of the light fluid, Pardies to the combined effect of attenuated intensity and selected frequency. Despite their thoroughly mechanical nature, all these explanations retained the Aristotelian idea that color had to do with the combination of brightness and darkness. This may explain why none of the seventeenth-century medium theorists defined color through frequency, despite Mersenne's and others' analogy between musical tones and colors.[73]

[73]Newton and Malebranche are two exceptions, if the former is regarded as a medium theorist and the second as a seventeenth-century philosopher. Their ways of associating color with frequency are described below in chap. 3, p. 87, and chap. 4, pp. 140–141.

3

NEWTON'S OPTICS

In a world still impregnated with the scholastic concept of light as the multiplication of intentional species through a medium, the most natural way to subsume the theory of light under mechanics was to adopt a mechanical medium similar to the air for sound propagation or to liquids for the transmission of pressure. This was indeed the choice of some of the greatest natural philosophers of the seventeenth century, as we saw in the previous chapter. Yet there was another option: a partial return to ancient Greek atomism, which could be reinterpreted as a kind of mechanical philosophy since it purported to deduce all phenomena and sensations from the collisions of immutable atoms. Although Newton's optics is far too complex to be reduced to a variety of atomism, its espousal of the corpuscular nature of light partly depended on the contemporary revival of this ancient philosophy.

The first section of this chapter briefly describes the new varieties of atomism. The second is devoted to Newton's early emissionist concept of light and to the accompanying insights into the nature of colors. The third section recounts the response of the main protagonists of the wave or pulse theory of light, with emphasis on a revealing polemics with Hooke. The fourth section describes Newton's public though unpublished hypothesis of an optical ether whose states affect the motion of the rays or of the light corpuscles. The structure and contents of the *Opticks* of 1704, as well as the queries appended to its successive editions, are described in the fifth and last section.

3.1 Neo-atomist theories

The chief protagonists of seventeenth-century atomism were Isaac Beeckman in the Netherlands, Pierre Gassendi in France, and Walter Charleton in England. Beeckman's variety of mechanical philosophy reduced all matter to arrangements of atoms and all sensations to the impact of atoms on the sensory organ. Although his interest in optics was mostly limited to the Dutch art of telescope making, he made clear that he espoused a neo-atomist doctrine in which light was a flux of atoms emitted by the luminaries. In his understanding of vision, the atoms of light were reflected by an illuminated object and impacted the retina to form a picture of it. That is to say: he rejected the flying effigies of the ancient atomists; he adapted his atomism to Alhazen's idea that light was responsible for vision; and he accepted Kepler's analysis of the eye. Beeckman's diary contains numerous allusions to this view. For instance, in 1616 he wrote: "[In the hearing process] the air itself … strikes our ear, in the same manner as the flame of a candle is dispersed through the entire room and [then] is called *light*." Thus, light and sound were equally material. In the same entry, Beeckman made clear that tiny particles of the air (torn off

and projected by the sounding body) conveyed sound, and that air was much coarser than light. In an entry of 1618, he explained refraction by a differential adherence of the "globules of rays" with the globules of the refracting surface. In other occasions he used the expression "atoms of light."[1]

The French astronomer and philosopher Pierre Gassendi met Beeckman in the late 1620s, and judged him to be "the best philosopher I have yet encountered." Gassendi criticized Descartes's pretense to reach truth by introspection, and opted for a revised Epicurean atomism as the best hypothesis for interpreting our sensorial experience. In his turgid *Physica*, written toward the middle of the century, he defined light as follows:

> It appears that light in a luminous body is nothing but tiny corpuscles which, configured in a certain configuration, then transferred from this body with an extreme velocity, and received by the organ of vision, are able to move this organ and to create the sensation called vision.

Gassendi then defined rays as the rectilinear trajectory of the corpuscles, explained reflection and refraction by analogy with the motion of a ball at the border between two porous media with unequal amounts of pores, colors as an Aristotelian mixture of light and shadow. Regarding the process of vision, he accepted the essentials of Kepler's theory, although he had earlier speculated with his patron Nicolas-Claude Fabri Peiresc that the crystalline served to correct the inversion of the retinal image.[2]

In England, Walter Charleton's *Physiologia* of 1654 propagated Gassendi's ideas, with a few nuances. Charleton's definition of vision was slightly more Epicurean than Gassendi's, as Charleton hesitated to abandon the idea of particles ejected from the surface of luminous bodies:

> The SIGHT ... discerns the exterior Forms of Objects, by the reception either of certain *Substantial*, or *Corporeal Emanations*, by the sollicitation of *Light* incident upon, and reflected from them, as it were Direpted from their superficial parts, and trajected through a diaphanous Medium, in a direct line to the eye: or, of *Light it self*, proceeding in streight lines from Lucid bodies, or in reflex from opace, in such contextures, as exactly respond in order and position of parts, to the superficial Figure of the object, obverted to the eye.

The analogy with sound induced him to favor the second alternative:

> As it is the property of *Light*, transfigured into colours, to represent the different Conditions and Qualities of bodies in their superficial parts, according to the different Modification and Direction of its rayes, either simply or frequently reflexed from them, through the Aer, to the Eye: so is it the property of *Sounds* to represent the different Conditions and Qualities of bodies, by the mediation of the Aer percussed

[1] Beeckman 1939–1953, vol.1, pp. 92 (1616), 211 (1618); vol. 2, p. 240 (1623, atoms of light); also vol. 1, p. 28 (light the agent of vision), and vol. 3, p. 49 (Lucretius's effigies refuted).

[2] Gassendi to Peiresc [late 1620s], in Tannizey de Larroque 1688–1698, vol. 4, pp. 178–81; Gassendi [1649], vol. 1, pp. 422–32 (*De luce*), 441–9 (*De simulacris*), citation on p. 422; vol. 2, pp. 369–82 (*De visu et visione*), esp. pp. 380–1 (Kepler's theory); 1642, pp. 16–18 (redressed retinal image), 59 (light corporeal). Cf. Brett 1908, pp. 73–8; Fischer 2005a, 2005b, pp. 33–7; Lodoro 2007, pp. 69–72.

> and broken by their violent superficial impaction, or collision, and configurate into swarms of small consimilar masses, accomodable to the Ear.

Charleton also hesitated on the nature of colors: in one section of his treatise he made them a contamination of the reflected light by particles of the surface of the illuminated body; in another he adopted the Aristotelian idea of a fine mixture of light and shadow.[3]

Atomism being commonly regarded as a threat to Christianism, Gassendi and Charleton strove to demonstrate the compatibility of their philosophy with the existence of God and the immortality of the soul. So too did the British supporters of Descartes's variety of mechanical philosophy. The Cambridge neo-Platonist Henry More praised Descartes's divide between *res cogitans* and *res extensa*, as he took it to confirm the Platonic notion of the immateriality of the soul. In a treatise of 1659 on this subject he nonetheless rejected Descartes's identification of matter with extension as well as the reduction of every interaction to the contact of rigid particles. More regarded matter as the continuous union of "indescerpible" (indivisible) parts with no definite shape. A space-filling natural spirit accounted for the interaction of these parts.[4]

3.2 Newton's early investigations

Globules and colors

In philosophical notes written around 1664, Isaac Newton paraphrased Charleton's deduction of the necessity of atoms, praised More's argument for the existence of indescerpible parts, and criticized much of Descartes's *Principia*. Newton agreed with Charleton that matter was necessarily composed of a finite number of indivisible parts with vacuous interstices and not of Descartes's space-filling elements. He rejected the Cartesian explanation of light as pressure between the globules of the second element by arguing that, if it were true, light would be generated when the globules were pressed by gravity, by the motion of the earth, or even by the motion of the observer. In a later manuscript on gravitation and the equilibrium of fluids, he showed that pressure was transmitted uniformly through the whole mass of a fluid in equilibrium. This result implicitly contradicted Descartes's explanation of the rectilinear propagation of light.[5]

When Newton began his researches on colors, he had already excluded Descartes's theory and adopted the atomist view of light as a stream of "globules" moving very quickly in straight lines. Accordingly, he reinterpreted Descartes's proofs of the laws of reflection and refraction in terms of the actual motion of individual corpuscles of light. In this view, refraction should alter the velocity of the corpuscles of light. In the earliest optical experiments described in the philosophical notes of 1664, Newton used a prism as a velocity analyzer of the corpuscles of light. From observations of a paper painted in two different colors through a prism, he concluded that color corresponded to the velocity (or

[3]Charleton 1654, pp. 136 (citation), 145 (colors), 209 (citation), 191 (colors). Cf. Kargon 1964.

[4]More 1659. Cf. Henry 2007.

[5]Newton [c. 1664], pp. 1–3 (atoms), 32 (against Descartes on light); *De gravitatione* [c. 1668?]. On the roots of Newton's atomism, cf. Westfall 1962. On his criticism of Descartes, cf. Shapiro 1974.

momentum) of the particles as they struck the retina while white and black corresponded to heterogeneous velocity. He sketched two theories of the colors of bodies, one in which color was produced by enhancement of a velocity class among particles of equal mass, another in which it resulted from enhancement of a momentum class among particles of different masses and equal initial velocity. At that stage, Newton already had the idea that white light was a heterogeneous mixture of lights of different colors and that a prism could be used to separate the various colored components.[6]

In a later manuscript "Of colours," Newton described a more direct consequence of this idea. He passed a narrow beam of sunlight, selected by a small hole on the shutter of his window, through a prism, and observed the resulting spot of light on a distant wall (see Fig. 3.1). The spot did not have the circular shape it would have had if all rays had been equally refracted (the prism being in the position of minimum deviation for which the angles of incidence and emergence are equal). It had an elongated shape and displayed a series of colors of which Newton estimated the characteristic refractions. In a posterior experiment, which he would later call *experimentum crucis*, he showed that the resulting colored rays had a well-defined refraction by a second prism (see Fig. 3.2), by an amount

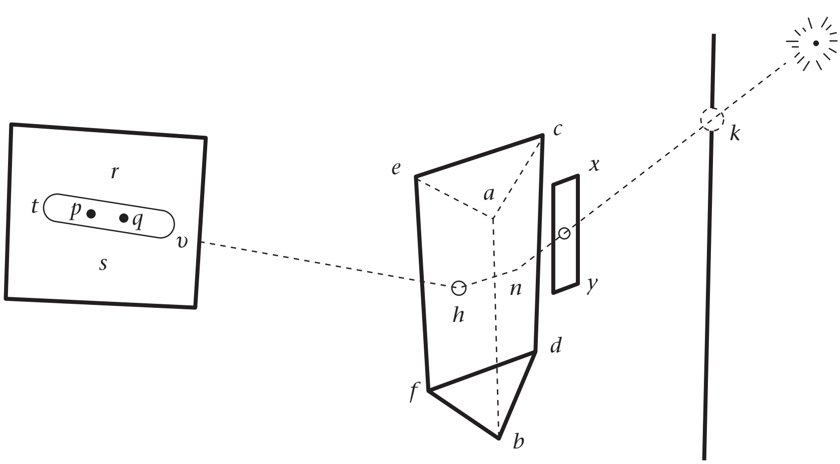


Fig. 3.1. Newton's setup for the spectrum of white light. From Newton [c. 1666], 2. Courtesy of *The Newton Project* (http://www.newtonproject.sussex.ac.uk). The light from the sun passes through the hole *k* in the blinds of the window and through the diaphragm *xy*. The resulting beam is deflected by the prism *eacbdf* (in the position of minimal deflection) and produces the oblong spectrum *tv* on the screen.

[6]Newton, "Of colours," in Newton [c. 1664], pp. 63–8. Cf. Westfall 1980, pp. 156–61. A large number of Newton's manuscripts are available through the excellent *Newton Project* directed by Rob Iliffe, http://www.newtonproject.sussex.ac.uk .The mechanisms that Newton then imagined to explain the colors of bodies allowed for a change of velocity of momentum of the globules of the reflected light, and therefore did not comply with the immutability of simple colors.

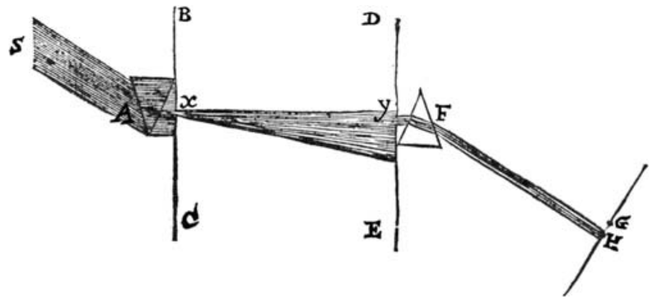


Fig. 3.2. Newton's *experimentum crucis*.[8] The light S from a small opening on the window of a dark room passes through the prism A and the hole x in the diaphragm BC. The portion of this beam selected by the second diaphragm DE enters the prism F at well-defined incidence and emerges at a well-defined angle, as verified from the narrowness of the spot on the screen GH. This angle depends on the simple color selected by the hole y. From Newton 1672d, p. 5016.

depending on the color. Also, he obtained white light by superposing the various spectral colors.[7]

In the same manuscript "Of colours," Newton described experiments with two prisms firmly pressed against each other in which he accidentally observed the colored fringes produced by trapped bubbles of airs. In order to control the thickness of the interstitial air, he used a convex lens pressed on a glass plate. He explained the resulting colored rings by "vibrations of the medium" with a well-defined "thickness of pulses" for each simple color. He presumably had in hand the essentials of his later theory of thin plates, according to which the corpuscles of light produce periodic progressive waves in the film of air or ether when entering it. The corpuscles are either reflected or transmitted by the second air/glass interface, depending on the length of the waves. These considerations are probably contemporary to Newton's reading Hooke's *Micrographia* (1665), although Newton does not cite Hooke in this regard.[9]

Newton's idea of waves by impact of bodies (the corpuscles of light) on an interface probably derived from analogy with the waves produced by a stone thrown into water or with the sound produced by hitting a bell. That Newton had an acoustic analogy in mind can be inferred from his contemporary analysis of the way visual information is transferred from the retina to the brain:

> Light seldom striks upon the parts of grosse bodys (as may bee seen in its passing through them), its reflection & refraction is made by the diversity of æthers, & therefore

[7]Newton [c. 1666], pp. 2, 12. Cf. Shapiro 1984, "Introduction"; Westfall 1980, pp. 118, 166–74. According to Newman [2011], Newton's new emphasis on the immutability of simple colors and on the synthesis of white light resulted from his familiarity with Boyle's anti-Aristotelian concept of chemical mixture, in which the corpuscles of the mixed substances survived the mixing.

[8]In the *experimentum crucis* the prisms need not be in the position of minimum deflection because the holes *x* and *y* select a precise ray between the two prisms. The color of this ray is simple to the extent that the direction of the rays of the incoming sunbeam is well defined.

[9]Newton [c. 1666], pp. 9–11, 21. Cf. Shapiro 1993, pp. 8–12, chap. 2.

> its effect on the Retina can only bee to make this vibrate which motion then must bee either carried in the optick nerve to the sensorium or produce other motions that are carried thither ... [This motion] can noe way bee conveyed to the sensorium so entirely as by the æther it selfe. Nay granting mee but that ther are pipes filld with a pure transparent liquor passing from the ey to the sensorium & the vibrating motion of the æther will of necessity run along thither. Ffor nothing interrupts that motion but reflecting surfaces, & therefore also that motion cannot stray through the reflecting surfaces of the pipe but must rush along (like a sound in a trunk) intire to the sensorium. And that vision bee thus made is very conformable to the sense of hearing which is made by like vibrations.

This citation shows that Newton believed that both vision and hearing implied the excitation of the vibrations of some part of the relevant organ, and the propagation of this vibration to the brain through a channeled medium. Nothing more can be said on his understanding of ether or air waves, which was probably precarious at this early stage of his natural philosophy.[10]

To the Royal Society

Newton integrated these results in the Optical lectures that he delivered in 1670–1672 from the Lucasian chair in which he succeeded Isaac Barrow. As an evident consequence of his theory, optical lenses have different foci for each spectral component of white light, a defect now called chromatic aberration. In late 1671, Barrow brought to the Royal Society a telescope that Newton had built with a spherical mirror instead of the objective lens, in order to avoid this defect (see Fig. 3.3). This instrument caused a sensation, and the

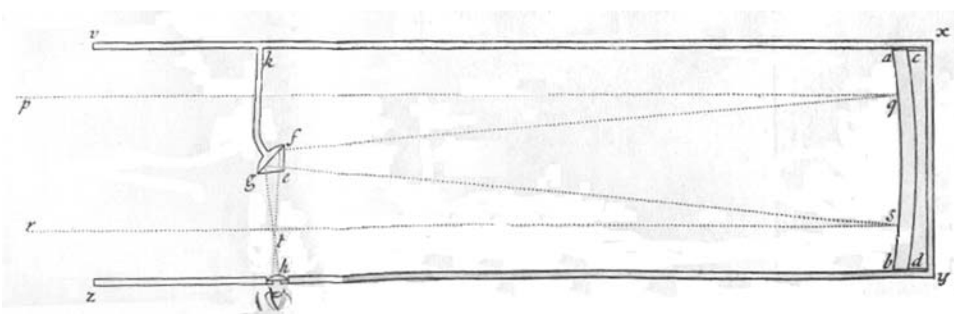


Fig. 3.3. Newton's refractor. The light beam pr from a distant object is reflected on the glass mirror acbd (the convex side cd is silvered) to converge on the glass prism fge. After reflection at the bottom fg of this prism the light passes through a semi-convex lens whose focus f coincides with the focus of the mirror-prism system. From Newton 1704, book 1, part 1, fig. 29.

[10]Newton [c. 1666], pp. 19–20. As can be judged from later writings (1675), the first obscure sentence of the citation means that light cannot travel along the curved path of a nerve by repeated scattering by the particles of the liquor contained in the nerves, because such scattering would imply opacity (besides, Newton explains reflection and refraction by a change of ether density, not by collisions between light corpuscles and material corpuscles, because this process would no yield a well-defined direction of reflection or refraction).

Society elected Newton on 11 January 1872. On 18 January, Newton promised to its secretary Henry Oldenburg soon to give the theory that had induced him to construct the telescope: “the oddest, if not the most considerable detection w^{ch} hath hitherto beene made in the operations of Nature.” Newton kept his word in the famous letter of 6 February 1672 that Oldenburg printed in the *Transactions* of the Society under the title “New theory about light and colours.”[11]

Newton's letter begins with the first spectrum experiment:

> SIR,
> **TO** perform my late promise to you, I shall without further ceremony acquaint you, that in the beginning of the Year 1666 (at which time I applyed my self to the grinding of Optick glasses of other figures than *Spherical*,) I procured me a Triangular glass-Prisme, to try therewith the celebrated *Phænomena* of *Colours*. And in order thereto having darkened my chamber, and made a small hole in my window-shuts, to let in a convenient quantity of the Suns light, I placed my Prisme at his entrance, that it might be thereby refracted to the opposite wall. It was at first a very pleasing divertisement, to view the vivid and intense colours produced thereby; but after a while applying my self to consider them more circumspectly, I became surprised to see them in an *oblong* form; which, according to the received laws of Refraction, I expected should have been *circular*.

Newton then carefully eliminated “suspicions” that this effect might be caused by irregularities in the glass of the prism, the finite size of the sun, or a curvature of the emerging rays. He went on:

> The gradual removal of these suspitions, at length led me to the *Experimentum Crucis*, which was this: I took two boards, and placed one of them close behind the Prisme at the window, so that the light might pass through a small hole, made in it for the purpose, and fall on the other board, which I placed at about 12 feet distance, having first made a small hole in it also, for some of that Incident light to pass through. Then I placed another Prisme behind this second board, so that the light, trajected through both the boards, might pass through that also, and be again refracted before it arrived at the wall. This done, I took the first Prisme in my hand, and turned it to and fro slowly about its *Axis*, so much as to make the several parts of the Image, cast on the second board, successively pass through the hole in it, that I might observe to what places on the wall the second Prisme would refract them. And I saw by the variation of those places, that the light, tending to that end of the Image, towards which the refraction of the first Prisme was made, did in the second Prisme suffer a Refraction considerably greater than the light tending to the other end. And so the true cause of the length of that Image was detected to be no other, than that *Light* consists of *Rays differently refrangible*, which, without any respect to a difference in their incidence, were, according to their degrees of refrangibility, transmitted towards divers parts of the wall.

[11]Newton to Oldenburg, 18 January 1672, in Turnbull 1959–77, vol. 1, p. 82; Newton 1672a. Cf. Westfall 1980, pp. 232–7. For the optical lectures, cf. Shapiro 1984. Newton did not doubt that the sine law of refraction applied to simple colors, although his experimental proof of this point was untypically floppy: cf. Lohne 1961.

Newton briefly described the chromatic aberration of lenses, and he recalled his success in building a reflecting telescope in which this cause of aberration did not exist. He formulated the heterogeneity of white light independently of the corpuscular interpretation: "Colours are not *Qualifications of Light*, derived from Refractions, or Reflections of natural Bodies (as 'tis generally believed,) but *Original* and *connate properties*, which in divers Rays are divers." His only allusion to the corporeality of light was brief and cautious:[12]

> These things being so, it can no longer be disputed, whether there be colours in the dark, nor whether they be the qualities of the objects we see, no nor perhaps, whether Light be a Body. For, since Colours are the *qualities* of Light, having its Rays for their intire and immediate subject, how can we think those Rays *qualities* also, unless one quality may be the subject of and sustain another; which in effect is to call it *Substance*. We should not know Bodies for substances, were it not for their sensible qualities, and the Principal of those being now found due to something else, we have as good reason to believe that to be a Substance also.
>
> Besides, whoever thought any quality to be a *heterogeneous* aggregate, such as Light is discovered to be. But, to determine more absolutely, what Light is, after what manner refracted, and by what modes or actions it produceth in our minds the Phantasms of Colours, is not so easie. And I shall not mingle conjectures with certainties.

3.3 Early response

Pardies and Huygens

The most successful of the results announced in this letter was the reflecting telescope, which the members of the Royal Society had already tested, and which Newton described in a subsequent letter. No one denied that Newton had produced the first working telescope of this kind, although James Gregory and Laurent Cassegrain had earlier made similar proposals. Newton's considerations on colors soon underwent criticism from three authorities in optics: Pardies, Huygens, and Hooke. Pardies politely suggested that the divergence of the refracted beam in Newton's first prism experiment might result from ordinary refraction. He promptly apologized after Newton told him that his publication already contained the answer: in the selected position of minimum deflection for the prism, the two refractions at the prism preserve the parallelism of the beam.[13]

After some initial praise, Huygens reproached Newton with introducing a continuous spectrum of simple colors as he agreed with Hooke that two base colors (yellow and blue) should be sufficient to generate all other colors. He added that Hooke's assumption would be easier to explain mechanically than Newton's. Newton replied that the simplicity of every component of the spectrum was an incontrovertible fact of experiment, and that the corpuscular hypothesis provided an easy mechanical interpretation of this diversity if the simple colors corresponded to the size or velocity of the corpuscles. The latter argument failed to convince Huygens, who favored a wave theory with none of the periodicity that

[12] Newton 1672a, p. 3081.

[13] Newton 1672b (reflector); Pardies 1672a, 1672b; Newton 1672c, 1672d.

could have represented simple colors. Nevertheless, he soon accepted Newton's experimental results as well as his analysis of chromatic aberration.[14]

A debate with Hooke

In a condescending letter, Hooke approved Newton's experiments so much as to suggest that he knew them all in advance. He nonetheless rejected the heterogeneity of white light and the corpuscular hypothesis. A clever acoustic analogy inspired his criticism:[15]

> But why there is a necessity, that all those motions, or whatever else it be that makes colours, should be originally in the simple rays of light, I do not yet understand the necessity of, no more than that all those sounds must be in the air of the bellows, which are afterwards heard to issue from the organ-pipes; or in the string, which are afterwards, by different stoppings and strikings produced; which string (by the way) is a pretty representation of the shape of a refracted ray to the eye; and the manner of it may be somewhat imagined by the similitude thereof: for the ray is like the string, strained between the luminous object and the eye, and the stop or fingers is like the refracting surface, on the one side of which the string hath no motion, on the other a vibrating one. Now we may say indeed and imagine, that the rest or streightness of the string is caused by the cessation of motions, or coalition of all vibrations; and that all the vibrations are dormant in it: but yet it seems more natural to me to imagine it the other way.

In his haughty reply, Newton maintained that the heterogeneity of white light was a direct consequence of his experiments and that it did not depend on any hypothesis on the nature of light. He added that his own preferred hypothesis, that light was made of corpuscles, was not so different from the objector's view:[16]

> For certainly it has a much greater affinity with his own *Hypothesis*, than he seems to be aware of; the Vibrations of the *Æther* being as useful and necessary in *this*, as in *his*. For, assuming the Rays of Light to be small bodies, emitted every way from Shining substances, those, when they impinge on any Refracting or Reflecting superficies, must as necessarily excite Vibrations in the *æther*, as Stones do in water when thrown into it. And supposing these Vibrations to be of several depths or thicknesses, accordingly as they are excited by the said corpuscular rays of various sizes and velocities; of what use they will be for explicating the manner of Reflection and Refraction, the production of Heat by the Sun-beams, the Emission of Light from burning, putrefying, or other substances, whose parts are vehemently agitated, the *Phænomena* of thin transparent Plates and Bubles, and of all Natural bodies, the Manner of Vision, and the Difference of Colors, as also their Harmony and Discord.

For the sake of comparison, Newton characterized Hooke's hypothesis as follows:

> *That the parts of bodies, when briskly agitated, do excite Vibrations in the Æther, which are propagated every way from those bodies in streight lines, and cause a Sensation of Light by beating and dashing against the bottom of the Eye, something after the manner*

[14]Huygens 1673a, 1673b; Newton 1673a, 1673b.

[15]Hooke [1672], p. 11. On this exchange, cf. Westfall 1980, pp. 241–7.

[16]Newton 1672e, p. 5087.

> *that Vibrations in the Air cause a Sensation of Sound by beating against the Organs of Hearing.*

He then proceeded to demonstrate that this hypothesis, if properly developed, justified his conception of white light and all his other considerations on colors:[17]

> Now, the most free and natural Application of this *Hypothesis* to the Solution of *phænomena* I take to be this: *That* the agitated parts of bodies, according to their several sizes, figures, and motions, do excite Vibrations in the *æther* of various depths or bignesses, which being promiscuously propagated through that *Medium* to our Eyes, effect in us a Sensation of Light of a *White* colour; but if by any means those of unequal bignesses be separated from one another, the largest beget a Sensation of a *Red* colour, the least or shortest, of a deep *Violet*, and the intermediat ones, of intermediat colors; much after the manner that bodies, according to their several sizes, shapes, and motions, excite vibrations in the Air of various bignesses, which, according to those bignesses, make several Tones in Sound: *That* the largest Vibrations are best able to overcome the resistance of a Refracting superficies, and so break through it with least Refraction; whence the Vibrations of several bignesses, that is, the Rays of several Colors, which are blended together in Light, must be parted from one another by Refraction, and so cause the *Phænomena* of *Prismes* and other refracting substances: And *that* it depends on the thickness of a thin transparent Plate or Buble, whether a Vibration shall be *reflected* at its further superficies, or *transmitted*; so that, according to the number of vibrations, interceding the two superficies, they may be reflected or transmitted for many successive thicknesses. And since the Vibrations which make *Blew* and *Violet*, are supposed shorter than those which make *Red* and *Yellow*, they must be reflected at a less thickness of the Plate: Which is sufficient to explicate all the ordinary *phænomena* of those Plates or Bubles, and also of all natural bodies, whose parts are like so many fragments of such Plates.

This remarkable sketch of the wave theory of light contains the first known suggestion that frequency is the parameter of color, based on analogy with the pitch of sounds. More exactly, Newton speaks of the "bigness" of the vibrations. Under this word, he certainly meant to include the wavelength, as the analogy with sound and the interpretation of the colors of thin plates in terms of this bigness clearly indicate. He also meant the amplitude of the vibration, as indicated by the proposed interpretation of dispersion. The reason for this seeming contradiction (which has puzzled many commentators) is to be found in the analogy with water waves. The waves created by throwing a stone into water have lengths and amplitudes both growing with the size of the stone, so that early wave theorists commonly assumed a correlation between length and amplitude.[18]

After arguing the compatibility of the wave hypothesis with his theory of colors, Newton proceeded to refute this hypothesis:

> For, to me, the Fundamental Supposition it self seems impossible; namely, That the *Waves* or Vibrations of any Fluid, can, like the Rays of Light, be propagated in *Streight* lines, without a continual and very extravagant spreading and bending every

[17] Ibid., p. 5088. [18] Cf. Sabra 1963; Blay 1980.

> way into the quiescent Medium, where they are terminated by it … What I have said of this, may easily be applied to all other *Mechanical Hypotheses*, in which Light is supposed to be caused by any Pression or Motion whatsoever, excited in the *æther* by the agitated parts of Luminous bodies. For, it seems impossible, that any of those Motions or Pressions can be propagated in *Streight* lines without the like spreading every way into the shadow'd Medium, on which they border.

Newton here expressed for the first time what he believed to be the definitive objection against any medium theory. As we saw, he probably reached this conclusion in a critical analysis of the concept of pressure in Descartes's *Principia*.[19]

In his next paragraph, Newton insisted that his theory of colors did not require any specific hypothesis on the nature of light. Somewhat surprisingly, he went on with an acoustic analogy for the colors of bodies:

> For if *Light* be consider'd abstractly without respect to any *Hypothesis*, I can *as* easily conceive, that the several parts of a shining body may emit rays of differing colours and other qualities, of all which Light is constituted, *as* that the several parts of a false or uneven string, or of uneavenly agitated water in a Brook or Cataract, or the several Pipes of an Organ inspired all at once, or all the variety of Sounding bodies in the world together, should produce sounds of several Tones, and propagate them through the Air confusedly intermixt. And, if there were any natural bodies that could *reflect* sounds of one tone, and stifle or *transmit* those of another; then, as the *Echo* of a confused Aggregat of all Tones would be that particular Tone, which the Echoing body is disposed to reflect; so, since (even by the *Animadversor's* concessions) there are bodies apt to *reflect* rays of one colour, and stifle or *transmit* those of another; I can as easily conceive, that those bodies, when illuminated by a mixture of all colours, must appear of that colour only which they reflect.

The reason why Newton here develops an analogy that would seem to favor the wave hypothesis becomes clear in the next few lines, in which he condemns Hooke's use of the acoustic analogy to disprove the heterogeneity of white light:[20]

> But when the *Objector* would insinuate a difficulty in these things, by alluding to Sounds in the string of a *M*usical instrument before percussion, or in the Air of an Organ Bellowes before its arrival at the Pipes; I must confess, I understand it as little, as if one had spoken of Light in a piece of Wood before it be set on fire, or in the oyl of a Lamp before it ascend up the match to feed the flame.

While Newton here caught a genuine weakness of Hooke's analogy, he failed to see that Hooke had unveiled a genuine logical error in his deduction of the heterogeneity of light. Newton's three basic facts, the splitting of a ray of white light by a prism, the stability of the resulting colored rays, and the possibility of synthesizing white light by superposing these colored rays, by no means imply that white light should be a mixture of these rays. As Hooke's own theory of colors already suggested, the superposition of lights of different colors does not need to preserve the individual properties of these lights. Hooke made

[19]Newton 1672c, p. 5089. [20]Ibid., p. 5091.

these points in an attempted reply to Newton. He also rehearsed the string and organ analogies, without answering Newton's objection (that the air from the bellow, not being a sound, could not be compared to white light), but adding a clever comparison between the refraction of simple colors and the resonance of pure tones:[21]

> I may as well conclude that all the sounds that were produced by the motion of the strings of a Lute were in the motion of the musitians fingers before he struck them, as that all colours wch are sensible after refraction were actually in the ray of light before Refraction. All that he doth prove by his *Experimentum crucis* is that the colourd Radiations doe incline to ye Ray of light wth Divers angles, and that they doe persevere to be afterwards by succeeding mediums diversly refracted one from an other in the same proportion as at first, all wch may be, and yet noe colourd ray in the light before refraction; noe more than there is sound in the air of the bellows before it passt through the pipes of ye organ—for A ray of light may receive such an impression from the Refraction medium as may distinctly characterize it in after Refractions, in the same manner as the air of the bellows does receive a distinct tone from each pipe, each of which has afterwards a powere of moving an harmonious body, and not of moving bodys of Differing tones.

In the modern wave optics based on Fourier analysis, the superposition of periodic signals may well be a signal that does not exhibit any periodicity whatsoever, and yet spectroscopes are able to separate the abstractly superposed components of this signal. Hooke's reasoning anticipated this point. Newton's and others' failure to appreciate it presumably derived from their unconscious integration of a minimal consequence of every corpuscular theory of light: any stable property of light should be represented as a stable individual property of the corpuscles. They implicitly reasoned as follows: since simple colors are stable (the color cannot be altered by any optical device), they must correspond to a stable, preexisting property of the individual corpuscles of light; therefore, white light is heterogeneous.[22]

3.4 An hypothesis

Corpuscles and ethers

Newton resumed his investigation of the colors of thin films in 1671–72 and communicated the improved results and the accompanying theory in 1675 to the Royal Society. On this occasion, he no longer refrained from hypotheses and developed a theory in which light corpuscles and ether waves both played a role. On the corpuscular or ray side, he assumed that light consisted of "swift corpuscles" or "any impulse or motion of any other medium" that propagated in rays. At any rate, light could not be ethereal vibrations, because these would fail to explain rectilinear propagation, the existence of opaque bodies, and the periodicity of the colors of thin films. The acoustic analogy illustrated the two first impossibilities: "Were [light] these vibrations, it ought always to verge copiously in

[21] Hooke to Lord Brouncker, June 1672, in Turnbull 1959–77, vol. 1, pp. 198–203, on pp. 202–3.

[22] This minimal consequence of the corpuscular view is what Buchwald 1989 (pp. xviii, 50–1) (following Thomas Young) called "selectionism" in the context of early nineteenth-century optics.

crooked lines into the dark or quiescent medium, destroying all shadows; and to comply readily with any crooked pores or passages, as sounds do."[23]

Newton nevertheless gave many reasons to assume a pervasive, multi-component ether in the explanation of thermal, gravitational, electric, magnetic, chemical, physiological, and optical phenomena. In the optical context, he explained the refraction of light by a different density of the ether contained in different substances. As in Descartes's reasoning (to which Newton referred), this difference implies a perpendicular force curving the rays in the vicinity of the "aethereal superficies" between two media. Newton then associated the refrangibility of a simple color with the "bigness or strength" of the corresponding rays. The violet rays, which are the most refracted, must be the weakest; the red ones the strongest. As refraction does not change the bigness, color cannot be altered by any further refraction.[24]

Ether waves

Newton then introduced ether waves as a means of explaining partial reflection:

> And for explaining this, I suppose, that the rays, when they impinge on the rigid resisting æthereal superficies, as they are acted upon by it, so they react upon it and cause vibrations in it, as stones thrown into water do in its surface; and that these vibrations are propagated every way into both the rarer and the denser mediums; as the vibrations of air, which cause sound, are from a stroke, but yet continue strongest where they began, and alternately contract and dilate the æther in that physical superficies … And so supposing that light, impinging on a refracting or reflecting æthereal superficies, puts it into a vibrating motion, that physical superficies being by the perpetual appulse of rays always kept in a vibrating motion, and the æther therein continually expanded and compressed by turns; if a ray of light impinge upon it, while it is much compressed, I suppose it is then too dense and stiff to let the ray pass through, and so reflects it; but the rays, that impinge on it at other times, when it is either expanded by the interval of two vibrations, or not too much compressed and condensed, go through and are refracted.

Newton here assumed more analogy between air and the ether than anyone had done before. Earlier in the same text he wrote: "There is an æthereal medium much of the same constitution with air, but far rarer, subtler, and more strongly elastic." Moreover, he clearly related sound and its propagation to the compressibility and elasticity of the air, a rare insight at that time.[25]

Newton next elaborated his earlier idea that light excited vibrations of the retina transmitted to the brain through the substance of the optical nerve:

[23]Newton [1675], pp. 254–5.

[24]Ibid., pp. 255–8. On p. 263, Newton writes: "And, because refraction only severs them, and changes not the bigness or strength of the rays, thence it is, that after they are once well severed, refraction cannot make any further changes in their colour." Thus, he implicitly identifies the "bigness" with the mass of the corpuscles. On Newton's ether, see also Newton to Boyle, 28 February 1679, in Birch 1756–7, vol. 1, pp. 70–3. The variety of spirits included in Newton's aether and its tensional quality may have reflected his alchemical concerns, although his optical ether was mostly mechanical: cf. Westfall 1980, pp. 269–71.

[25]Newton [1675], pp. 258, 253. See also Newton to Oldenburg, 10 January 1676, in Turnbull 1959–77, vol. 2: "That aether is a finer degree of air and air a vibrating Medium are old notions and ye principles I go upon."

> I suppose, that as bodies of various sizes, densities, or sensations, do by percussion or other action excite sounds of various tones, and consequently vibrations in the air of various bigness; so when the rays of light, by impinging on the stiff refracting superficies, excite vibrations in the æther, those rays, whatever they be, as they happen to differ in magnitude, strength or vigour, excite vibrations of various bigness ... And therefore the ends of the capillamenta of the optic nerve, which pave or face the retina, being such refracting superficies, when the rays impinge upon them, they must there excite these vibrations, which vibrations (like those of sound in a trunk or trumpet) will run along the aqueous pores or crystalline pith of the capillamenta through the optic nerves into the sensorium (which light itself cannot do) and there, I suppose, affect the sense with various colours, according to their bigness and mixture; the biggest with the strongest colours, reds and yellows; the least with the weakest, blues and violets; the middle with green, and a confusion of all with white, much after the manner, that in the sense of hearing, nature makes use of aereal vibrations of several bignesses to generate sounds of divers tones; for the analogy of nature is to be observed.

Newton here emphasized the acoustic analogy and related it to a general principle, "the analogy of nature," which he would often evoke in his philosophy. As a reflection of God's perfection, nature had to be conformable to itself, to present all sorts of analogies between different phenomena and different scales.[26]

The music of colors

In this frame of mind, Newton did not hesitate to revive the Pythagorean analogy between the harmonies of colors and sounds:

> And further, as the harmony and discord of sounds proceed from the proportions of the aereal vibrations, so may the harmony of some colours, as of golden and blue, and the discord of others, as of red and blue, proceed from the proportions of the æthereal. And possibly colour may be distinguished into its principal degrees, red, orange, yellow, green, blue, indigo, and deep violet, on the same ground, that sound within an eighth is graduated into tones.

Newton believed his observations of the spectrum of white light to confirm this musical pattern. He drew the diagram of Fig. 3.4, in which the lines separating two different colors are placed as the frets of a monochord that would yield the notes DEFGABC of the modern English scale. He had earlier reflected on the best means to divide the octave, although his interest in this problem was more mathematical than musical.[27]

[26]Newton [1675], p. 262. See also Newton to Briggs, 25 April 1685, in Turnbull 1959–77, vol. 2, pp. 417–19: "Nature is after all simple, and is normally self-consistent throughout an immense variety of effects, by maintaining the same mode of operation. But how much more so in the causes of the related senses?" On Newton's "analogy of nature," cf., e.g., Shapiro 1993, pp. 43–4.

[27]Newton [1675], 262–3; Newton [c. 1665], ff. 137r–143v; Musical calculations, Cambridge University Library, Add. 4000, ff. 104r–113v and Add. 3958 (B), f. 31r. Cf. Wardhaugh 2006, pp. 103–7, 253–9; McGuire and Rattansi 1966; N. Hutchison 2004; Gouk 1986, 1988, 1999 (chap. 7). Hooke had compared musical and chromatic harmony a few weeks earlier at the Royal Society: see above, chap. 2, p. 55. Newton also used a musical division for the colors of thin plates: cf. Shapiro 1993, pp. 89–92, 174.

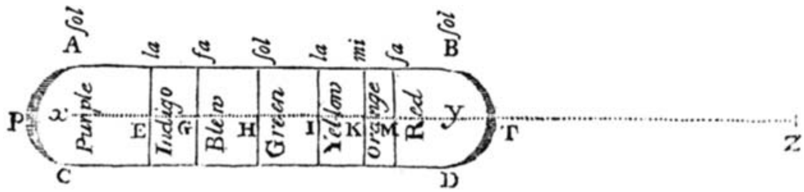


Fig. 3.4. Newton's analogy between spectrum and musical scale. From Newton [1675], p. 262. The points x, E, G, H, I, K, M mark the ends of a vibrating string beginning at z. The corresponding sounds (marked in a contemporary notation that has nothing to do with the present naming of notes in Latin languages; DEFGABC in the modern English scale) yield the limits between two successive colors of the spectrum ABCD. For instance, y is an octave higher than x and I is a fifth higher than x because yz/xz = 1/2 and Iz/xz = 2/3.

Newton then described his investigation of the colors of thin plates, and the ensuing theory of the colors of bodies. He did so to greater length and precision in another paper of the same year. As in his manuscripts of the mid-1660s, he imagined the impact of the rays on the first surface of the plate to produce ether waves. Transmission or reflection of the rays at the second surface depended on whether the waves, having reached this surface faster than the rays, produced a condensation or a rarefaction of the ether in its neighborhood. For the colors of bodies, Newton recycled Hooke's analogy with the colors of thin plates. He imagined the bodies to be made of transparent particles immersed in a medium of smaller optical index. This picture provided a link between the size of the particles of bodies and their color, in the spirit of Boyle's *Touching colours* of 1664. It implied that colors by transmission should be complementary to colors by reflection and was therefore compatible with Newton's more phenomenological theory of 1666, according to which the colors resulted from selective reflection or transmission of the simple components of white light.[28]

Diffraction

Newton briefly discussed diffraction, about which he knew from Honoré Fabri's account of Grimaldi's discovery and from Hooke's more recent experiments on this matter. Despite Grimaldi's evidence to the contrary and despite Hooke's comparison with the "straying of sound," Newton argued that diffraction was only a special case of refraction, caused by ethereal atmospheres near the surface of bodies, and perhaps involving thin-film effects. Whereas Hooke took diffraction to confirm the analogy between light and sound, Newton took it to confirm the essential role of the ether in refraction. In his opinion, the

[28]Newton [1675], pp. 263–7 (rings), 268 (colors of bodies); Newton [1676]; Boyle 1664. Cf. Shapiro, 1993, chaps. 2–4. As Newton understood, the color generated by the particles does not depend much on the angle of observation if their size is of the order of the optical wavelength.

diffraction implied by the wave theory of light was so large that it completely excluded shadows.[29]

3.5 The *Opticks*

The papers that Newton read in 1675 to the Royal Society did not appear in print until 1757. Although he began to work on a systematic treatise, the controversy around his first publications so irked him that he withheld his manuscript until 1704: "To avoid being engage in disputes over this matters, I have hitherto delayed the printing, and I should still have delayed it, had not the importunity of friends prevailed upon me." This gave him time to consolidate his main results and to include new materials on thick plates, the colors of bodies, diffraction, and extraordinary refraction. In the main text of the *Opticks*, Newton refrained as much as possible from hypotheses on the nature of light, as he had done in the short communication of 1672. He favored a neutral language of rays, defined as "the least parts [of light], and those as well successive in the same lines as contemporary in several lines." He left most of his speculations on light corpuscles and ether waves to a series of appended queries whose number increased at each new edition of his treatise.[30]

Geometrical optics

Newton devoted an unusually small amount of space to what we would now call geometrical optics: eight definitions for rays, reflection, refraction, and homogeneous light, and nine "axioms" giving the laws of reflection and refraction, the condition of approximate stigmatism, and the locus of images for homogeneous light. The most remarkable features of his presentation are the restriction of the usual laws to homogeneous light (simple colors), and a definition of virtual images (axiom VIII) borrowed from Barrow: "An object seen by reflection or refraction, appears in that place from whence the rays after their last reflexion or refraction diverge in falling on the spectator's eye." Newton gave a terse justification: "For these rays do make the same picture in the bottom of the eyes as if they had come from the object really placed at [that place] … ; and all vision is made according to the place and shape of that picture."

In his optics lectures of 1670–72, Newton had integrated and even improved some of Barrow's more refined results. In his treatise, he ignored stigmatic surfaces and caustics, as these concepts became secondary in an optics dominated by chromatic dispersion and aberration. He confined himself to the basic principles: "This may suffice for an introduction to readers of quick wit and good understanding not yet versed in optics."[31]

In conformity with the phenomenological tone of his treatise, Newton gave only a brief and fragmentary derivation of the law of refraction. He assumed the refraction to be caused by a force perpendicular to the interface and depending only on the distance from

[29]Newton [1675], pp. 268–9 (diffraction, Hooke's opinion); Fabri 1669, first dialogue. Hooke announced his own discovery of diffraction (which he called "inflection") on 18 March 1675 at the Royal Society: cf. Birch, 1756–7, vol. 3, pp. 194–5, and the more detailed MS in Hooke 1705, pp. 186–90. Fabri anticipated the idea of a refracting atmosphere. Cf. Hall 1990.

[30]Newton 1704, p. 1: "Advertisement." Cf. Blay 1983; Hall, 1993.

[31]Newton 1704, pp. 1–13.

the interface. In this case, he asserted, the parallel component of "the motion or moving thing whatsoever" is conserved, and the square of the normal component changes by a constant amount. The sine law of refraction evidently follows from these two mechanical results. Newton left their derivation to his sagacious reader. They correspond to the conservation of the parallel component of the momentum, and to a simple generalization of Galileo's law of fall. Newton had already given a more geometrical proof of the sine law in his *Principia* of 1687, based on the parabolic shape of the trajectory in bands of constant force. Newton thus justified Descartes's result through a precise mechanical reasoning in which the velocities of the deflected "bodies" became identical to the velocity of light. He accepted Rømer's measurement of this velocity, and noted that it was larger in (optically) denser media.[32]

Newton's derivation of the sine law of refraction does not require any assumption about the way color is related to force, mass, and velocity, besides the implicit requirement that, for a given color, these three parameters must have a definite value. In modern terms, energy conservation yields

$$m(\upsilon'^2 - \upsilon^2) = 2\Delta$$

where υ is the initial velocity, υ' the final velocity, m the mass, and Δ the integral of the force over its range. The conservation of the parallel component of momentum yields

$$\upsilon \sin i = \upsilon' \sin r.$$

Consequently, the sine law of refraction $\sin i = n \sin r$ holds with

$$n^2 - 1 = 2\Delta/m\upsilon^2.$$

The index varies with m, υ, and Δ. In his derivation, Newton did not decide to which of these variations a change of (simple) color should be traced. The probable reason for this silence is his hesitation about the dispersion law, as we will see in a moment.

Chromatic dispersion

Newton devoted the first book of his optics to the demonstration of the heterogeneity of white light, to its consequences on the design of optical instruments, to the theory of the rainbow, and to the perception of colors. Having learned how easily he could be misunderstood, he gave detailed descriptions of numerous experiments meant to prove the heterogeneity of white light and to refute the theories that made colored light a modification of white light. These were refinements of the early experiments of 1666, as shown for instance in Figs. 3.5 and 3.6.

In applications of the dispersion of light to the theory of optical instruments, it is important to know how the index of refraction varies with the simple color and with the nature of the refracting interface. Newton addressed these questions through his musical division of the spectrum and through the following "experiment":

[32]Ibid., book 2, part 3, props. 10–11. Newton 1687, pp. 227–30 (props. 94–96), 231 (scholium).

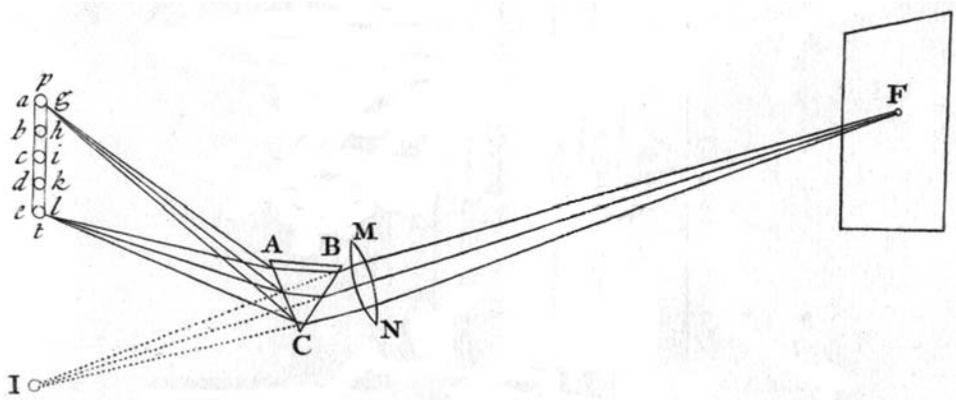


Fig. 3.5. Newton's improved contrivance for the analysis of white light. Without the prism, the lens MN would focus the light from the tiny hole F to the intersection of the dotted lines. With the prism, it would focus homogeneous light on a point of the segment pt. The tiny circles g, h, i, k, l represent the foci for five simple colors. From Newton 1704, book 1, part 1, fig. 24.

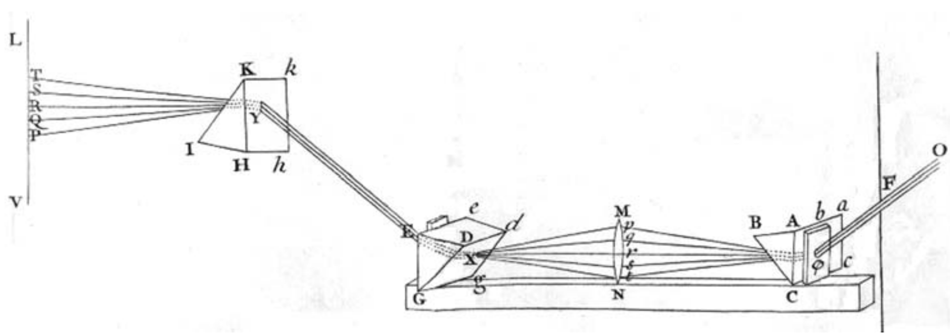


Fig. 3.6. Newton's synthesis of white light from simple colors. The prism FDG recombines the colors separated by the first prism ABC, so that the beam FY has all the properties of white light. The blocking of one simple color at the points p, q, r, s, t of the lens MN implies a missing color P, Q, R, S, T in the spectrum produced by the prism KIH. From Newton 1704, book 1, part 2, fig. 16.

I found … that when light goes out of air through several contiguous refracting mediums as through water and glass, and thence goes out again into air, whether the refracting superficies be parallel or inclin'd to one another, that light as often as by contrary refractions 'tis so corrected, that it emergeth in lines parallel to those in which it was incident, continues ever after to be white. But if the emergent rays be inclined to the incident, the whiteness of the emerging light will by degrees in passing on from the place of emergence, become tinged in the edges with colours. This I try'd by refracting light with prisms of glass placed within a prismatic vessel of water.

This means that a ray of white light cannot be deviated without loosing its whiteness. Hence achromatic lenses or prisms cannot be built. Newton "gathered" two "theorems" from this experiment. Calling n the modern index of refraction, the first theorem states that for any two colors a and b the ratio $(n_a-1)/(n_b-1)$ is the same for any refraction occurring at the interface between a given medium (say air) and another medium (water, glass, etc.). The second theorem gives the relation $n_{13} = n_{12}n_{23}$ between the indexes for the various interfaces of three media labeled 1, 2, 3.[33]

The second theorem is an immediate consequence of the Newton–Descartes derivation of the sine law of refraction, which makes the index a ratio of velocities. The first theorem is more mysterious. As Samuel Klingenstierna showed some fifty years later, it is only compatible with the "experiment" to the extent that all angles of incidence are small. It is compatible with the musical division of the spectrum; but we will now see that it does not square well with the Newton–Descartes derivation of the law of refraction, on the basis of which Newton had earlier favored another dispersion law.

As we saw, the mechanics of corpuscular refraction yields the relation

$$n^2 - 1 = 2\Delta/m\upsilon^2,$$

where m is the mass of the deflected body, υ its initial velocity, and Δ what we would now call the total variation of the potential of the deflecting force. This relation implies the universality of $(n_a^2 - 1)/(n_b^2 - 1)$ for refraction from a given medium to any other medium, granted that the force parameter Δ does not depend on the selected color.[34] Newton gave this dispersion law in his optical lectures of 1670–72. Although he did not provide the derivation, his statement of this law clearly reflects a construction in which the parallel component of velocity is conserved and the acquired perpendicular component is the same for every color at grazing incidence. The latter condition is only met if the ratio Δ/m is the same for all colors while the initial velocity υ is the parameter of color. At that time Newton favored this assumption in analogy with gravitation, for which the acceleration of a body does not depend on its mass.[35]

At some point, Newton nonetheless opted for the universality of $(n_a-1)/(n_b-1)$, presumably because he had stakes in the impossibility of achromatic lenses. It is not clear whether he ever performed the prism-in-prism experiment. He described this experiment in a draft letter to Hooke of 1672, with a result opposite to that given in the *Opticks*! His settling for the linear dispersion law may perhaps be related to its approximate validity for water and the kind of glass that Newton was using. The incompatibility with his earlier mechanical analysis did not necessarily bother him. He may have tolerated a variable Δ/m,

[33]Newton 1704, book 1, part 2, exp. 7 (musical division), exp. 8 (prism in prism). Cf. Bechler 1973, 1975; Shapiro 1979, 2005.

[34]As this parameter depends on the choice of the initial medium, the universality cannot be extended to any pair of medium. This is the probable reason why Newton keeps one of the two media constant in his first theorem.

[35]Newton (optical lectures), in Shapiro 1984, pp. 198–201, 335–7. The construction of Newton's derivation is confirmed in the MS "Of refraction," in Turnbull 1959–1977, vol. 1, p. 103. Cf. Shapiro 1984, pp. 199n–200n.

as he was aware of forces (for instance magnetic forces) for which different masses undergo different accelerations.[36]

The circle of colors

The *Opticks* goes on with a rule for determining the sensible color produced by the mixture of simple colors. In his seminal letter of 1672, Newton had already explained that the superposition of two (or more) simple colors could produce a color perceptively equivalent to another simple color but nevertheless analyzable through a prism. Hooke's and Huygens's claims that two base colors could generate all other colors prompted him to elaborate the distinction between compound colors and simple colors. The celebrated circle of colors gave a precise empirical rule for determining the colors produced by any given mixture of simple colors (Fig. 3.7).[37]

Newton divided a circle into arcs proportional to the intervals of the successive notes of his musical division of the spectrum. For each component of the mixture, he drew small disks of size proportional to the amount of the component. He then marked the gravity

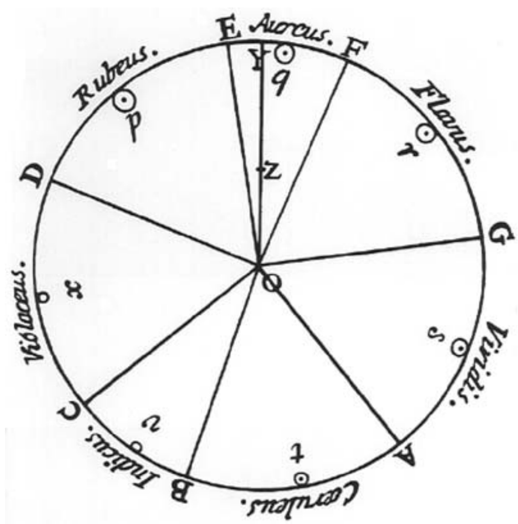


Fig. 3.7. Newton's circle of colors. From Newton 1704, book 1, part 2, fig. 11.

[36]Draft of Newton to Hooke (1672), MS Add. 3970, f. 529r, discussed in Shapiro 1979; Newton 1687, p. 411 (magnetic forces). On the approximate validity of the linear dispersion law, cf. Boegehold 1928. To assume an invariant υ and to make m the parameter of color (as Newton did in Query 21 of the *Opticks*) leads to the same dispersion law as the velocity model. The only possible way out is to assume that the refractive force depends on color. This option is compatible with the indefiniteness of the force law assumed in Newton's derivations of the sine law of refraction. It may be justified by making color depend on the inner structure of the light particles, as for instance Bošković later did (see chapter 4, p. 128, note 46). For a different view, see Bechler 1974.

[37]Newton 1704, book 1, part 2, props. 3 (musical division of the spectrum), 6 (circle of colors). The color circle does not appear in Newton's earlier writings. One important issue, on which Newton's had to change his mind, is the number of simple colors necessary to produce whiteness (from an infinite number to two or three): cf. Shapiro 1980b.

center, z, of these disks and the radius OY passing through it. The point Y on the circle gives the resulting color. The ratio Oz/OY gives the "fullness or intenseness" of the color. The idea of mapping the spectrum on a circle may seem artificial, since it implies the contiguity of the most extreme colors of the spectrum (red and violet). Newton possibly drew this idea from contemporary representations of the division of the octave through points on a circle. The division is natural in the musical case because of the optimal consonance of octaves.[38]

Thin plates

In the second book of his treatise, Newton gave a detailed account of his experiments on thin plates, with accurate determinations of the effects of thickness and inclination (see Figs. 3.8 and 3.9). He carefully separated the observations from the theory, and he kept the

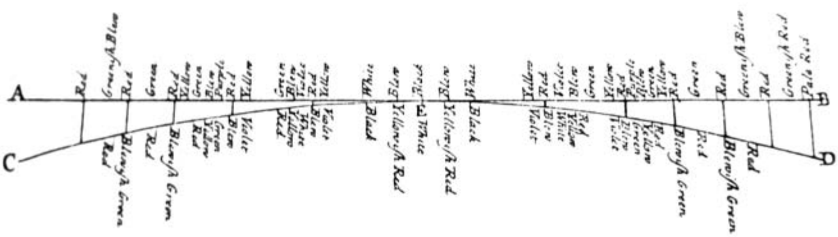


Fig. 3.8. The colors of Newton's rings. AB represents the inferior surface of a large plane-convex lens, and CD the superior surface of a biconvex glass lens. The colors of reflected (above) and transmitted (below) light depend on the thickness of the interstitial air. From Newton 1704, book 2, part 1, fig. 3.

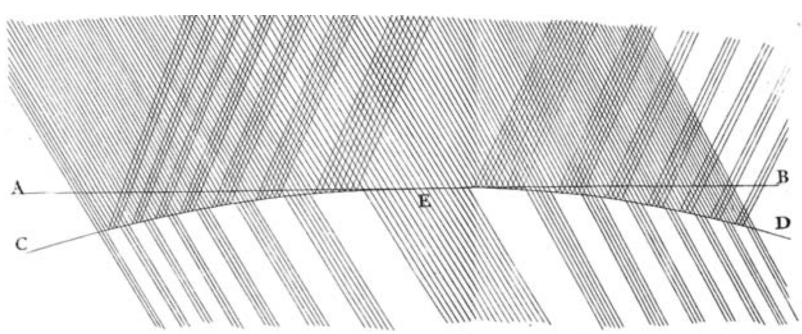


Fig. 3.9. Newton' s drawing of the reflection and transmission of (inclined) rays of monochromatic light by the thin layer of air trapped between a spherical and a plane glass surface. The light from above is either transmitted or reflected by the second surface it encounters, periodically depending on the thickness of the layer. From Newton 1704, book 2, part 1, fig. 4.

[38]Cf. Wardhaugh 2006, pp. 253–6.

theory as close as possible to the phenomena. He rephrased his older, ether-based explanation of colored rings and partial reflection in terms of the more economical notion of "fits of easy reflection or transmission" for the rays themselves:

> Prop. XII. Every ray of Light in its passage through any refracting surface is put into a certain transient constitution or state, which in the progress of the ray returns at equal intervals, and disposes the ray at every return to be easily transmitted through the next refracting surface, and between the returns to be easily reflected by it.
> Prop. XIII. The reason why the surfaces of all thick transparent bodies reflect part of the Light incident on them, and refract the rest, is, that some rays at their Incidence are in fits of easy reflexion, and others fits of easy transmission.

In the case of light entering a thin layer of transparent material normally, transmission occurs at the second surface whenever the thickness of the layer is equal to a whole number of intervals of the fits. In the case of Newton's rings, the thickness varies as the square of the distance from the contact point of the two glasses. Therefore, the radii of the successive rings of monochromatic light (as seen through a prism) are to each other as the square roots of successive integers.[39]

Newton briefly indicated a wave-based explanation of the fits:

> What kind of disposition this is? Whether it consist in a circulating or vibrating motion of the ray or a vibrating motion of the ray, or of the medium, or something else? I do not here enquire. Those that are averse from attending to any new discoveries, but such as they can explain by an Hypothesis, may for the present suppose, that as Stones by falling upon Water put the Water into an undulating motion, and all Bodies by percussion excite vibrations in the Air: so the rays of Light, by impinging on any refracting surface, excite vibrations in the refracting or reflecting medium or substance, and by exciting them agitate the solid parts of the refracting or reflecting Body, and by agitating them cause the Body to grow warm or hot; that the vibrations thus excited are propagated in the refracting medium or substance, much after the manner that vibrations are propagated in the Air for causing sound, and move faster than the rays so as to overtake them; and that when any ray is in that part of the vibration which conspires with its motion, it easily breaks through a refracting surface, but when it is in the contrary part of the vibration which impeded its motion, it is easily reflected; and, by consequence, that every ray is successively disposed to be easily reflected, or easily transmitted, by every vibration which overtakes it. But whether this Hypothesis be true or false I do not here consider. I content my self with the bare discovery, that the rays of Light are by some cause or other alternately disposed to be reflected or refracted for many vicissitudes.

This is a variant of the hypothesis of 1675, with the same acoustic analogy, and with a new insistence that the related fits of easy reflection or transmission are experimental facts that do not depend on this hypothesis.[40]

[39]According to Shapiro (1993, pp. 171–2), Newton designed the theory of fits in the early 1890s after studying thick plates and while rewriting Part 4 of his treatise. He borrowed the term "fit" from contemporary medical language, in which it meant a recurrent attack of a periodic ailment such as malaria: cf. Shapiro 1993, p. 180.

[40]Newton 1704, pp. 78, 80. In this variant, the vibrating entity is the matter of the body, not the ether; although the explanation of partial reflection in Prop. 13 would rather require ethereal vibrations.

As he had done in 1675, Newton related the colors of bodies to those of thin plates. He placed the relevant considerations after the observations on thin plates and before the theory of fits: he valued the resulting insights into the corpuscular structure of matter so much that he did not want their reception to depend on any specific theory of the colors of thin plates. This was a wise decision, as we will see in the next chapter.[41]

Thick plates

The mirrors of Newton's reflecting telescopes were made of spherical shells of glass silvered on the convex side. Newton accidentally discovered that the diffuse light outside the geometrical image of a distant source of light presented colors analogous to the colors of thin plates. The precise setup (Fig. 3.10) with which he studied these colors involves a white screen perpendicular to the axis of the mirror and containing the center of its spherical shell. The light from the distance source passes through a small hole on this screen around the center. It is then reflected on the mirror. A small portion of the returned light is diffusely reflected and casts colored rings on the screen. Newton gave much importance to this little phenomenon, for he regarded it as a confirmation of his theory of fits. In his explanation, a ray from the source enters the glass shell of the mirror in a fit of easy transmission and returns to the concave surface after diffuse reflection on imperfections of the silvered surface, in a fit that depends on the thickness of the shell and on the inclination of the reflected ray. From his earlier determination of the effect of inclination in thin plates, he could quantitatively determine the rings produced on the screen. The result matched his observations:[42]

> And thus I satisfy'd my self, that these rings were of the same kind and original with those of thin plates, and by consequence that the fits or alternate dispositions of the rays to be reflected or transmitted are propagated to great distances from every reflecting and refracting surface.

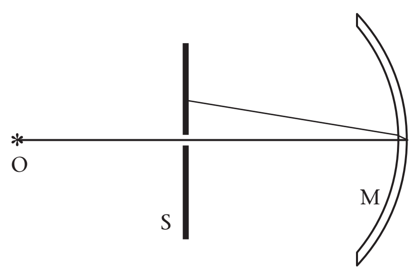


Fig. 3.10. Newton's setup for observing the rings of thick plates. The light from the source O, passing through the hole in the screen S, is diffusely reflected on the spherical glass mirror M. The rings appear on the (white) face of the screen facing on the mirror.

[41]Newton 1704, book 2, part 3, props. 1–7.

[42]Newton 1704, book 2, obs. 8. Thomas Young's later explanation of this phenomenon involves two-ray interference and scattering by dust particles on the external surface of the mirror. See below, p. 176.

Inflexion

Newton devoted the third and last book of his treatise to a short account of a few quantitative observations regarding diffraction, which he called "inflexion" in conformity with his view that this phenomenon corresponded to a bending of rays in the vicinity of a material body. In contrast with the other books of his treatise, Newton here offered no theoretical analysis beyond the broad assertion of inflected rays. The probable reason for this reticence is the difficulties he encountered in imagining the possible paths of inflected rays.[43]

Newton's first observations were made with the "hair of a man's head," placed in a beam of sunlight from a tiny hole in a shutter. The observed shadow was larger than the geometrical shadow, and it presented three colored fringes on each side. Newton did not see the internal fringes included in Grimaldi's account. There is manuscript evidence that he first assumed that the fringes were made of thin, flat bundles or rays (*fasciae*) and therefore propagated rectilinearly from the inflexion zone to the screen. This assumption, together with his later observation that the relative distances of the fringes are the same at any distance, leads to the implausible consequence that the inflexion of the rays increases with their distance from the hair. In his published account, Newton emphasized that the shadow became broader in proportion to the distance of the screen from the hair when this distance decreased. He explained this fact by a rapid decrease of the rays' inflection with their distance from the hair (see Fig. 3.11).[44]

Newton also described diffraction by the edge of a blade, by two parallel blades, and by two intersecting blades. For a sufficient narrowness of the space between the two blades,

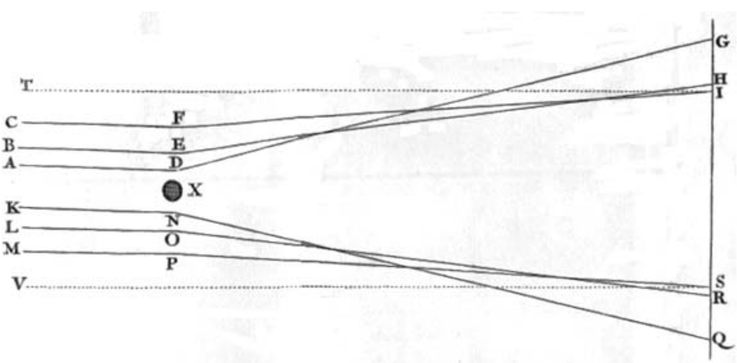


Fig. 3.11. The inflexion of light by a hair (X) according to Newton. From Newton 1904, book 3, plate 1. The ray CFG is less inflected than the ray ADG, which is closer to the hair. Consequently, the ratio of the shadow's breadth IS over the distance of the screen from the hair increases when the screen is closer to the hair.

[43]Newton 1704, book 3. [44]Ibid., obs. 1–4. Cf. Stuewer 1970; Shapiro 2001.

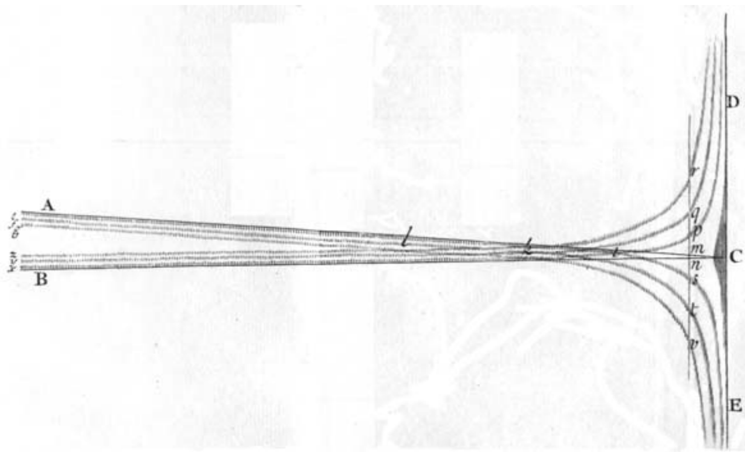


Fig. 3.12. Newton's drawing of the diffraction fringes from two intersecting knives. From Newton 1704, book 3, plate 1, fig. 3. The straight lines AC and BC represent the limits of the geometrical shadow of the knives. For a large spacing of these edges (on the left side of the figure), the fringes (x, y, z; e, f, g) are outside the shadow, close to each edge. For a small spacing (on the right side of the figure), the fringes (p, q, r; s, t, u) are in the shadow.

he found that the light bent into the shadow. He used the two intersecting blades (Fig. 3.12) in an attempt to determine the distance from the hair at which the rays responsible for a given fringe were inflected (assuming that the intersection of two symmetrical fringes corresponded to undeflected rays). The result again contradicted his earlier assumption that the fringes propagated rectilinearly from the source:

> I gather, that the light which makes the fringes upon the paper is not the same light at all distances of the paper from the knives, but when the paper is held near the knives, the fringes are made by light which passes by the edges of the knives at a less distance, and is more bent than when the paper is held at a greater distance from the knives.

This circumstance, the very existence of the fringes, and the fact that bending occurred sometimes into, sometimes from the geometrical shadow were formidable theoretical challenges. Newton left them to posterity.[45]

Queries

At the end of his treatise, Newton proposed a series of queries, "in order to a further search to be made by others." In the first edition of 1704, he refrained from any assumption on the nature of light. He assumed only that a universal action at a distance between rays of light and material bodies explained reflection, refraction, inflexion, the heating of bodies

[45]Newton 1704, book 3, obs. 5–10, citation from obs. 9. Newton's observations are compatible with the predictions of Fresnel's theory of diffraction, with a precision of about 2%: cf. Nauenberg 2000. In queries 3, Newton suggested that the fringes might result from an eel-like motion of the rays.

by light, the emission of light by heated bodies, and vision. He repeated his old idea that the rays of light excited vibrations of the retina transmitted to the brain through the optic nerve, and compared color and pitch:[46]

> *Qu.* 13. Do not several sorts of rays make vibrations of several bignesses, which according to their bignesses excite sensations of several Colours, much after the manner that the vibrations of the Air, according to their several bignesses excite sensations of several sounds? …
> *Qu.* 14. May not the harmony and discord of Colours arise from the proportions of the vibrations propagated through the fibres of the optick Nerves into the Brain, as the harmony and discord of sounds arises from the proportions of the vibrations of the Air? For some Colours are agreeable, as those of Gold and Indico, and others disagree.

In the Latin translation of his treatise, published in 1706, Newton added seven new queries in which he at last came out as a supporter of the corpuscular concept of light: "Are not the rays of Light very small bodies emitted by shining substances?" He asked this question after rejecting any medium theory for a variety of reasons: colors would need to be explained by modifications of the rays, the rays would bend in the shadow, the asymmetry (polarization) of the rays from a doubly refracting crystal would remain unexplained, the colors of thin plates would require two ethers (one for the rays, one for the fits), and the ether would slow down the motion of celestial bodies. In favor of the corpuscular view, he cited the easy mechanical explanations of rectilinear propagation, reflection, and refraction; the interpretation of colors as referring to the size of the corpuscles of light; the interpretation of the fits in terms of vibrations excited by the impact of the corpuscles; the interpretation of double refraction and polarization by "some attractive virtue lodged in certain sides both of the rays, and of the particles of the crystal."[47]

Lastly, in the second English edition of his treatise (1718), Newton inserted eight queries in which he introduced the ether as a medium that was useful to explain the fits, heat transfer through a vacuum, refraction, inflexion, gravitation (by the tendency of bodies to go from the denser parts to the rarer parts of the ether), vision, propagation along nerves; and he now judged that the ether could be so rare as to allow the unimpeded motion of celestial bodies. The chronological order of writing of the queries clearly depended on their explanatory level: queries concerning the distance action between rays and matter came first, those concerning the corpuscular representation of rays came second, and those concerning the ether-based explanation of all action at a distance came last.[48]

[46]Newton 1704, pp. 132, 136.

[47]Newton 1706, queries 17–23 (citations from query 21), renumbered 25–31 in Newton 1718. After much hesitation, Newton chose the "size" (mass) of the corpuscles as the parameter of color, and not the velocity, because in 1692 the Astronomer Royal John Flamsteed had failed to observe a consequence of the velocity model: Jupiter's satellites should turn blue at the instants preceding their occultation (since a larger velocity implies a lesser refraction in the Descartes–Newton model): cf. Shapiro 1993: 145–6. As was mentioned above, p. 97, note 36, both choices are compatible with Newton's derivation of the law of refraction, and neither is compatible with his linear law of dispersion.

[48]Newton 1718, queries 17–24.

The analogy with sound played an important role in justifying ether waves produced by light, as it already did in the early optical papers:[49]

> If a stone be thrown into stagnating Water, the Waves excited thereby continue some time to arise in the place where the Stone fell into the Water, and are propagated from thence in concentrick Circles upon the Surface of the Water to great distances. And the Vibrations or Tremors excited in the Air by percussion continue a little time to move from the place of percussion in concentrick Spheres to great distances. And in like manner, when a Ray of light falls upon the Surface of any pellucid Body, and is there refracted or reflected, may not Waves of Vibrations, or Tremors, be thereby excited in the refracting or reflecting Medium at the point of Incidence, and continue to arise there, and to be propagated from thence as long as they continue to arise … , and are not these Vibrations propagated from the point of Incidence to great distances?
>
> Is not Vision perform'd chiefly by the Vibrations of this Medium, excited in the bottom of the Eye by the Rays of Light, and propagated through the solid, pellucid, and uniform Capillamenta of the optick Nerves into the place of Sensation? And is not Hearing perform'd by the Vibrations either of this or some other Medium, excited in the auditory Nerves by the Tremors of the Air, and propagated through the solid, pellucid, and uniform Capillamenta of those Nerves into the place of Sensation? And so of the other Senses.

Newton also used the acoustic analogy to exclude medium-based theories of light. In Query 19 of 1706, he wrote:

> If [light] consisted in Pression or Motion, propagated either in an instant or in time, it would bend into the Shadow … The Waves on the Surface of stagnating Water, passing by the sides of a broad Obstacle which stops part of them, bend afterward and dilate themselves gradually into the quiet Water behind the Obstacle. The Waves, Pulses or Vibrations of the Air, wherein Sounds consist, bend manifestly, though not so much as the Waves of Water. For a Bell or a Cannon may be heard beyond a Hill which intercepts the sight of the sounding Body, and Sounds are propagated as readily through crooked Pipes as through straight ones. But Light is never known to follow crooked Passages nor to bend into the Shadow.

As we saw earlier, the apparent impossibility of explaining the rectilinear propagation of light in a medium theory probably determined Newton's original preference for the corpuscular view.[50]

The second book of the *Principia mathematica* (1687) contained several arguments of this kind. In Proposition 41, Newton reproduced his old anti-Descartes remark that the rectilinear transmission of pressure along contiguous balls required an improbable alignment of the balls (see Fig. 3.13). He also argued that the isotropy of pressure in a fluid implied that the pressure transmitted beyond a diaphragm necessarily acted on the sides of the cone delimited by the diaphragm (see Fig. 3.14).[51]

[49] Ibid., queries 17, 23. [50] Newton 1706, query 19. [51] Newton 1687, pp. 354–6.

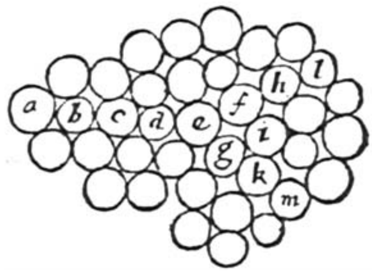


Fig. 3.13. Newton's diagram for excluding rectilinear propagation of endeavor in a hard-ball model. From Newton 1687, 354. A pressure applied on the ball *a*, is transmitted rectilinearly along *abcde*, then obliquely along *efhl* and *egkm*.

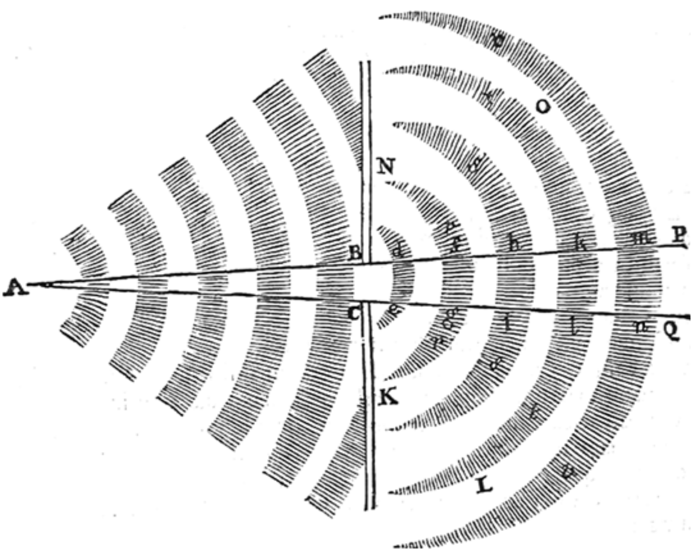


Fig. 3.14. Newton's drawing for the straying of pressure (or water waves, or air waves, or continuous flow) beyond a diaphragm. From Newton 1687, pp. 355, 357. The isotropy of hydrostatic pressure implies that the pressure transmitted in the truncated cone BPCQ across the surfaces *de*, *fg*, etc. must also act across the surfaces BP and CQ. In the case of water waves, the water heaps *de*, *fg*, etc. imply a flow toward the intermediate valleys and beyond the lines BP and CQ as well.

In Proposition 42, Newton gave a proof that "*All motion propagated through a fluid diverges from a rectilinear progress into the unmoved space*." In the case of water waves, he argued that water had to descend from the heaps that passed the diaphragm at equal speed in the forward and sideways directions. In the case of pulses in an elastic fluid, he similarly argued that truncated zones of compressed fluid beyond the diaphragm had to expand

with equal speed in the forward and sideways directions. In the case of any flow through a diaphragm, he argued that the isotropy of pressure implied divergence. In the elastic-fluid case, Newton commented:

> We find the same by experience in sounds which are heard through a house interpose; and if they come in to a chamber through the window, dilate themselves into all parts of the room, and are heard in every corner; and not as reflected from the walls, but directly propagated from the window.

Newton then gave his theory of wave propagation in an elastic medium, the first mathematical theory of plane elastic waves that yielded the propagation velocity as a function of the elasticity and the density of the medium. An optical scholium followed:[52]

> The last propositions respect the motions of light and sounds; for since light is propagated in right lines, it is certain that it cannot consist in action alone (by Props. 41 and 42). As to sounds, since they arise from tremulous bodies, they can be nothing else but pulses of the air propagated through it.

In a discourse on the cause of gravity appended to his *Traité de la lumiere*, Huygens addressed the incompatibility of his theory of light with Newton's Proposition 42:

> I answer that what I have brought up to prove that light (excepting reflection and refraction) spreads directly does not impair the validity of the said Proposition [42 of Book 2]. For I do not deny that when the Sun shines through a window, there is motion strayed beyond the lit space; but I say that these detoured waves are too weak to produce light. And even though [Newton] wants to believe that the emanation of sound proves that these sideways effusions are sensible, I take it for certain that this emanation rather proves the contrary. Indeed if sound, after passing an opening, would spread sideways, as Mr. Newton wants it to be, it would not so exactly respect the equality of the angles of incidence and refraction in echo … As for his argument that wherever one stands in a room whose window is open, one can hear the sound from outside not by reflection on the walls but coming directly from the window; it is easily seen to be misleading, because of the multitude of repeated reflection occurring as in one instant … I admit that in the case of the undulations or circles formed on the surface of water, things are roughly as Mr. Newton asserts … But for sound, I say that the sideways emanations are nearly insensible to the ear, and that in the case of light they have no effect whatsoever on the eyes.

Huygens here suggested that sideways propagation occurred for any kind of waves, but to a different degree depending on the kind of waves. He did not give any reason for this difference. As we saw, in the *Optice* of 1706 Newton admitted Huygens's contention that the straying of sound was inferior to that of water waves; yet he remained convinced that this straying made rectilinear propagation impossible.[53]

[52]Ibid., pp. 356–9, 363–72, citations from pp. 358, 369. More will be said on Newton's theory of sound propagation in chapter 4, pp. 153–4.

[53]Huygens, *Discours de la cause de la pesanteur*, in Huygens 1690, p. 164.

3.6 Conclusions

To summarize, Newton's optics involves three levels of description, which we may call the heuristic, phenomenological, and observational levels. At the heuristic level, Newton has light corpuscles interact with matter and ether, and also produce waves by impact on the interface between two different media. The mass (or velocity) of the corpuscles corresponds to the simple colors and determines the length of the waves. As we just saw, this heuristic level has three sublevels corresponding to the successive queries of 1704, 1706, and 1718. Even though this level undoubtedly permeated Newton's investigations from the earliest trials to the latest refinements, it did not appear in print before these queries were published. The phenomenological level is the level of rays endowed with selective refrangibility and fits of easy reflection or transmission. The observational level corresponds to theory-neutral accounts of experiments, which Newton calls "Observations" in his *Opticks*. Newton was very careful in separating these three levels, to make sure that future controversy at the heuristic level would not contaminate the other levels.

The main investigative tools through which Newton reached this elaborate conceptual system were the laws of mechanics as Newton understood them, precise and quantitative experiments, some mathematics, and the creative use of various analogies. His understanding of mechanics determined his rejection of Cartesian optics and his emissionist reinterpretation of Descartes's derivation of the law of refraction. With Huygens, Newton was the only optician of the seventeenth century to perform precise quantitative experiments in support of his theories. He was well ahead of his time in his control of the imaginative, material, and interpretive dimensions of experimentation. His definition of color through measurable parameters of refraction or selective transmission (through thin plates) contributed to make physical optics a quantitative science.

However, Newton's use of mathematics in optics was essentially limited to the derivation of the law of refraction and to some geometrical optics. He rather relied on "the analogy of nature," according to which nature should be conformable to itself at different scales or for different kinds of phenomena. For instance, he applied the laws of macroscopic mechanics to the motion of the light corpuscles; or he explained the colors of bodies by analogy between their parts and thin plates. Most frequently and most systematically, he relied on acoustic analogies or disanalogies. The reason for this characteristic of Newton's optics is not to be found in his predilection for music, for he reportedly declared music to be an "ingenious nonsense" and "when hearing Haendel play on the harpsichord, could find nothing worthy of remark but the elasticity of his fingers." Rather, the colors of thin plates and Hooke's challenges called for the consideration of ether waves; and sound waves, as they began to be understood in Newton's times, were the best illustration of such waves.[54]

Newton used the acoustic analogy in four different manners. First, he used it *ab absurdo*: if light were similar to sound, it would bend in the shadow. Secondly, in his reply to Hooke he used this analogy to conciliate the wave theory with his concept of the heterogeneity of white light. He thus pioneered the correspondence between color and frequency, as well as the idea that white light was a mixture of components with various frequencies. Thirdly,

[54]On Newton and music, cf. Gouk 1986, p. 101.

Newton used the acoustic analogy to explain aspects of light that the corpuscular or ray theory could not capture. He explained the fits of easy reflection and the propagation of visual signals along the optical nerves by analogy with the aerial vibrations produced by the impact of two bodies. Fourthly, he assumed that colors and tones obeyed the same rules of harmony: he compared the rainbow to a musical scale, and the harmony of two colors to consonance. This fourth mode of analogy was not unrelated to the third mode, for Newton associated both (simple) color and tone with frequency, the perception of colors being conditioned by the traveling of periodic waves through the optical nerve.

When Newton developed these analogies, the concept of sound as a compression wave was gaining ground, as a consequence of Boyle's pneumatic experiments and of the general progress of experimental acoustics. In his *Principia* of 1687, Newton gave the first mathematical theory of a plane periodic progressive wave of compression. He thus obtained the relation between velocity, density, and compressibility by balancing the pressure gradient on a slice of the fluid with its inertial force. In this concept of wave propagation, there is a spatial periodicity of the compression and of the velocity of the fluid, in contrast with the earlier concepts of sound in which the only periodicity was temporal. This aspect is essential in Newton's wave interpretation of the fits of easy transmission.[55]

Beyond plane-wave propagation, Newton could only guess the behavior of waves from analogy with the observed properties of sound or water waves. Thus, he inferred the straying of light in the wave theory of light from the straying of sound behind obstacles. He drew the relation between the mass of the light corpuscles and the length of the waves produced by their impact from analogy with water waves. Had he known the relation between wavelength and diffraction, he would have lost the main argument in favor of the corpuscular interpretation of light. Had he better understood the acoustic vibrations caused by impact, he would have lost his wave interpretation of the fits of easy transmission.

In sum, a large amount of Newton's heuristics crucially depended on the contemporary state of the theory of vibrations, and also on his related preference for the emissionist concept of light. At the same time, his phenomenology of rays, colors, and fits was solidly anchored on numerous experiments of unprecedented quality. In conformity with his famous *Hypotheses non fingo*, Newton believed he could cleanly separate this phenomenological level from the heuristic level. In reality, some of his phenomenology bore unconscious traces of the emissionist viewpoint, such as the preexistence of rays of simple color in white light or the idea that the second interface of a thin plate controlled the selection of colors. Thus, the various levels of Newton's optics were not as cleanly separated as he wished them to be. The distinction is nevertheless useful in understanding the structure and the reception of his work.

[55]In earlier concepts of sound, the analogy between sound and water waves did not imply spatial periodicity. The analogy was mostly used to illustrate circular perturbation from a center.

4

THE EIGHTEENTH CENTURY

Eighteenth-century philosophers of nature inherited a multiform and multilayered optics from the previous century. In elaborating their own views and theories, they drew on these sources in a selective, combinative, and elaborative manner. Several levels of description must be distinguished in this process, as Newton did in his own optics. The first level builds on a concept of ray that is largely independent of the deeper nature of light: it implies Kepler's theory of vision, his geometrical optics with various additions including the sine law of refraction, principles for the location of images, Newton's theory of colors, and photometric considerations. This level was shared by anyone concerned with optical instruments and astronomical observations. A large amount of the optical writings of the eighteenth century is confined to this level. This is not to say that their authors had no opinion on the essence of light. Rather, they judged that most of the optics they needed did not depend on it.

Deeper levels of descriptions involved the nature of light. Their main sources were Newton's queries, Cartesian reduction to contact action, Cartesian optics, Ango's and Huygens's treatises, acoustics by analogy, and a variety of chemical and theological considerations defining the origin and purpose of light. Many authors adopted the concept of light as a flux of corpuscles emitted by the source, thus turning a Newtonian query into a certainty. A large fraction of these emissionists had the light corpuscles directly interact with matter through distance forces. The rest traced this interaction to ether processes, either in the manner suggested in Newton's queries, or in a Cartesian manner based on contact action between ether globules and light corpuscles. Lastly, a significant number of the optical writers denied the corpuscular nature of light and adopted a pure medium theory of a more or less Cartesian kind. Historians of optics usually classify the eighteenth-century theories into emissionist and medium-based (or wave) theories. In this chapter, I prefer to classify the theories according to their Newtonian or neo-Cartesian character. The cultural interconnections of the various actors will thus be more evident.[1]

By convention, I call Newtonians the authors who accepted the corpuscles of light, who borrowed all other important concepts of their optics from Newton, but did not necessarily accept all Newton's speculations. This includes authors who rejected Newton's ether; and this excludes authors who accepted the corpuscles of light but integrated them in a non-Newtonian framework. The other important category, which I call neo-Cartesian, comprehends authors who pursued Descartes's project of reducing all interactions to

[1]For instance, I discuss Mairan together with other neo-Cartesians (Malebranche, Johann II Bernoulli, Nollet, Le Cat ...), whereas he is usually grouped with Dutch or British emissionists.

mechanical contact but did not necessarily follow the details of his theory of light and were willing to adopt some concepts of Newton's optics, even the light corpuscles in some cases. The historian Geoffrey Cantor introduces a third category: the fluid theorists, mainly chemists and theologians, who regarded light as another imponderable fluid in a broad, qualitative philosophy of nature and neglected geometrical or physical optics. As far as physics is concerned, this category only matters for the support it occasionally brought to Newtonian emissionism.[2]

The first section of this chapter is devoted to the ray level of description, arguably the most productive in eighteenth-century optics. The second section is a history of Newtonian theories, and the third a history of neo-Cartesian theories. The fourth section is devoted to the genesis, contents, and reception of Leonhard Euler's vibrational theory of light.

4.1 Ray optics

The gradation of light

In his optics, Ptolemy stated that visual clarity depended on the intensities of light and visual flux on the surface of the object, and that these intensities decreased with the distance of the object from the source or the eye, as "an aggregation of … rays is weakened when they are extended out to a great distance." In his *Perspectiva*, Witelo similarly explained the smaller illumination of a larger room by the larger distance of the objects from the summit of the pyramids of rays emanating from the source. Kepler, who regarded the rays as the lines of flow of an emanation from the source, had the intensity of light vary as the inverse square of the distance from the (point) source. Although many optical writers later adopted this assumption, it was not unchallenged at the beginning of the seventeenth century. The Jesuit scholar François de Aguilón considered both linear and exponential decrease. He decided in favor of the latter by comparing the distances at which a light and a double light gave equal illuminations. Peter Paul Rubens illustrated his contraption (Fig. 4.1).[3]

As most optical writers at least since Ptolemy had a concept of the intensity of light, one may wonder why they did not try to measure it before the second half of the seventeenth century. Besides the long neglect of quantitative experimentation, a plausible reason may have been the early recognition that the appreciation of the intensity of light depended on the luminous environment of its source. Ptolemy and Alhazen insisted on this subjective effect of contrast. Renaissance painters consciously compensated for it by means of the *claro oscuro*. Galileo Galilei struggled against the resulting misjudgments of intensity, as they hampered what he believed to be the proper interpretation of his telescopic observations. For instance, Galileo argued that sun spots were brighter than the brightest parts of the moon's surface, because they were at least as bright as the vicinity of the sun and

[2]Cantor 1983, chap. 4. For instance, we will see that Boerhaave enthusiastically supported Newton's theory of colors; and the late nineteenth-century resurgence of Newtonian optics in Germany has to do with the rise of the chemical view of light among physicists.

[3]Ptolemy, *Optics*, book 2, §§17–18, in Mark Smith 1996, p. 76; Witelo 1535 (from MS of the 1270s), theorems 22, 24 of part II, in Risner 1572, pp. 69–71(denser rays illuminate more); Kepler 1604, p. 9 (see chap. 1, p. 32); Aguilón 1613, book 5. Cf. Morère 1965, pp. 358–9, 368–76.

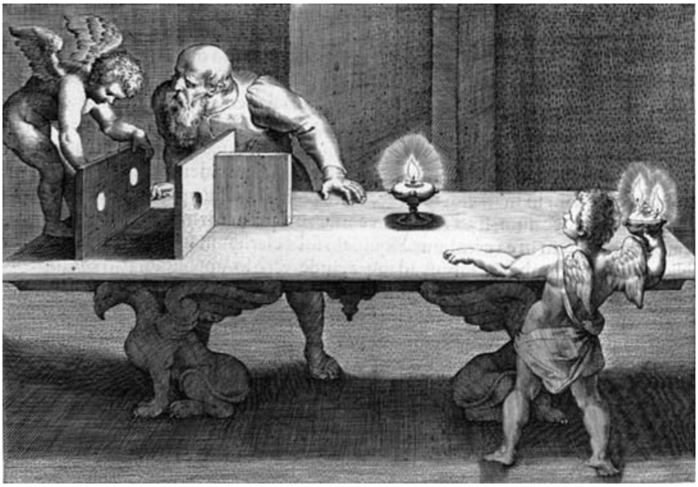


Fig. 4.1. Rubens's illustration of Aguilón's setup for determining how illumination varies with the distance (Aguilón 1613, P. 356). The single lamp and the double lamp project two luminous spots on the dark screen held by the upper angel. The other angel moves the double lamp until the philosopher judges that the two spots are equally bright.

because Venus, which is about as bright as the moon, disappeared in the vicinity of the sun. Galileo tacitly assumed that the luminosity of adjacent surfaces could be compared, whereas distant luminous objects immersed in different contexts could not.[4]

Marin Mersenne's posthumous *Optique* of 1651 contains pioneering photometric considerations, including the inverse square law, the effect of inclination of the rays, the question of the additivity of the light from several sources, and an estimate of the brightness of the surface of the sun by comparing its image in a *camera obscura* with the light from a candle. It seems to have been largely unnoticed. In a long letter published in 1665, the French astronomer Adrien Auzout called for a comparison of the light from the stars with the light from the sun and the planets, and for a comparison of the illuminations of the various planets. He also discussed the contrast between the dark and bright parts of the surface of the moon, and he vaguely estimated the intensity ratio of direct and reflected (or scattered) light (*lumieres premiere et seconde*). In the lack of any definite method of measurement, he entreated his readers "to perform every conceivable experiment to shed light on this matter that might serve the art of painting more than one thinks." A few artists, including Leonardo da Vinci, had pleaded for an "aerial perspective" in which the illumination of objects would be correctly and quantitatively represented.[5]

In his *De aberratione* written in 1666 and posthumously published in 1703, Christiaan Huygens pioneered the consideration of the brightness of images in the theory of optical

[4]Ptolemy, *Optics*, book 2, §§ 90–1, in Mark Smith 1996, p. 107; Alhazen, *De aspectibus*, book 1, chap. 6, §§ 108–16, in Mark Smith 2001, vol. 2, pp. 385–7; Galilei 1613, p. 13. On Galileo's remarks, cf. Piccolino and Wade 2008.

[5]Mersenne 1651, pp. 24–38; Auzout 1665. Cf. Morère 1965, pp. 346–9 (Auzout), 343–6 (painting). On optics and painting in the Renaissance, cf. Summers 1987. Bouguer and Lambert both mentioned the possible application of photometry to painting.

instruments. He showed that in a telescope for which the objective controls the amount of light, the brightness of the retinal image was proportional to the surface of the objective and inversely proportional to the square of the linear magnification. He proved that for direct vision the perceived brightness did not depend on the distance of the object from the eye (if absorption can be neglected, and if the size of the pupil does not change). And he proved that this brightness was unchanged when the object was seen through a system of lenses such that the emergent beam covered the whole surface of the retinal pupil. The explanation of the second point is simple for a distant object: by definition, the brightness is the ratio between the incoming luminous flux and the surface of the retinal image, which both vary as the inverse square of the distance of the object from the eye.[6]

Huygens's posthumous Κοσμοθεωρός (1698), a many-world cosmology in which all stars are suns, included a rough attempt to determine which fraction of the light of the sun is equivalent to the light of a bright star (Sirius). To this end, Huygens observed the light from the sun through a tiny hole on the bottom of a long tube and determined the size of the hole that reproduced the appearance of Sirius. Combining the result with Kepler's inverse-square law, he estimated the distance of Sirius to be 27 664 times the distance of the sun from the earth. In 1700, the Capuchin father François Marie independently invented the "lucimètre," an optical counterpart to the thermometer, barometer, and hygrometer. There were two versions of this instrument, one in which Marie counted the number of equal glass plates through which the measured light was no longer visible, another in which he counted a number of attenuating reflections. Marie accepted the subjective character of his method as an inevitable consequence of the individuality of visual sensations. He hoped it would serve to determine the atmospheric absorption of sunlight according to the hour, day, and season; the dimming of light during eclipses; and the visual sensitivity of a person.[7]

Some twenty years later the French Academician Jean-Jacques Dortous de Mairan revived the question of the atmospheric absorption of sunlight, in the context of a study of the cause of summer heat and winter cold. As he believed heat and light to obey the same law of absorption, he called for an investigation of the variation of the intensity of sunlight according to the elevation of the sun over the horizon. His protégé Pierre Bouguer, an expert in naval architecture and a supporter of Newtonian mechanics, answered the call in 1726. In order to avoid the subjectivity inherent in Huygens's procedure, Bouguer reduced every measurement of light to the balancing of illuminations produced by the investigated source and by a candle or torch placed at a variable distance. As the light of a candle is more comparable to moonlight than it is to sunlight, he first measured moonlight for different elevations, and then multiplied the results by the ratio of sunlight to moonlight. He estimated this ratio to 300 000, by balancing the light of the same candle first with

[6]Huygens [1666], prop. 10, pp. 332–8; [1685–1692], prop. 7, pp. 480–1. Cf. Korteweg 1916, LXVI–LXVIII, XCIII. From a modern point of view, Huygens's theorems result from the conservation of the luminous flux, $L\mathrm{d}S\mathrm{d}\Omega = E'\mathrm{d}S'$ (for small inclinations of the rays), and from the conservation of the optical etendue of a beam, $n\mathrm{d}S\mathrm{d}\Omega = n'\mathrm{d}S'\mathrm{d}\Omega'$ ($\mathrm{d}S$ and $\mathrm{d}S'$ denote the surfaces of the object and of the retinal image, $\mathrm{d}\Omega$ and $\mathrm{d}\Omega'$ the solid angles of the corresponding beams, n and n' the relevant optical indices, L the luminance of the object, E' the illuminance (brightness) of the retinal image), which is a consequence of the sine law of refraction. In the first case, the objective of the telescope controls $\mathrm{d}\Omega$. In the other two cases, the pupil of the eye controls $\mathrm{d}\Omega'$.

[7]Huygens 1698; Marie 1700. Cf. Morère 1965, pp. 339–40 (Marie), 349–52 (Huygens). The correct Sun/Sirius ratio is about 200 000 (cf. Andriesse 2005, pp. 397–8). Marie's procedure leads to a logarithmic scale of measurement, in conformity with Fechner's law. Bouguer condemned it in his *Traité*.

the light of the moon and secondly with sunlight passing through a diaphragm and a concave lens.[8]

In 1729, Bouguer published his influential *Essai d'optique sur la gradation de la lumiere*, which he aimed to found a new department of knowledge: "As optics has so far been confined to examining the situation or direction of rays, it still lacks an entire department the object of which would be the force or vivacity of light." This essay contained a precise statement of the null method of measurement, an application to the absorption by glass and water for various thicknesses, his solution of Mairan's problem, and a theoretical investigation of the absorption of light in transparent media. In connection with Malebranche's vibrational theory of light (on which more later), Bouguer traced absorption to the rays' partial reflection by and transmission through the molecules of the absorbing matter. After this concession to the ambient Cartesianism, he emphasized that his basic absorption law depended only on the intuition that the extinguished fraction of the incident rays did not depend on the initial numbers of rays. According to this law, successive equal layers of a homogenous transparent medium absorb a constant ratio of the light that they receive. Bouguer then deduced the exponential decrease of the intensity of parallel light with the thickness of the (homogenous) absorbing matter. He also treated the cases of divergent beams and heterogeneous absorbers.[9]

Although Bouguer's interest in the gradation of light never declined, his hydrographic activities and his long and tiring expedition to Peru long prevented him to publish more on this matter. Other optical writers nonetheless recognized the importance of his essay. The Cambridge professor Robert Smith reproduced Bouguer's results in notes appended to his influential *Opticks* of 1738, as a comment to photometric considerations of his own. Smith also developed Huygens's arguments about the brightness of direct and indirect images, and he noted that distinct vision was possible under widely different lighting conditions, including moonlight and sunlight. The latter point required an estimate of the ratio of the amounts of light from the sun and the moon. In addition to Bouguer's measurement, Smith gave two estimates of his own. The first was based on the observation that the moon is about as luminous as an average cloud. Since daylight is about the same under a clear sky and under a completely clouded sky, the light of the sun roughly is to the light of a full moon what the surface of a half-sphere centered on the earth and containing the moon is to the surface of the lunar disk. The result is $r^2 / 2d^2$, if r denotes the radius of the moon, and d its distance from the earth. Smith reobtained the same formula by assuming that the moon's surface reflected the light from the sun perfectly and isotropically. He calculated the light from any phase of the moon under the same assumptions.[10]

In 1750, Leonhard Euler independently considered the same problem under the same assumptions. With characteristic clarity, he distinguished the brightness of sources from

[8]Mairan 1719, 1721; Bouguer 1726. Cf. Wilde 1843, pp. 294–338; Morère 1965, pp. 352–7. On Bouguer's biography, cf. Lamontagne 1964. For a full history of photometry, cf. Johnston 2001.

[9]Bouguer 1729, p. vii. Cf. Morère 1965.

[10]Smith 1738, remarks 85–6 (Bouguer), §93 (brightness of retinal images), §255 (brightness through instruments), §349 (brightness of telescopic images), §95 (first moonlight calculation), remarks 87–9 (second moonlight calculation). Smith's first reasoning may have been inspired by Galileo's earlier comparison between the moon and a cloud.

the illumination of objects, and he properly took into account the effect of the inclination of the illuminating beam on the illumination. There is, however, an error of a factor 4 in his calculation. With Smith's formula, moonlight is about 1/90 000th of sunlight; with Euler's formula, the ratio is 1/360 000. This is smaller than Bouguer's measurement (1/300 000), so that absorption by the moon surface cannot explain the discrepancy. Undisturbed by this difficulty, Euler went on to calculate the light from the various phases of the moon and other planets. For experiments, he referred to Bouguer's "excellent treatise."[11]

When, a few years later, Bouguer resumed his photometric researches, he detected serious flaws in Smith's and Euler's evaluations of reflected light. These authors assumed that the amount of light sent by a surface element dS did not depend on the angle θ that the direction of observation makes with the plane of the element. The corresponding illumination of the retina is the ratio between the amount of received light and the surface of the image of the element, which is proportional to its apparent surface dS sin θ. It therefore becomes infinite when the angle θ reaches zero. Bouguer concluded that under Euler's assumption the moon or the sun should appear infinitely bright on their rim. With a "heliometer" of his own, he found that the center of the sun was somewhat brighter than its periphery. *En passant*, he noted that emission proportional to sin θ (Lambert's law) would imply a uniform brightness of the sun. For the light reflected by the moon, planets, or any rough surface, Bouguer modeled the asperities as a set of tiny plane mirrors with variable orientation. He determined the macroscopically reflected light through the *numératrice des aspérités*, a curve that gave the number of micro-mirrors as a function of the azimuth of their orientation. He obtained the *numératrice* itself by measuring the angular distribution of the reflected light in the case of normal incidence.[12]

Bouguer integrated these considerations in a voluminous *Traité* published posthumously in 1760 by Abbot de la Caille. The first part described improvements of his earlier techniques of measurement (see, e.g., Fig. 4.2), new instruments such as the *lucimètre* (see Fig. 4.3) and the *héliomètre*, and numerous applications to reflected, absorbed, and

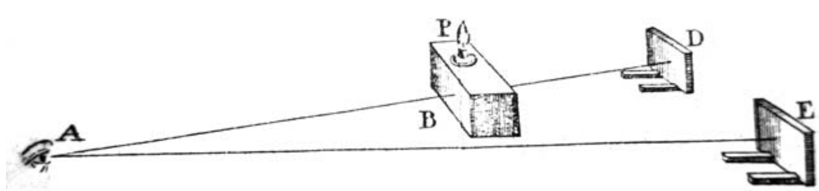


Fig. 4.2. Bouguer's setup for measuring the absorption of light. From Bouguer 1760, plate 1, fig. 5. The candle P illuminates the two white tablets D and E. Tablet E is seen directly; tablet D is seen through the absorbing body B. The amount of absorbed light is deduced from the distance ratio PD/PE for which the perceived brightnesses are equal.

[11] Euler 1750b. Aguilón and Mersenne had long ago noted that illumination depended on the inclination of the illuminated surface.

[12] Bouguer 1757; 1759; 1760, pp. 91–6, 161–97. On the origins of the heliometer, cf. Bouguer 1748; Fauque 1983.

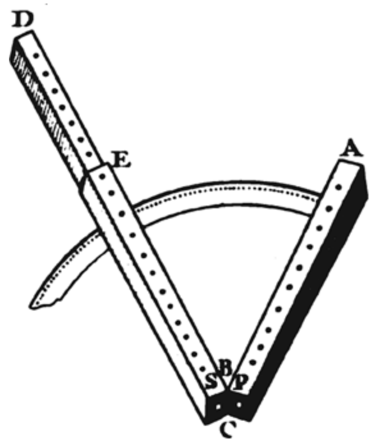


Fig. 4.3. Bouguer's *Lucimètre*. From Bouguer 1760, plate 2, fig. 10. The rigid tube AC and the sliding tube DEC have open ends at D and A, a blackened interior, and close ends P and S with two small holes covered with a thin white paper. The luminosity ratio (of the sky) in their directions is given by the square of the length ratio for which the holes at P an S are equally bright.

scattered light. He briefly remarked that the smallest detectable difference of illumination was proportional to the illumination, an early statement of Ernst Heinrich Weber's law about the threshold of sensibility. The second part of the treatise provided theories for the reflection of light by spherical mirrors (with a clever solution to Newton's problem of double caustics), by egg-box shaped mirrors (as a failed model of rough surfaces), and by rough surfaces in the aforementioned model. The third and last part was an amplification of Bouguer's early theory of the absorption of light, without the outdated molecular considerations, and with a tentative estimate of the "aerial colors" caused by atmospheric scattering. Bouguer favored geometrical methods and saved his analytical prowess for the most complex cases. In a casual but sufficiently precise manner, he introduced the main concepts of photometry (brightness and illumination) and its main laws (the additivity of light sources, the inverse-square law, the sine law of illumination, and the constant-ratio law of absorption). Any vagueness of formulation was compensated by the clarity of the supporting picture of a beam of rays.[13]

While Bouguer was striving to complete his treatise, the Mulhouse-based savant Johann Heinrich Lambert was working on a *Photometria*, which was published in the same year, 1760. Lambert knew Bouguer's essay, Abraham Kästner's critical edition of Smith's *Opticks*, and Euler's memoir on the light from celestial bodies. His main innovation was the sine law of emission, according to which the intensity of the light emitted by a surface element varies like the sine of the angle of emission (with respect to the plane of the element). Lambert drew this law from his observation of the uniformity of the solar disk as seen through an attenuating helioscope, by reasoning similar to Bouguer's. He believed it to be as solidly established as the other laws of photometry, and he extended it to diffuse

[13]Bouguer 1760, p. 53 (Weber's law). Cf. Middleton 1961.

reflection. Lambert's other concepts, laws, and theorems were mostly borrowed from his predecessors: Bouguer for the additivity law, the inverse-square law, and the extinction law; Euler for the sine law of illumination; Smith for the brightness of images.[14]

The main originality of Lambert's treatise was the systematic character of his exposition. In conformity with his philosophical inclination, Lambert brought the axioms of the new science to the fore, and strove to separate their empirical and their a priori components. Eager to display his mastery of Leibnizian calculus, he filled his treatise with numerous calculations of illumination for specific configurations of sources and receptors. Being a less patient experimenter than Bouguer, he did not realize that many of his results were spoiled by a false emission law.

Bouguer's geometrico-experimental style and Lambert's philosophico-mathematical style appealed to different sorts of readers. In his *History*, Priestley declared that "no philosopher since the time of Sir Isaac Newton ha[d] given so much attention to the subject of light as M. Bouguer." The German translator of this history, Georg Simon Klügel, judged Lambert's treatise to be widely superior to Bouguer's, and so did most of nineteenth-century German writers on photometry. They accepted Lambert's emission and diffuse-reflection laws, at least until the astronomers Friedrich Zöllner and Hermann Carl Vogel confirmed Bouguer's contradictory observations. In truth, Bouguer's own model of diffuse reflection also failed, despite its higher adjustability. In photometry as in other domains of physics, it was not easy to decide which aspects of the phenomena obeyed simple regularities.[15]

The heterogeneity of white light

Another component of the ray level of description is Newton's theory of colors, considered independently of the deeper nature of light (as Newton meant it to be). During the thirty years following Newton's letter of 1672 to Oldenburg, the acceptance of this theory was mostly limited to a few British astronomers, including Richard Towneley, the Astronomer Royal John Flamsteed, and the Irish author of the *Dioptrica nova*, William Molyneux. The theory was taught in Scottish universities, owing to James Gregory's early embracement. On the continent, Newton's theory was coolly received owing to Newton's failure to provide precise directions for decisive experiments and owing to the seeming incompatibility of a result reported in 1682 in Edme Mariotte's *De la nature des couleurs.* Mariotte passed a sunbeam through a first prism and a distant diaphragm followed by a second prism, and saw three colors in the issuing beam (violet, blue, and yellow) when the color selected by the diaphragm was violet. Newton did not respond at that time. Some thirty years later, he explained that Mariotte's result was to be expected in his theory, because the finite diameter of the sun and the light reflected from clouds here implied an incomplete separation of simple colors by the first prism.[16]

[14]Lambert 1760. Cf. Wilde 1843, pp. 338–84. On Lambert's biography, cf. Scriba 1973. Lambert's interest in pyrometry antedated his interest in photometry. His first measurements of light were done with a thermometer.

[15]Priestley 1772, p. 405; Klügel, note in Priestley 1776, p. 294; Zöllner 1865; Vogel 1877. For a critical (somewhat biased) assessment of Lambert's results, cf. Ernst Anding's edition of the *Photometrie* (Lambert 1892). On 19th and 20th century photometry, cf. Johnston 2001.

[16]Mariotte 1681, pp. 227–8; Newton, unsigned introduction to Desaguliers 1714, p. 435. Cf. Schaffer 1989, who insists on the weaknesses of Newton's original reports and on the failed replications of his results; Shapiro

Only in his *Opticks* of 1704 did Newton describe more refined techniques for purifying rays. The precision and abundance of the experiments described in this treatise impressed continental readers. In France, large sections of the *Optice* of 1706 were read in public meetings of the Academy in the year following its publication. Nicolas Malebranche and Jean-Jacques Dortous de Mairan reported success in private repetitions of some of Newton's experiments. In Germany, Gottfried Wilhelm Leibniz manifested interest in Newton's theory and his disciple Christian Wolff championed it in a textbook published in 1710. In a review that Wolff probably wrote for the *Acta eruditorum* of 1713, "the extremely sagacious Mr. Newton" was prayed to "condescend to devote attention to the problem that had been raised about this theory by the highly ingenious Mariotte." Newton consequently asked the demonstrator of the Royal Society, John Theophilus Desaguliers, to publicly perform (improved versions of) his most important experiments as well as a repetition of Mariotte's. The second of these demonstrations occurred in 1715 in the presence of three French Academicians. By 1720, the doctrine that white light can be separated into simple, immutable colors with a specific index of refraction was broadly accepted. Attempts to save the Aristotelian concept of color or to better represent the subjective aspects of color, for instance Father Castel's *Optique des couleurs* of 1740 or Wolfgang Goethe's turgid *Farbenlehre* of 1810, failed to remove Newton's theory from mainstream natural philosophy.[17]

As we saw in the previous chapter, Newton's theory of colors included a musical division of the spectrum both for prismatic colors and for the colors of thin plates. This analogy was far less successful than the general concept of simple colors. Its reception did not follow the Newtonian/Cartesian divide. As we will see in a moment, the Cartesian Malebranche praised the musical division and took it to imply that Newton had only seven simple colors, an influential error among later French optical writers. Three of the most widely read Newtonian writers, Willem 's Gravesande, Pieter van Musschenbroek, and Robert Smith simply ignored this analogy, perhaps because of Newton's remark that the border between two successive colors of the spectrum could not be appreciated with much precision. In a memoir of 1737 on sound, Mairan argued that in order to be justifiable by similarities in the perceiving organs, the analogy should be directly between the notes of the scales and the spectral colors, whereas Newton's analogy worked for intervals only. Mairan also noted that colors that differed by a tone in Newton's analogy, for instance green and yellow, did not form an unpleasant combination. Jean le Rond d'Alembert echoed Mairan's criticism in his encyclopedia article on color, published in 1754:

> The proportional extension of these seven intervals of *colors* answers fairly well to the proportional extension of the seven tones of music: this is a singular phenomenon. Whence one should not conclude that there is any analogy between the sensations of

1996, who demonstrates the early British interest in Newton's theory. As Shapiro rightly insists, Newton's *experimentum crucis* (unlike Mariotte's variant) was not intended to prove the immutability of simple colors; its true purpose was to show that refrangibility depended on the selected color.

[17] Wolff 1710, vol. 3, pp. 24–7; [1713]; Castel 1740; Goethe 1810. Cf. Montucla 1758, vol. 2, pp. 620–4, and Guerlac 1981 on the French reception; Hakfoort 1995, pp. 19–26 on the German reception; Sepper 2007 on Goethe.

> *colors* and those of tones: for our sensations are not at all similar to the objects that cause them.

In 1767, the French editor of Smith's optics similarly noted: "[Mairan's] deeper investigation of this alleged analogy, having revealed that it failed in the most essential points, has shown how ill-founded it is."[18]

Aberrations

As Newton emphasized in his *Opticks*, chromatic dispersion implies that lenses have different foci for different simple colors. Newton judged this aberration unavoidable and therefore recommended the use of reflecting telescopes. His dispersion law, according to which the ratio $(n_a - 1) / (n_b - 1)$ for the refractive indices of any two colors, a and b, does not depend on the refracting medium, seems to have been adjusted to his conviction that achromatic lenses were impossible. Indeed, in the paraxial approximation this law is easily seen to imply that the dispersion of foci in any compound thin lens is the same as it would be in a simple thin lens. Since Newton presented it as a consequence of his prism-in-prism experiment, it was commonly accepted until Euler challenged it in 1848.[19]

In Newton's statement of his dispersion law, the universality of the ratio $(n_a - 1) / (n_b - 1)$ is restricted to refraction from a given medium to another variable medium. The probable origin of this restriction is in the dispersion law implied by Newton's mechanical model of refraction: as we saw in chapter 3, in this model the universality of the ratio $(n_a^2 - 1)/(n_b^2 - 1)$ is subjected to the same restriction. Euler, who had no interest in a corpuscular theory of refraction, ignored this restriction and assumed that n_b was the same function f of n_a for any pair of media. He then exploited the relation $n_{13} = n_{12}n_{23}$ between the indexes for the various interfaces of three media labeled 1, 2, 3 in order to determine the form of the universal function. This relation was equally obvious for Newton and Euler, as they both identified the optical index with the ratio or inverse ratio of the velocities of light in the two media. For the function f, it implies the functional equation $f\,(xy) = f\,(x)f(y)$ whose continuous solutions have the form $f(x) = x^a$. Consequently, the ratio $\ln n_b / \ln n_a$ is universal.[20]

Euler showed that this new dispersion law permitted achromatic lenses, and cited the human eye as a proof of this possibility. He adjusted the curvatures of the four successive interfaces of a compound lens so that the foci should be the same for two different spectral colors. And he indicated how to satisfy this criterion by enclosing water between two glass menisci (Fig. 4.4). The suggestion was impractical, because the suppression of chromatic aberration required a large curvature of the glass surfaces for which spherical aberration

[18]Mairan 1737, pp. 24–33 ("Sur l'analogie particuliere des tons et des couleurs prismatiques"), 34–45 ("En quoi l'analogie du son et de la lumiere, des tons et des couleurs, de la musique et de la peinture, est imparfaite, ou nulle"); Smith 1767a, p. 196n. Bošković (1766, pp. 141–4) reproduced the analogy as well as the color circle. Inspired by Kircher's analogy between voices and colors, but rejecting Newton's theory of colors, Castel (1725) planned an ocular harpsichord that would play colors instead of notes. Voltaire acclaimed the idea, and Diderot judged it worth an entry of the *Encyclopédie*. Cf. Hankins 1994.

[19]See above, chap. 3, pp. 94–97.

[20]Euler 1748. Cf. Clairaut 1761, pp. 380–7; Priestley 1772, pp. 456–84; Speiser 1962, pp. XL–XLV; K. Hutchison 1991; Pedersen 2008.

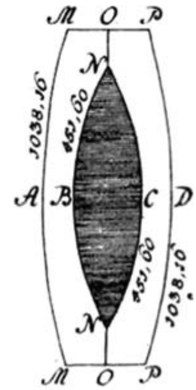


Fig. 4.4. Euler's drawing of an achromatic lens made of water (shaded) trapped between two glass menisci (white). From Euler 1748, plate.

became important. The great English optician John Dollond soon rejected it in the name of Newton's authority. Euler responded that Newton's dispersion law differed from his logarithmic law by amounts undetectable at the precision of Newton's index measurements, and that it was incompatible with his concept of universality (which he confused with Newton's). He repeated that the human eye implemented his own dispersion law, in a manner so ingenious as to provide a new proof of the existence of god.[21]

Things stood still until in 1755 the Swedish natural philosopher Samuel Klingenstierna briefly and elegantly proved that Newton's dispersion law impied that a series of refractions had a dispersive effect when the emerging ray was parallel to the original ray (unless the successive angles of incidence were very small). This meant that either the law was wrong or the prism-in-prism experiment was wrong. After reading Klingenstierna, Dollond decided to repeat this experiment and to measure the dispersions of water and various kinds of glasses. He found that the dispersions of flint and crown glass differed by a factor 3/2 incompatible with both Newton's and Euler's laws. By trial and error, he successfully built achromatic lenses and telescopes based on combining these two kinds of glass. This was the beginning of a true revolution in instrumental optics.[22]

Descartes's project to produce perfectly stigmatic lenses and Huygens's project to compensate the spherical aberration of lenses had lost most of their appeal under the impact of Newton's assertion that chromatic aberration necessarily exceeded spherical aberration. Dollond's invention lifted the Newtonian prejudice and induced a wave of theoretical studies of the various optical aberrations. In 1760, Klingenstierna gave the

[21]Euler 1748, 1753a, 1753b; Dollond 1753. At the end of his *Elementa* of 1695, David Gregory (James's nephew) had the idea of an achromatic objective lens imitating the several humors of the eye. The amateur-optician Chester Moor Hall conceived a flint-crown achromatic lens around 1729 and built it in 1733. His discovery was little noticed until Dollond's rediscovery prompted a patent dispute. Cf. Court and Rohr 1929, part 4; Fellmann 1973, pp. 304–5.

[22]Klingenstierna 1755; Dollond 1757. On Klingenstierna's optics, cf. Nordenmark and Nordstrøm 1938.

theoretical conditions of minimal (longitudinal) spherical aberration for a combination of lenses, as this kind of aberration became important for the large curvatures implied in Dollond's lenses. Two years later he won the prize that the Saint-Petersburg Academy offered for a theory of optical aberrations. Around that time, Euler completed a thorough study of both chromatic and (longitudinal) spherical aberration in which he still employed his logarithmic dispersion law. He soon changed his mind after reading a lucid memoir by the French *géomètre* Alexis Clairaut and becoming aware of the highly dispersive glass of the Petersburg mechanician Johann Ernst Zeiher. Clairaut argued that Euler's assumption of universality, though plausible, was contradicted by a very simple model of refraction. Namely, Clairaut proved that Newton's attraction model implied that the ratio $(n_a^2 - 1)/(n_b^2 - 1)$ was the same for refraction from one given medium to any other medium, as Newton had found long ago. He showed that the dispersion measurements by Dollond and those done by himself with Charles Penot de Tournières were incompatible both with the latter law and with Euler's law.[23]

Most of Clairaut's memoir was devoted to the precise determination of compound lenses and objectives in which chromatic and spherical aberrations were nil or minimal. As Dollond had kept his own reckoning secret and as French glasses differed from English glasses, Clairaut wanted to provide the missing directions to instrument makers in his own country. In a sequel of 1764, he extended his study of both kinds of aberration to the case when the object point is not on the axis of the optical system (transversal aberration). D'Alembert did the same independently, as part of a thorough study that filled the third volume of his *Opuscules*. His pinches to Euler, Clairaut, and Klingenstierna triggered useless polemics. Despite his declining sight, Euler kept writing abundantly on the theory of optical instruments, with insights into the optimization of the optical field and into the positioning of the eye for which chromatic dispersion is minimal. The sum of his reflections appeared in 1769–71 as the *Dioptrica*.[24]

In retrospect both Clairaut and d'Alembert obtained valid formulas for the longitudinal aberrations and for the transversal aberrations now called coma and astigmatism, although d'Alembert had to correct an error he originally made in the calculation of the coma. D'Alembert's study was more detailed and more general than Clairaut's, in several respects including the consideration of the eyepiece of the instruments, multiple strategies for the simultaneous minimization of the various kinds of aberrations, and estimates of the resolving power. Euler unfortunately neglected the transversal kinds of spherical aberration, which Clairaut and d'Alembert correctly judged important. For this reason Euler's lens designs could only fail. Although Clairaut's and d'Alembert's instructions were in principle better, French instrument makers ignored them and rather imitated Dollond's

[23]Klingenstierna 1760, 1762; Euler 1759 (field and spherical aberration in telescopes), 1762 (double lenses without spherical and chromatic aberrations); 1765 (reply to d'Alembert, giving up the logarithmic law); 1768a (favoring the constancy of $n^2 - 1$ ratios; ref. to Clairaut on p. 287); 1768b (objectives made of two kinds of glass; ref. to Clairaut and Zeiher on p. 104); Clairaut 1761, pp. 403–5, 422–3; Clairaut 1762 (measurements). Klingenstierna and Euler seem to have been unaware of the thorough study of spherical aberration in Smith 1738.

[24]Clairaut 1761, 1764; D'Alembert 1764a; Euler 1769, 1770, 1771. Euler's optimal positioning of the eye is that for which the images of the same object for different simple colors are seen under the same angle: cf. Herzberger 1969. Bošković and Béguelin also contributed to the theory of aberrations: cf Ferlin 2008, p. 26.

designs. The three greatest *géomètres* of the time thus failed to alter the course of the history of optical instruments. As we will see later, improvement over Dollond's designs required more collaboration with instruments makers, better optical glass, and better dispersion measurements.[25]

At the more fundamental level of optics, the discovery of achromatic lenses challenged the received theories of dispersion. As we will see in a moment, Clairaut and d'Alembert proved its incompatibility with the simplest forms of the Newtonian model of refraction. D'Alembert insisted on the necessity to determine dispersion empirically, without any theoretical preconception. At the same time, Euler could not quite renounce the logarithmic law. Late in his life, he confined its violation to colored transparent media such as Dollond's crown glass. In general, the two *géomètres* disagreed on the extent to which nature obeyed simple mathematical regularities. They nonetheless agreed that the discovery of achromatic lenses had revolutionized instrumental optics while unveiling a serious misconception in Newton's optics.[26]

4.2 Newtonian optics

In Britain

The earliest British supporters of Newton's theory of colors did not necessarily embrace the Newtonian concepts of light corpuscles and attractions. On the contrary, the Scottish natural philosophers who taught this theory in the late seventeenth century tended to prefer the Cartesian concept, properly modified to include the diversity of motions required by Newton's analysis of white light. For instance, in 1682 the Edinburgh professor Gilbert MacMurdo taught that Newton's doctrine of colors "can be easily accommodated to the Cartesian hypothesis, if we consider the globules of the second element to be unequal, and some pressures of the globules to be stronger than the others, and thus to be diversely refrangible and apt to excite sensations of diverse colours." On the Irish side, Molyneux's *Dioptrica nova* of 1692 stated that "Light is a body" since "the various properties of light, that necessarily belong to a body, are so many and evident, that they leave no room for any further doubt in this matter." Molyneux nonetheless favored Leibniz's and Barrow's derivations of the sine law of refraction, because they made the more natural assumption of a higher resistance of denser media to the propagation of light.[27]

The turning point was the publication of Newton's *Optice* of 1706, which included Query 21 about light corpuscles and attractions. In the 1710 edition of his widely used *Lexicon technicum*, the English writer John Harris turned the query into an affirmation:

[25] D'alembert 1764b [1766] (coma corrected). Cf. Boegehold 1935 (testing of Clairaut's and d'Alembert's achromatic designs), 1939, 1943; Herzberger 1969, pp. XII–XIII (failure of Euler's objectives); Fellmann 1973, p. 314 (attitude of French instrument makers); Ferlin 2008 (thorough analysis of Clairaut's and d'Alembert's contributions, of their origins and reception). None of the eighteenth-century theorists considered the effects of diffraction (which invalidate d'Alembert's estimates of the resolving power).

[26] Euler 1777b. Cf. K. Hutchison 1991.

[27] MacMurdo, unpub. MS [1682], Edinburgh University Library, cited in Shapiro 1996, p. 86; Molyneux 1692, p. 198, cited in Cantor 1983, p. 11.

> The *rays of light* are therefore certain *little particles* actually emitted from the *lucent body*, and *refracted* by some attraction, by which *light* and the *bodies* on which it falls do mutually act upon one another.

In his influential *Physico-mechanical lectures* of 1717, Desaguliers integrated the materiality of light in a clear and concise statement of Newton's general doctrine of attractions. He gave special weight to the alleged impossibility of rectilinear propagation in medium-based theories:

> That light is a body, appears from its *reflection, refraction, composition, division*, and *moving in time*; but especially from its being *propagated in right lines, and being stopp'd by an obstacle*, (how thin soever, if not transparent) which shews, that it cannot be an *action upon the medium*, which wou'd be communicated beyond an obstacle, as in the case of sound.

Desaguliers invoked the attraction of light by matter in his account of refraction: "The physical cause of the refraction, is the greater or less attraction of the new medium."[28]

From then on, British authors commonly asserted the materiality of light and cited its attraction by matter as the common cause of refraction, reflection, and inflection. The most widely used optical treatise of the century, Smith's *Opticks* of 1738, opened with the statement (a footnote points to Newton's relevant query):

> Whoever has considered what a number of properties and effect of light are exactly similar to the properties and effects of bodies of a sensible bulk, will find it difficult to conceive that light is any thing else but very small and distinct particles of matter: which being incessantly thrown out from shining substances, and every way dispersed by reflection from the others, do impress upon our organs of seeing that peculiar motion, which is requisite to excite in our minds the sensation of light.

Smith immediately noted that the concept of ray was sufficient for his main concern, the theory of optical instruments. He nonetheless reproduced Newton's explanation of reflection and refraction by attraction, as well as the corpuscular interpretation of simple colors: "According to this theory nothing more is requisite for producing all the variety of colours and degrees of refrangibility, than that the rays of light be bodies of different sizes." The corpuscles of light thus became an integral part of Newton's system, despite Newton's own reticence. Two reasons may be invoked for this evolution: the pedagogical value of a concrete picture of light, and the growing success of the new mechanics of attractions developed in Newton's *Principia*.[29]

The British Newtonians generally admitted that the particles of light were projectiles traveling all the way from the source to the eye. There is an interesting exception to this rule. In 1748, the English inventor of an important kind of compass needle, Gowin Knight, published a treatise in which he attempted to reduce every known phenomenon to the repulsion or attraction of particles. He regarded this action at a distance as the "immediate

[28]Harris 1710, article "Light," cited in Cantor 1983, p. 12; Desaguliers 1717, pp. 42, 46. Desaguliers's lecture 22 on colors does not contain the corpuscular interpretation of specific refrangibility.

[29]Smith 1738, pp. 1, 92. Smith's treatise got translated into Dutch (1753), German (1755), and French (twice, in 1767a, 1767b). Cf. F. Dijksterhuis 2008, pp. 316–20.

effect of God's will," and he accepted Newton's idea that reflection, refraction, and inflection resulted from such forces acting between the particles of light and the particles of matter. Yet he rejected the concomitant idea that the particles of light traveled freely in a vacuum. He filled space with a static array of mutually repelling particles of light, which could only vibrate around their equilibrium position:

> It is much easier to comprehend how a tremor may be propagated from one end of a series of elastic bodies to the other, in the same time that light takes up in coming from the sun to us, than to conceive how a particle of light can continue its motion and direction unaltered, through so vast a space, and with so great a velocity; whilst innumerable other particles are everywhere moving in different and often contrary directions.

Knight did not regard the propagated tremor as an undulation, for this view would have implied unwanted diffraction by analogy with water waves. Instead he argued that the vibrations of a particle of light on a given line only implied significant vibrations for the neighboring particles situated on this line.[30]

In analogy with the pitch of sounds, Knight identified the frequency of the tremor with the color. He conciliated this interpretation with Newton's by assuming the proportionality between the frequency and the velocity of the oscillating particles (and distinguishing this velocity from the constant propagation velocity):

> For in the same manner as the tones in musick make a difference in sound, according as these vibrations are quicker or slower; so the momenta of the parts of light will differ, according as they vibrate with more of less force.

Thus, Knight's theory of light relied on analogy with a theory of sound without yet being a wave theory; and it was Newtonian without being emissionist. There is no trace of any impact of this oddity on other Newtonians, except for a favorable mention by the Irish instrument-maker James Short.[31]

On the Continent

On the continent, the earliest and most read Newtonians were Dutch. In 1708, Herman Boerhaave embraced Newton's theory of colors in his medical lectures at Leyden. His disciples Willem Jacob 's Gravesande and Pieter van Musschenbroek befriended Desaguliers and combined Newton's optics with Boerhaave's concept of light as projected fire. In his widely read physics text, first published in 1721, 's Gravesande taught the rays of different colors and their specific attractions by matter in agreement with Newton's first queries and in analogy with gravitational attraction. Musschenbroek followed suit in his

[30]Knight 1748, pp. 53–5. Knight believed that a similar process applied to sound: for he accepted the French idea that the visible undulations of sonorous bodies were not the direct source of sound but caused a "tremulous motion" of their particles and of the adjacent particles of air.

[31]Knight 1748, pp. 53–5. Cf. Cantor 1983, pp. 65–6, 123, 127. On Short's remark, see note 45 below. There are similarities between Knight's views and Johann II Bernoulli's neo-Cartesian theory of 1736 (see below, pp. 146–9). J. II Bernoulli's and Mairan's memoirs may have been the source of Knight's awareness that making propagation velocity the parameter of color would imply a coloration of Jupiter's satellites at emersion.

equally popular treatises. He growingly referred to light as the "particles" of a "very subtle matter," in conformity with some of Newton's queries of 1706.[32]

There is not much to say on the German case, as it only implies elementary and derivative considerations. In the first half the century, German opinions on the nature of light were equally divided between medium and emission. The earliest supporters of Newton's theory of colors, Christian Wolff and Johann Jakob Scheuchtzer, favored an impoverished version of Huygens's system, without Huygens's principle and without double refraction. Later exponents of this system, such as Johann Heinrich Winkler, did not hesitate to combine it with an attractionist account of refraction. Other authors, such as Georg Erhard Hamberger, adopted Newton's system as seen through the Dutch Newtonian lens.[33]

France was the least favorable ground for the penetration of Newtonian attractions, owing to the stronghold of Cartesianism.[34] Consequently, the first few French Newtonians were most militant in their defense of light corpuscles and attractions. In his *Lettres philosophiques* of 1734 and in his popular account of Newton's philosophy (1738), Voltaire attacked Cartesian theories and defended the necessity of Newton's attractions. After reviewing older derivations of the law of refraction, Voltaire declared:

> At last, Newton alone found the true reason [of refraction]. His discovery surely merits the attention of all centuries. For it not only a property specific to light that is here at stake, although he would already have done much in this case; we will see that this property belongs to all bodies.

Like the Dutch Newtonians he had read, Voltaire defined light as a fire projected from luminous sources and attracted by matter as gravitating bodies are attracted by the earth. He regarded the atomic constitution of every matter, including the luminous fire, as a consequence of its porosity. In order to explain how refraction depended on color and refracting substance, he assumed the optical attraction to depend both on the mass of the atoms of light and on the mass of the material substance.[35]

In 1739, Clairaut politely rejected the neo-Cartesian explanations of refraction in the name of mechanical absurdities in their concept of medium "resistance." He then cited James Bradley's discovery of the aberration of stars as a proof of the corpuscular nature of light. Most importantly, he simplified Newton's derivation of the sine law of refraction thanks to the French Leibnizian reformulation of Newton's laws of mechanics, with a

[32]'S Gravesande 1720–1721, vol. 2; Musschenbroek 1734; 1739–1751, vol. 2, p. 500; 1741, p. 359. In his earlier *Epitome* (1726), Musschenbroek made no use of the corpuscular hypothesis, although he accepted it (on p. 249). Cf. Hakfoort 1995, pp. 42–9; F. Dijksterhuis 2008, pp. 310–15.

[33]Cf. Hakfoort 1995, pp. 117–26.

[34]Newton's mechanics (without the doctrine of attractions) and his theory of colors were better and earlier received. Cf. Shank 2008.

[35]Voltaire 1734, letters 14–16; Voltaire 1738, p. 97. Like Malebranche, Voltaire mistook Newton's analogy between musical scale and optical spectrum to imply seven colors only in the spectrum.

preamble meant to conciliate his mostly Cartesian audience at the French Academy of Sciences:[36]

> I pray you, while listening to me, to keep in mind that I declare that I do not want to establish Attraction as an essential property of matter. I do not have any preference on a question that exceeds my forces. My only aim here is to explain how Mr. Newton uses Attraction when he attempts to explain Refraction.

Neo-Cartesian views of light prevailed for a few more years. In 1742, the budding Newtonian Jean le Rond d'Alembert was still investing the refraction of a solid moving in a fluid of variable density, with intended application to light. The wind turned around the middle of the century. In 1752, the Marquis de Courtivron published his thoroughly Newtonian *Traité d'optique*, which included a praise of Newtonian attractions, Clairaut's Newtonian derivation of the sine law of refraction, and large sections of Smith's treatise. The *Secrétaire perpétuel* of the Academy of Sciences, Grandjean de Fouchy (who succeeded to the neo-Cartesian Mairan in 1744), praised this treatise as the best to be found in this department of knowledge.[37]

Another French Newtonian, Pierre Louis Moreau de Maupertuis, brought indirect support to the emissionist theory of light through his principle of least action. He began with the contradiction between Fermat's principle of least time and most derivations of the law of refraction, which required a larger velocity of light in the denser medium. In 1682, Leibniz had solved this contradiction by returning to Fermat's original idea of least "resistance" and making resistance proportional to the velocity of light. Leibniz justified this choice in a vaguely Cartesian manner, by tracing greater resistance to smaller pores in the matter of the medium, which implied greater velocity or impetus for the interstitial fluid. In 1744, Maupertuis similarly required the total "action" to be a minimum, the action in a given medium being defined by the product of the path length by the velocity of light. In this communication he did not have to decide between the competing theories of light, for he knew that both the emissionist and the neo-Cartesian theories assumed a higher velocity in a denser medium. There is little doubt, however, that he preferred the emissionist viewpoint that his friend Voltaire so strongly supported. This would explain, together with a theological belief in final causes, how easily he extended his principle to the motion of material bodies. In his *Essai de cosmologie* of 1751, Maupertuis clearly endorsed Newton's substantialist concept of light:[38]

> We have always regarded the sun as the cause of light. But it is only in recent times that we discovered that light is the very matter of the sun: inexhaustible source of this precious matter, flowing for a multitude of centuries, it does not seem to have in the least abated.

[36]Clairaut 1739, p. 263. Mairan's concept of resistance implies that velocity is the parameter of color, and should therefore imply changes of color whenever the particles of light are retarded by collision with the particles of matter. Clairaut 1737 included a discussion of stellar aberration based on the rain-tube analogy. On Bradley's discovery, see below p. 129.

[37]D'Alembert 1741–42; 1744, part III; Courtivron 1752; Grandjean de Fouchy 1752.

[38]Leibniz 1682; Maupertuis 1744; 1751, pp. 153–4. Cf. Dugas 1988, pp. 260–73; Pulte 1989; Panza 1995; Terrall 2002. Maupertuis misrepresented Leibniz's view as implying a lower velocity of light in the denser medium.

When d'Alembert wrote the article *Lumiere* for the *Encyclopédie* in the 1760s, he gave a detailed account of the Newtonian system, including estimates of the size and number of the light corpuscles and multiple evidence of their attraction by matter. After describing the various Cartesian and neo-Cartesian theories (and ignoring Euler's theory!), he gave much weight to Newton's objection regarding the impossibility of shadows, and rejected Huygens's wavelet-based derivation of rectilinear propagation. Yet he did not regard the question of the nature of light as entirely settled:[39]

> On the one hand, it is certain that the opinion of Descartes and his followers on the propagation of *light* cannot be conciliated with the laws of hydrostatics. On the other hand, it is equally certain that the continual emissions thrown by luminous bodies according to Newton and his followers boggle the mind ... It must be admitted that neither of these two opinions is proved; and the wisest answer to the question of the matter and propagation of light would perhaps be to say that we now nothing about it.

Force, mass, and momentum

Most Newtonians accepted Newton's derivation of the law of refraction by means of a deflecting force normal to the interface of the two media and uniformly spread on this interface, although they rarely provided the full mechanical reasoning.[40] Opinions varied on the nature of this force. For Musschenbroek and a growing variety of Newtonians, it was primitive and God given. For Smith, it did not matter "whether this force be a real attraction, or whether it be an impulse upon light, caused by the spring or elastick power of a subtil fluid which pervades the medium." For a negligible faction, truth resided in the elastic ether of Newton's latest queries. For Clairaut and many others, it had a deeper foundation that did not need to be known for descriptive purposes.[41]

Toward the end of the century pure attractionism prevailed, sometimes in Boscovichian garb. In the mid-century, the Croatian professor of mathematics at the *Collegium Romanum*, Ruđer Josip Bošković, had developed a concept of point atoms defined as centers of force, a Leibnizian improvement over Newton's hard atoms and attractions. In his fully Newtonian optics, Bošković answered a most common objection to the emissionist concept of light: he related transparency to the undisturbed course of the fast particles of light in the force field of the material atoms. His admirer Joseph Priestley endorsed this explanation in his *History* of 1772. The musician–astronomer William Herschel soon improved it by exploiting the oscillatory character of Boscovichian forces. In the 1790s, the Scottish natural philosopher John Robison vented much praise of Bošković's theory, although he used it in a mostly negative manner: as a means to discard Newton's ether and neo-Cartesian atmospheres.[42]

[39] D'Alembert 1765a, p. 722.

[40] They sometimes described other derivations, though only to condemn them. Smith (1738, pp. 69–70) gave Fermat's; Musschenbroek (1739–1751, vol. 2, p. 509) gave the Hobbesian derivation, which he learned from Barrow and from Dechales's *Cursus* (1674, vol. 2, p. 617).

[41] Smith 1738, pp. 90–1. On Newtonian ether theories, cf. Cantor 1983, p. 70.

[42] Bošković 1758, §§475–83; 1766, part 2; Priestley 1772, pp. 390–4; W. Herschel [1780]; Robison 1803. Cf. Cantor 1983, pp. 71–5.

Newtonian attraction includes diffraction, understood as an "inflection" of the trajectory of the light particles. Although a review of 1752 by Thomas Melvill emphasized the need for further research in this domain, the first Newtonians to seriously experiment and theorize on diffraction were the Scottish aristocrat Henry Brougham and the Reverend Gibbes Walker Jordan at the end of the century. Brougham's project was to extend Newton's idea of specific (color-dependent) refraction to all sorts of "flections," including reflection, inflection, and deflection. In the case of diffraction, he determined how the flection in Newton's hair and knives setups depended on color and concluded that the red rays were the most bent and the violet rays the least bent (in contrast with refraction). But he did not reach his aim of tracing the observed figures to forces emanating from the surface of the screens. Jordan's purpose was to trace all of Newton's observations to attractions between rays and matter, despite Newton's appeal to repulsion in the case of diffraction by a hair. He performed variants of Newton's experiments and traced the results to the combined effect of forces from the diffracting object and from the rim of the hole that produced the incoming beam.[43]

Opinions varied on the parameter of the light corpuscles that defined their color. British and Dutch Newtonians usually chose mass as Newton had done in his queries. Some authors preferred velocity, especially in France. As we will see in a moment, Mairan's neo-Cartesian theory of light implied that the refraction of the light corpuscles did not depend on their mass, so that velocity had to be the parameter of color. In 1737, an unidentified Academician told Mairan that this choice implied a never observed coloration of Jupiter's satellites during their emersion (a point already made in 1692 in Newton's correspondence with Flamsteed). In reply, Mairan argued that the emerged portion of the satellite (about 1/20) during the estimated time of coloration was too small for the coloration to be visible. In his Newtonian treatise of 1752, the Marquis de Courtivron again favored velocity as the parameter of color, because he believed that Newtonian dynamics implied a mass-independent deflection of the light particles (as it does for free fall). Like Mairan, he judged that this choice would have no detectable effect on the color of the satellites of Jupiter.[44]

In the same year, 1752, the short-lived Irish natural philosopher Thomas Melvill independently chose velocity as the parameter of color, arguing like Courtivron that analogy between gravitational and optical forces implied the mass-independence of deflection. He asked the Irish telescope-maker James Short to look for an alteration of the color of Jupiter's satellite at emersion. The result was again negative. In the article "Couleur" of the *Encyclopédie*, published in 1754, d'Alembert adopted Mairan's assumption and speculated that divers should see redder light because of the higher velocity of light in water,

[43]Melvill 1756 [1752]; Brougham 1796, 1797; Jordan 1799. Cf. Cantor 1983, pp. 78–82; Kipnis 1991, pp. 69–74. Brougham's work included dubious considerations on color-dependent reflection by transparent bodies. Some neo-Cartesians, mainly Mairan and du Tour, pursued Newton's idea of reducing inflection to atmospheres of variable density: see below pp. 145–6. The Newtonian Robison (1797a, p. 261) condemned du Tour's attempt, judging that atmospheres were "mere gratuitous assumptions."

[44]Mairan 1717, p. 50; 1738, pp. 12–13, 34–7; Courtivron 1752, 31–3. For mass as the parameter of color, see, e.g., Desaguliers 1717; Smith 1738; Musschenbroek 1739–1751, p. 539. Montucla (1758, vol. 2, pp. 638–9) did not decide between the two options. On the Newton-Flamsteed exchange, see above, chap. 3, note 46. The unnamed Academician probably got his idea from Johann II Bernoulli's prize memoir of 1736: see below p. 148.

even though Mairan had earlier explained that perceived color only depended on the velocity of light in the last transparent medium, the vitreous humor of the eye.[45]

That neither velocity nor mass could be the parameter of color should have become clear after Clairaut's aforementioned study of achromatism (1761). Clairaut showed that the attraction model of refraction led to the same unacceptable dispersion law as long as the deflecting force solely depended on intrinsic characteristics of the light corpuscles (such as their mass or their velocity). In his words, Dollond's measurements "destroyed the possibility of tracing the different refrangibility of the various colors to the different velocity of colored rays, or to a different tendency that would depend only on the nature of the corpuscles and not on the constitution of the refracting materials." In his memoir of 1764 on the same subject, d'Alembert confirmed Clairaut's judgment and recalled the older argument based on the lack of coloration of Jupiter's satellites at emersion. He also made clear that Newton's theory of refraction could be saved by making the deflecting forces depend on the build-up of the light particles. Although this did not prevent the French translator of Smith's optics from reproducing Mairan's idea, a growing number of writers on Newtonian optics recognized that the dispersion observed for different materials required specific optical forces for each simple color.[46]

Whether or not the light corpuscles varied in mass or velocity, they necessarily carried momentum and *vis viva*. Some Newtonians including Musschenbroek and Bishop Samuel Horsley showed that by properly adjusting the mass and number of the corpuscles they could avoid any observable pressure on illuminated bodies without losing the spatial and temporal continuity of luminous impressions. On the contrary, other Newtonians tried to detect the recoil of an illuminated object as a proof of the materiality of light. So did the Cambridge natural philosopher John Michell, in an experiment probably performed in the 1750s and reported in Priestley's *History* of 1772. Michell focused intense light on a copper plate attached to a sensitive balance, and saw the plate move. The non-Newtonian emissionist Mairan had earlier attempted, with Charles du Fay's help, to demonstrate the momentum of light by means of some sort of light mill. Mairan ended up recognizing that the heating of the air next to the plates of the mill was the likely cause of the mill's rotation. The context of this inquiry was the deviation of the tails of comets which he and Euler attributed to the pressure of sunlight. As Mairan rejected Euler's contention that compression waves carried momentum, he meant his experiment to refute Euler's wave theory of light. Conversely, when in 1792 the inventor of the gold-leaf electroscope, Abraham Bennet, failed to detect any non-thermal effect of light focused on a balance

[45]Melvill 1753 [read in early 1752]; Short 1753; D'Alembert 1754; Mairan 1738, p. 31. Cf. Pav 1964, pp. 125–33; Cantor 1983, pp. 64–6; Eisenstaedt 1996, pp. 136–45; 2005a, pp. 92–6. In the "Réfrangibilité" entry of the *Encyclopédie*, D'Alembert (1765b) made clear that for Newton mass was the parameter of color. Short regarded his negative result as supporting Gowin Knight's version of the vibrational theory of light. Upon reading Melvill, Clairaut wrote to the *Philosophical transactions* (1753) to claim Courtivron's priority about the color of Jupiter's satellites at emersion (without naming Mairan).

[46]Grandjean de Fouchy 1762 (after Clairaut), p. 124; Clairaut 1761, p. 423; D'Alembert 1764a, pp. 341–67; Duval-Le Roy, in Smith 1767a, pp. 225n–230n. Priestley referred to Clairaut's analysis in his *history* (1772, pp. 403–5). Bošković (1766, p. 132) gave other reasons for velocity not being the sole parameter of color, and considered the inner structure of particles of light as affecting their interaction with matter. On the necessity of color-specific forces, cf. Cantor 1983, pp. 67–9.

in an evacuated vessel, he made light a vibration of the caloric. For twenty more years, the Newtonians kept referring to Michell's experiment as a proof of the material character of light, and wave theorists to Bennet's as a proof of their own view.[47]

Playing with the velocity of light

In Newtonian theories of light, the finite velocity of light corresponds to the free motion of the light corpuscles. Consequently, any alteration of the direction or speed of this movement should have observable effects. The astronomer James Bradley pioneered the study of this kind of effect in the late 1720s. While attempting to detect stellar parallax[48] at the home of his friend Molyneux, Bradley discovered a larger anomaly which he soon interpreted as a consequence of the motion of the earth around its orbit. He imagined the light from the star to be observed through a narrow tube, and reasoned that a particle of light could only reach the eye (without hitting the walls of the tube) if the tube was inclined with respect to the true direction of the star by an angle depending on the velocity of the earth (see Fig. 4.5). Although Euler later regarded this effect as a mere consequence of the vector composition of velocities, it was often regarded as a proof of the corpuscular nature of light. At any rate, it removed the last doubts on the validity of Rømer's result.[49]

In a letter to the astronomer Jérôme Lefrançois de Lalande written in 1766, Bošković stated that a water-filled telescope would produce a different aberration from which the velocity of light in water could be induced. He reached this conclusion by simply replacing

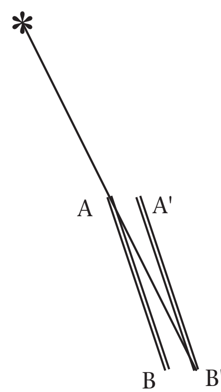


Fig. 4.5. Stellar aberration according to Bradley. During the travel of a light particle from one extremity of the observation tube AB to the other, the earth carries this tube from the position AB to the position A′B′. Consequently, the tube makes the (small) aberration angle $\alpha = A\hat{B}'A' \approx (U/c)\sin\theta$ with the true direction of the star if U denotes the velocity of the earth, c the velocity of light, and θ the angle $A'\hat{A}B'$ between this motion and the direction of the star.

[47]Musschenbroek 1739–1751, vol. 2, pp. 499–500; Horsley 1770, pp. 430–43; Priestley 1772, pp. 309–10; Mairan 1747, p. 427; Bennet 1792, pp. 87–8. Cf. Badcock 1962, p. 101; Schagrin 1974; Worrall, 1982; Cantor 1983, pp. 52–9. On Mairan, Euler, the aurora borealis, and comets, cf. Briggs 1967.

[48]Owing to the finite distance of the stars from the sun, the direction from which we see them should depend on the position of the earth on its orbit (in the Copernican system).

[49]Bradley 1729; Euler 1746b. Cf. Pedersen 2000, pp. 500–2; Eisenstaedt 2005a, pp. 74–84. Euler also considered the aberration of planets, which depends both on the velocity of the planet and on the velocity of the earth.

the velocity of light in air with the velocity of light in water in Bradley's formula, ignoring refraction at the surface of the water. Lalande published an account of Bošković's letter in 1781, in the second edition of his *Astronomie*. The following year, the Scottish astronomer Patrick Wilson published an independent discussion of the water-filled telescope in which he reached the opposite conclusion: if Newton's law of refraction holds in absolute space, then the aberration observed in a water-filled telescope is the same as in a normal telescope. Indeed, if $\mathbf{c}$ and $\mathbf{c}'$ denote the absolute velocities of a light particle before and after entering the water and $\mathbf{U}$ denotes the velocity of the earth, the relative velocity $\mathbf{c}' - \mathbf{U}$ of this particle must be parallel to the axis of the telescope in order to be seen; Newton's theory of refraction implies the equality of the components of $\mathbf{c}$ and $\mathbf{c}'$ parallel to the interface between air and water; therefore, the relative velocity $\mathbf{c} - \mathbf{U}$ is also parallel to the axis, in agreement with Bradley's value for the angle of aberration. Wilson concluded that a test of the water-filling independence of stellar aberration would provide "very strong additional evidence for [Newton's] principles."[50]

Bošković finally published his considerations in 1785, with a derogatory comment on Wilson and the additional suggestion of a diurnal aberration for terrestrial objects seen through the water-filled telescope. Again, his reasoning ignored refraction at the interface between air and water. Robison, who generally admired Bošković's writings, planned to test the diurnal aberration with a glass rod (a simple substitute for the water-filled telesope) but gave up after convincing himself that the effect should not exist. He intuitively grasped that there should be no aberration when there is no relative motion between the observer and the object. In conversations with his friend Wilson, he also understood that refraction at the interface between air and water annihilated Bošković's effect. In the ensuing memoir of 1790, he showed that the same law of refraction applied to the absolute and relative motions of the light particles, the index being given by the ratio of absolute velocies in one case and the ratio of relative velocities in the other. This being understood, the absence of additional aberrations for a water-filled telescope becomes obvious, because the validity of the law of refraction in the observer's frame implies the lack of refraction at normal (axial) incidence. Robison concluded:

> We have a direct proof of the acceleration of light in the above mentioned proportion, and of its refraction being produced by forces acting perpendicularly to the refracting surface, and almost a demonstration that light consists of corpuscles emitted by the shining body.

In modern terms, Robison understood the Galilean invariance of the dynamics of Newton's light corpuscles. His trust in this invariance increased his trust in Newtonian optics so much that he ceased to look for the effects predicted by Bošković.[51]

[50]Lalande 1781, p. 687; Wilson 1782, p. 58. Wilson may have been stimulated by Melvill's (unpublished) idea that stellar aberration depended on the velocity of light in the aqueous humor of the eye. Cf. Pedersen 1980; 2000, pp. 518–20 (Wilson), 530–3 (Bošković).

[51]Bošković 1785; Robison 1790, p. 95. Cf. Pedersen 1980; 2000, pp. 522–9; Cantor 1983, pp. 75–6. In modern terms, the invariance of the law of refraction results from the equivalence of $(\mathbf{c}-\mathbf{U}) \times \mathbf{n} = (\mathbf{c}'-\mathbf{U}) \times \mathbf{n}$ and $\mathbf{c}\times\mathbf{n} = \mathbf{c}'\times\mathbf{n}$, $\mathbf{n}$ denoting the normal to the interface.

Another way to alter the velocity of the light corpuscles is to subject them to gravitation, as should be done if gravitation is truly universal. Michell inaugurated considerations of this kind in the early 1770s. In a contribution to Priestley's *History*, he estimated that the velocity of a light corpuscle lost a fraction 1/500 000 of its velocity when escaping from the sun's attraction. In modern terms, the half-square of the velocity of the corpuscle varies by an amount equal to the value of the gravitational potential on the surface of the sun (from which it is emitted). Consequently, there is a well-defined numerical relation between the velocity loss and the ratio of the mass of the sun to its radius:

$$c^2 - (c - \Delta c)^2 = 2GM/R,$$

if G is the gravitational constant, M the mass of the sun, R its radius, c the normal velocity of light, and Δc the velocity loss. In 1783, Michell thought of exploiting this relation to determine the mass and diameter of stars. Assuming the density of the star to be the same as that of the sun, the velocity loss varies like the square of the diameter. It therefore becomes measurable for large enough stars: according to the Newton–Descartes derivation of the law of refraction, the velocity loss implies a variation of the optical index of the same order of magnitude as the relative velocity loss (this is true regardless of the relation between optical force and color). At the end of his memoir Michell suggested an experiment in which the light from the two unequal partners of a binary star would be observed through an achromatic prism. To his disappointment, the Astronomer Royal Nevil Masekelyne and William Herschel failed to detect any shift of the refracted light for the stars they observed.[52]

Michell's reasoning further implies that for a large enough star (about 500 times the diameter of the sun, with the same density) the light corpuscles do not have enough initial velocity to escape from the star's gravitation field. Hence Michell deduced the possibility of bodies about which "we could have no information from sight." He understood that, despite their invisibility, these bodies could still have observable gravitational effects. For instance, they could have visible satellites orbiting around them. In his *Système du monde* of 1796, the Marquis de Laplace similarly imagined bodies that "by virtue of their attraction would not let any of their rays reach us." Laplace also indicated that for normal bodies the emitted light should be slowed down in a way measurable through the resulting increase in their aberration. Seduced by Laplace's idea of subjecting the light corpuscles to gravitation, in 1801 the German astronomer Johann Georg von Soldner computed the resulting deviation of a light ray traveling near the surface of a celestial body, with an error of factor two. Michell's friend Henry Cavendish had privately obtained the correct value of the gravitational deflection ($2GM / Rc^2$) some twenty years earlier. As Soldner noted, the effect was too small to be detected by contemporary means.[53]

[52]Priestley 1772, pp. 786–91; Michell 1784 [1783]; ibid. on pp. 54–5 for Masekelyne and Herschel. Cf. McCormmach 1968; Cantor 1983, pp. 62–4; Eisenstaedt 2005a, pp. 117–38. As Arago later explained, an achromatic prism is preferable because it is much easier to observe the shift of a single bright line than the shift of a continuous spectrum.

[53]Michell 1784, p. 50; Laplace 1796, vol. 2, pp. 304–6; Soldner 1804 [written in 1801]. Cf. Badcock 1962, p. 107; Schaffer 1979; Eisenstaedt 1991; 2005a, pp. 120–50. Soldner criticized the assumption of a fixed initial velocity in Laplace's reasoning. For the gravitational deviation of light, general relativity gives $4GM / Rc^2$, in agreement with Soldner's erroneous result! Many years later, Philipp Lenard exploited this coincidence to claim Soldner's priority over Einstein.

Gravitation is not the only way in which the velocity of the light corpuscles may depend on their source. In an unpublished memoir read to the Royal Society in 1786, the Scottish astronomer Robert Blair noted that light corpuscles emitted by a moving source should travel with a velocity obtained by compounding their normal velocity with the velocity of the source, and also that the motion of the earth implies a periodic variation of the apparent velocity. Before Robison, Blair understood than in the Newtonian theory of light, optical phenomena depended only on the relative motion of objects and observer. He even suggested a crucial test of this property by comparing the apparent velocities of light from candles placed in different directions: these velocities should be the same in the emission theory, whereas in the (stationary) ether theory they should vary in a manner depending on the velocity of the earth through the ether.[54]

For the velocity measurements, Blair suggested an improvement of Michell's technique, namely, refraction through a series of achromatic prisms. For cases of altered velocity, he considered the light emitted from the opposite sides of a rotating planet (Jupiter), and the observation of two fixed stars directed along and against the motion of the earth. He thus hoped to confirm the Newtonian theory. His friend Robison publicized his suggestions, adding a test on Saturn's rings. In the early years of the nineteenth century, Robison's faith in Newtonian optics had somewhat declined. In 1804, shortly before his death, he pressed Herschel to perform the test on Saturn's rings, which he believed to be "decisive of the question Is light an emission of matter, or is it an elastic undulation."[55]

In the same spirit, the French astronomer François Arago later tried to determine whether the motion of the earth around its orbit affected the refraction of stellar light through an achromatic prism. In 1810, he reported to the French Academy that the dispersion of his measurements was inferior to the theoretical shift. At that time he still believed in Newtonian optics and in the resulting correlation between velocity and refraction. He did his best to save Newton:

> There seems to be no other way of explaining [my result] than assuming that luminous bodies emit rays of every velocity, provided that these rays are only visible within certain limits. Under this assumption, indeed, the visibility of the rays depends on their relative velocity, and, as this velocity also determines the amount of refraction, the visible rays will always be equally refracted.

Arago mentioned the recent discovery of ultraviolet and infrared rays to support this original hypothesis. By the same means he explained why Michell's gravitational shift of stellar light had never been observed.[56]

[54]Blair 1786. Cf. Cantor 1983, pp. 87–8; Eisenstaedt 2005a, chap. 9; 2005b.

[55]Robison to Herschel, 14 April 1804, quoted in Cantor 1983, p. 88. A velocity change implies a change of refraction, whether or not velocity is the parameter of color. Michell's and Blair's recourse to achromatic prisms to measure velocity excludes its being the parameter of color, since in this case different velocities would yield the same deviation by the compound prism. According to modern wave theory, a regular prism yields a refraction modified by the Doppler shift of the stellar light, which is of the same order of relative magnitude as the Newtonian velocity effect; an achromatic prism is of course insensitive to this Doppler shift.

[56]Arago 1853 [read in 1810], p. 563. Cf. Eisenstaedt 2005a, chap. 10. Early in the century, William Herschel and Marc Auguste Pictet had tentatively identified thermal radiation with infrared light; and Wilhelm Ritter had

Thin plates and the colors of bodies

The most neglected aspect of Newton's optics was his study of the transmission of light through thin plates or films, despite the popularity of his theory of the color of bodies. The Newtonians disliked the implication of a wave process that seemed at odds with the rest of the *Opticks*. They paid little attention to the fits of easy transmission and reflection, or they reinterpreted them within the restricted paradigm of light corpuscles and attractions.[57] 'S Gravesande simply ignored them. Musschenbroek briefly mentioned them in the first edition (1634) of his treatise, and removed them from the second edition (1641). Smith cited Newton's observation: "The air between the glasses, according to its various thickness, is disposed in some places to reflect, and in some others to transmit the light of any colour," without giving the fit-based explanation. In 1748, Bošković invoked a periodic dilation and contraction of the light particles, which he regarded as a compound of point atoms. In 1753 Melvill similarly interpreted the fits through a rotating polarity of the light corpuscles. Michell and Priestley in the 1770s, and Brougham in the 1790s imagined periodic zones of force within the film.[58]

In his *Encyclopédie* articles on light and color d'Alembert mentioned Newton's fits, their usefulness in explaining partial reflection, and their ether-based explanation, with a skeptical comment:

> Ingenious though it is, this theory is avowedly very far from having all that is needed to entirely convince and please the mind. We must here limit ourselves to the simple facts and wait, before knowing or seeking the causes, until we are better informed about the nature of light and bodies, which means that we may have to wait very long, perhaps forever.

In the *Encyclopedia Britannica* (1797), Robison paid lip service to the fits but judged that their ether-based explanation conflicted with Scottish common sense. To Newton's explanation of partial reflection in terms of fits, he substituted his own based on an agitation of the particles of the surface impacted by the rays. His account of thin plates was purely descriptive and mostly based on Boyle's and Hooke's earlier observations.[59]

On the experimental side, the most detailed studies of thin plates were those of Abbot Guillaume Mazeas in the 1750s and those of Etienne François du Tour in the two

discovered ultraviolet light through its chemical action on silver chloride: cf. Chappert 2004, pp. 282–5; Hentschel 2002, pp. 60–4; Frercks, Weber, and Wiesenfeldt 2009.

[57]Cf. Hakfoort 1995, pp. 46–8.

[58]'S Gravesande 1720–1721; Smith 1738, p. 98; Musschenbroek 1734, pp. 342–3; Bošcović 1748b; 1758, §§495–9; 1766, pp. 121–9; Melvill 1753; Priestley 1772, pp. 310–11 (with report of Michell's views). Cf. Cantor 1983, pp. 83–6. On Musschenbroek's rendition and its connection to Desaguliers's, cf. Hakfoort 1995, pp. 47–8. On Michell's alternative, cf. Steffens 1977, p. 73. Carlo Benvenuti's *Dissertatio* (1754) had the fullest discussion of Newton's fits, with mention of Bošković's interpretation (on p. 92).

[59]D'Alembert 1754, p. 330 (citation); 1765a, p. 721; Robison 1797a, p. 307. The moderate Newtonian Montucla (1758, vol. 2, pp. 639–42) gave a favorable review of Newton's fits.

following decades. Mazeas rubbed two plates of glass against each other and observed rings that did not depend on the thickness or nature of the intermediate substance, against Newton's observations. Despite numerous experiments and some antipathy with Mazeas, du Tour failed to elucidate the matter. In the late 1750s, Michel Ferdinand d'Albert d'Ailly, Duc de Chaulnes, experimented on Newton's thick plates and accidentally noticed that the rings became more visible as he breathed upon them. He concluded that the rings depended on diffraction on (irregularities of) the first surface of the plate, and not on diffuse reflection on the second (as Newton assumed). Despite his general commitment to Newton's system, Priestley later judged that this French work had discredited Newton's own:

> In no subject to which he gave his attention does he seem to have overlooked more important circumstances in the appearances he observed, or to have been more mistaken with respect to their causes.

Although this criticism later proved to be excessive, Priestley's contemporaries tended to share his suspicion. In 1800 Jordan declared Newton's observations on thin plates "absolutely illusive" and suggested that the true phenomena of thin plates belonged to inflection.[60]

Although Newton's theory of the colors of bodies relied on analogy between body particles and thin plates, it did not involve his explanation of the colors of thin plates nor any assumption on the nature of light other than the heterogeneity of white light. Hence one might expect this theory to have been as broadly accepted as the concept of simple color. In reality it was not, because it implied additional assumptions on the microstructure of colored bodies. The neo-Cartesians ignored it or replaced it with interpretations that depended on the vibrational theory of light.[61] Most of the Newtonians accepted it for most of the eighteenth century, although they were skeptical about Newton's attempt to determine the size of the corpuscles of colored bodies. 'S Gravesande, Musschenbroek, and Smith included it in their influential textbooks; D'Alembert mentioned it in his *Encyclopédie* articles on light and colors.[62]

A long time elapsed, however, before anyone performed decisive experiments on this matter. In 1853, Melvill queried whether a powder of broken thin plates would behave like a colored body, and tried and failed to make such a powder by freezing soap bubbles. At last, in 1777 the British scholar Edward Hussey Delaval published a number of experiments which he believed to prove the correlation between color and corpuscular size. In the most influential of these experiments, he observed the color change of a solution of purple flowers when an acid or an alkali was gradually added and found it to agree with the color change of a thin plate of decreasing or increasing thickness. This

[60]Mazeas 1752, 1755; Du Tour 1763, 1773; Chaulnes 1755 [read 1758]; Priestley 1772, p. 510; Jordan 1800. Cf. Shapiro 1993, pp. 204–7. As Thomas Young later observed, Mazeas confused the thin-film effect with diffraction by the scratches caused by rubbing the two plates together. Mariotte (1681, p. 293) had already described the colors of rubbed plates: cf. Arago 1817, p. 7.

[61]Malebranche and Mairan did not address the color of bodies; Euler and Nollet relied on the resonance of globules: see below, pp. 152, 158.

[62]Cf. Shapiro 1993, pp. 226–9.

confirmed Newton's theory, granted that the corpuscles were "united into larger masses by the alkali, and divided into smaller, by the acid."[63]

Around that time, chemical theories of the color of bodies began to threaten Newton's theory. The still dominant phlogiston chemistry had inherited from Georg Ernst Stahl the correspondence between color and amount of phlogiston. Around 1880, the French pharmacist and chemist Christophe Opoix supported this view by observing color change under heat, the Genevan botanist Jean Senebier by studying color and chemical changes induced by light. They both suggested that colored light and chemical substances had variable affinities depending on the amount of phlogiston they included. Senebier's authority as the founder of photosynthesis, the contemporary growth of the dyeing industry, the multiplication of photochemical discoveries, and Claude Louis Berthollet's powerful support boosted the new chemistry of light. This theory easily accommodated Lavoisier's chemical revolution, for the phlogiston impregnation could easily be replaced with oxygen depravation.[64]

In a world increasingly dependent on the chemical industry, the chemists challenged the physicists' theories, and the physicists were more willing to listen to them. As we will see at the end of this chapter, the new photochemistry converted Swiss and German followers of Euler's vibrational theory to the Newtonian materiality of light. Ironically, this movement also endangered Newton's physicalist explanation of the colors of bodies. In 1785, its former champion Delaval gave it up in favor of the chemical view after finding that a basic consequence of Newton's theory contradicted observation. This theory evidently implies that, for colored transparent bodies, the color of the reflected light should be complementary to the color of the transmitted light (as is the case for thin plates). Although Newton was well aware that in most cases the reflected and transmitted lights appeared to be of the same color, he saved his theory by assuming that a good part of the transmitted light was reflected back at the end of the transparent body. Delaval eliminated this possibility by blackening the end of the body and proving the absence of reflected light in this case. Few, however, realized the devastating power of Delaval's finding. Newton's theory still had a few more years to live.[65]

Let us briefly see how the story finished in the next century. In France, the chemist–physicist Jean Henry Hassenfratz and the military engineer Claude Antoine Prieur-Duvernois studied the absorption spectra of colored transparent bodies in 1807–8 and found them to be incompatible with Newton's theory and compatible with the chemical doctrine of affinities. A fervent follower of Laplace's neo-Newtonian doctrine, Jean-Baptiste Biot, nonetheless defended Newton's theory in his physics treatise of 1816. In Britain, there was little resistance to the chemical view. Thomas Young, who originally embraced Newton's theory, soon realized (around 1803) its incompatibility with the observed spectrum of transmitted light. Lastly, around 1830, John Herschel and David Brewster demonstrated that selective absorption determined the color of most bodies.[66]

[63]Melvill 1756 [1753], p. 66; Delaval 1777, p. 16. Cf. Shapiro 1993, pp. 229–34.

[64]Cf. Shapiro 1993, chap. 6.

[65]Delaval 1785. Cf. Shapiro 1993, chap. 7.

[66]Cf. Shapiro 1993, chaps. 8–9; Levitt 2009, pp. 20–31. Giovanni Battista Venturi's earlier study of absorption spectra (1789) remained unnoticed.

4.3 Neo-Cartesian optics

We now come to the neo-Cartesian theorists who pursued the Cartesian reduction of physical phenomena to contact action but took a few liberties with Descartes' own optics. As we saw in chapter 2, seventeenth-century Cartesians like Pardies and Huygens already felt free to contradict Descartes on the instantaneous propagation of light. Most of the eighteenth-century neo-Cartesians did so, especially after Bradley's discovery of stellar aberration had confirmed Rømer's measurement of the speed of light. Yet they did not follow the details of Huygens's system. They only retained the elasticity of the balls of the second element (which is required for finite-speed propagation), and the foundational analogy with sound. Huygens's name was rarely mentioned, except by a few German textbook writers. His principle of elementary waves and its application to rectilinear propagation and double refraction were ignored. The few of Huygens's readers who mastered the needed geometry believed that this principle was actually incompatible with rectilinear propagation. There was almost no interest in the marginal phenomena of crystal optics, on which Huygens reigned supreme; and there was a huge interest in the nature of colors, on which he remained silent.[67]

The eighteenth-century neo-Cartesians nonetheless followed Huygens, Pardies, and Ango in giving a prominent role to the analogy between sound and light. This is true to such an extent that it is impossible to discuss their works without previously recalling rudiments of contemporary acoustics.

Some eighteenth-century acoustics

Eighteenth-century writers on acoustics inherited from the previous century the idea that sound was a vibration propagated through air and that the pitch of musical sounds (tones) corresponded to the frequency of the vibrations. Their opinions otherwise varied. A few important authors, including Philippe de la Hire, Mairan, Le Cat, 's Gravesande, and Musschenbroek believed the frequency of the observable vibration of sonorous bodies to differ from the frequency of the emitted sounds. This idea originated in Perrault's report, in 1680, that a sensibly but silently vibrating string produced an audible sound when a solid obstacle was brought next to the string. On propagation, the Newtonians usually adopted a qualitative rendering of Newton's theory. Early in the eighteenth century, William Derham verified two singular consequences of this theory: that the velocity of light does not depend on pressure and that it does not depend on the kind of sound. A few neo-Cartesians (Le Cat, Nollet), and even one Newtonian (Robert Smith) felt free to follow the different theory of sound propagation that Mairan propounded in 1720.[68]

[67]Cf. F. Dijksterhuis 2004, pp. 249–54; Hakfoort 1995, pp. 53, 123–4 (German reception). In 1713, the French *géomètre* Antoine Parent praised Pardies as the originator of the wave theory of light, and rejected Huygens's principle as leading to the straying of light in the shadow (Parent 1713): cf. Shapiro 1973, pp. 253–6. Johann II Bernoulli also criticized this principle: see below, p. 146. Exceptionably favorable were Fontenelle (1733) and Montucla (1758, vol. 2, p. 197).

[68]Fontenelle 1716 [after de la Hire]; 1720 [after Mairan]; Le Cat 1739, p. 262; 'S Gravesande 1720–1721, vol. 1, p. 339; Musschenbroek 1739–1751, vol. 2, p. 650; Derham 1708; Le Cat 1739, pp. 266, 585; Nollet 1745, p. 477; Smith 1759, p. 106 (Mairan is not named).

In agreement with Perrault, Mairan assumed that the quivering of the small parts of a sonorous body, not its global oscillation, was responsible for its sound. Between the sonorous body and the ear, he imagined a succession of particles of air that played the role of resonators at the quivering frequency. Propagation then resulted from resonance between successive particles. Mairan held different sorts of particles (with different proper frequencies) responsible for the propagation of sounds of different pitches, in alleged analogy with Newton's idea that different particles of light were necessary to the transmission of different colors. He thus purported to explain the transmission of several tones by the same portion of air.[69]

Resonance, then called "sympathetic vibration," played a central role in Mairan's acoustics, as it did in a few later optical theories. This phenomenon had been known since Greek antiquity for vases and musical strings. In the seventeenth century, the resonance of two strings was commonly regarded as a consequence of the synchronic action of the aerial pulses from the first string on the second string. The only case originally considered was that of a resonant string tuned at a multiple of the frequency of the stimulating string, until in 1677 the British mathematician John Wallis reported that a string tuned at submultiples of the stimulating frequency experienced very small vibrations that imitated a succession of strings resonating at the exciting frequency. Most eighteenth-century writers on acoustics and the theory of music were familiar with these results. Jean Philippe Rameau and d'Alembert used resonance as one of the empirical facts on which to base the theory of harmony. Like most of their contemporaries, they took the commensurability of the frequencies to be the condition for the resonance of two oscillators, although they knew the resonance to be much stronger when the frequency of the excited resonator was a multiple of the exciting one. Their understanding of resonance still depended on the periodic-pulse idealization of tones. It did not involve any equation of motion, although in 1739 Euler obtained the theory of the sinusoidally driven harmonic oscillator in the context of his prize-winning work on tides. Euler did not apply this result to acoustic resonance, presumably because he refused to privilege harmonic oscillations in this domain. No one else did in the eighteenth century.[70]

In the course of that century, the progress of experimental and mathematical acoustics somewhat bridled theoretical speculation, enough to eliminate Perrault's and Mairan's concepts of vibration and propagation. Joseph Sauveur opened the century with a masterful study of musical strings and organ pipes, including an influential study of the higher modes of vibrations. In 1713, Brook Taylor applied Newtonian Mechanics to the small motions of a tense elastic string, and obtained the fundamental mode of vibration in which the string has a sine shape at any time, with amplitude varying as a sine function of time. In a series of memoirs published in the 1730s and 1740s, Daniel Bernoulli generalized the notion of a simple mode of oscillation to any vibrating system. These modes are sine functions of time with various discrete frequencies, not necessarily commensurable to each other. Daniel

[69]Fontenelle 1720 [after Mairan].

[70]Wallis 1677, pp. 839–42; Euler 1740, pp. 301–4; 1739b [1750]. On early concepts of resonance by Vitruvius, Francis Bacon, Jeronimo Fracastoro, Isaac Beeckman, Marin Mersenne, and Galileo Galilei, cf. Truesdell 1960, pp. 19, 22, 26–7, 34–5, 118. On Rameau and d'Alembert, cf. Bailhache 2001. Resonance already played an important role in Mersenne's discussion of consonance, the two notions being bridged by the idea of synchronized pulses.

Bernoulli also enunciated the principle of superposition according to which more general vibrations of an elastic body could be obtained by superposing simple sine modes.[71]

In his deductions of simple modes, Bernoulli presupposed that the restoring force acting on a given mass point was proportional to the displacement of this point, so that he only needed ordinary differential equations. In 1746, d'Alembert derived the partial differential equation of the general (small) motion of a vibrating string, which we would now write as

$$\sigma \frac{\partial^2 y}{\partial t^2} = T \frac{\partial^2 y}{\partial s^2},$$

if y denotes the transverse displacement of the string at time t at the distance s from its extremity, σ its mass per unit length, and T its tension. D'Alembert expressed the most general solution in the form

$$y = \Psi(ct + s) + \Gamma(ct - s),$$

where $c = \sqrt{T/\sigma}$ and the functions Ψ and Γ are only restricted by the boundary conditions. Euler produced a similar theory in the following year. There followed a long controversy between Daniel Bernoulli and the other string theorists on the meaning and generality of sums of harmonic oscillations. This controversy has some importance in the history of optics, as it conditioned the interpretation of white light in the vibrational theories.[72]

In 1759–60 Euler and Lagrange applied the new calculus of partial differential equations to sound propagation in one and three dimensions. This beautiful theory allowed some insights into the functioning of wind instruments. It also implied that plane waves of any shape could be propagated without any change of shape, and that the velocity did not depend on the shape or frequency of the waves, against Euler's earlier opinion on this matter. Very few of Lagrange's and Euler's readers mastered the mathematics needed to understand this theory. Most writers on acoustics until the early nineteenth century favored Daniel Bernoulli's more elementary methods. His influential memoir on organ pipes of 1762 ignored Lagrange's earlier study, despite much overlap in the results. Bernoulli reasoned directly in terms of proper modes (stationary waves) analogous to the modes of vibrating strings, as he had done since the beginning of his involvement in vibration theory. Most strangely, he assumed that the waves propagated from a source were analogous to the stationary waves in a conic organ pipe, with an antinode at the source and a periodic series of nodes henceforth; he obtained the velocity of sound by assuming that the sound took half a period of oscillation to travel from one "concameration" of the wave to the next.[73]

[71]Cf. Burckhardt 1901–1908; Truesdell 1960, part II; Dostrovsky 1975; Cannon and Dostrovsky 1981, pp. 15–20; Darrigol 2007, pp. 351–60.

[72]D'Alembert 1747; Euler 1749; Bernoulli 1753a, 1753b. Cf. Burckhard 1901–1908; Truesdell 1960; Darrigol 2007 and further reference there to studies by Rudolf Langer, Ivor Grattan Guinness, and Umberto Bottazini.

[73]Lagrange 1760; Euler 1766. Cf. Burhardt 1901–1908; Truesdell 1960; D. Bernoulli 1762. A "concameration" is a part of the wave delimited by two successive antinodes. In 1736, Johann II Bernoulli had applied a similar notion to sonorous fibers in the air: see below p. 148.

Lagrange and Euler did not try to solve the daunting problem of the propagation of waves passing an obstacle. They could not decide whether or not the waves spread out behind obstacles as Newton predicted. Most acoustic writers ('s Gravesande, Musschenbroek, Le Cat, Nollet, Euler, Diderot, Lambert, Robison, Chladni, and Haüy) assumed ray propagation, in conformity with Mersenne's and Kircher's opinion in the previous century. They usually referred to the observed direction of echoes and to the common explanation of speaking and hearing trumpets by successive reflections on their walls. And they dared question Newton's arguments for the straying of light behind obstacles. For example, in the Trumpet entry of 1797 for the *Encyclopaedia Britannica* Robison pondered:[74]

> Whoever considers the Newtonian theory of the propagation of sound with intelligence and attention, will see that it is demonstrated solely in the case of a single row of particles; and that all the general corollaries respecting the lateral diffusion of the elastic undulations are little more than sagacious guesses, every way worthy of the illustrious author, and beautifully confirmed by what we can most distinctly and accurately observe in the circular waves on the surface of still water.

A few authors including Mairan, d'Alembert, Matthew Young, and James Wood accepted Newton's arguments. They usually did so while repeating Newton's main objection against the wave theory of light: that light, if it were analogous to sound waves, would curve into the shadow. Some supporters of the vibrational theory of light like Le Cat or Benjamin Franklin shared Hooke's and Huygens's view according to which the lateral propagation of waves beyond an obstacle depended on the density of the medium: it was very important of water waves, intermediate for sound in air, and negligible for light.[75]

Malebranche's optics

The most influential Cartesian philosopher at the turn of the seventeenth and eighteenth century, the Oratorian priest Nicolas Malebranche, produced a theory of light that was in some respects closer to the modern wave theory than Descartes'. Malebranche adopted the general scheme of reduction to matter and motion, in a freely modified form. Whereas Descartes gave ideal hardness to the particles of his three elements, Malebranche explained every hardness, that of the particles and that of gross matter, by the confining effect of pressure from the subtle matter of the second element. As he believed every force to derive from moving matter, he replaced Descartes' hard balls with tiny vortices, and traced the pressure of subtle matter to the centrifugal force of these vortices. In 1675, he devoted the last chapter of his influential *Recherche de la vérité* to this theory of matter.[76]

[74]'S Gravesande 1720–1721, vol. 1, p. 341; Musschenbroek 1739–1751, vol. 2, p. 701; Le Cat 1739, p. 274; Nollet 1745, p. 436; Euler 1746a, pp. 175–6; Diderot 1748, p. 46; Lambert 1763, pp. 90–3; Robison 1797b (citation); Chladni 1802, p. 239; Haüy 1803, pp. 298–9; Kircher 1646, p. 1650. On the history of speaking trumpets, cf. Hunt 1978, pp. 121–30. On Lambert's theory, cf. Hakfoort 1995, p. 138.

[75]Mairan 1717, p. 11; D'Alembert 1765a, p. 718 (with a rejection of Huygens's theory); M. Young 1784, sect. 4; Wood 1801, pp. 1–2; Le Cat 1739, p. 308. Cf. Cantor 1983, pp. 125–6 (Franklin), 115–17 (Le Cat), 125 (popularity of Newton's shadow argument against the wave theory). On the gradualist view, cf. Hakfoort 1995, pp. 136–7.

[76]Malebranche 1674–1675, vol. 2, chap. 9. Cf. Robinet 1970. Descartes explained the rigidity of a body by the (God given) persistence of the mutual rest of its parts. On Malebranche's philosophy and theology, cf. Hankins

Malebranche first expounded his reflections on light and colors in a memoir read on 4 April 1699 to the French Academy of Sciences. From Descartes he retained the idea that light was a pressure instantaneously transmitted through the contiguous particles of subtle matter. But he rejected the hardness of the particles of this matter and the interpretation of colors as the rotation of these particles. He found inspiration in the micro-vortex picture and in the analogy between sound and light. As the *Secrétaire perpétuel* Bernard de Fontenelle announced:

> Philosophy has shaken off the yoke of authority, and the greatest philosophers persuade only through their reasons. No matter how ingenious Mr. Descartes's system on light might be, the Father Mallebranche [sic] has given it up to establish a new one, conceived on the model of the system of sound, and this analogy can pass for a sign of truth in the eyes of those who know how uniform nature is in regard to general principles.

According to Malebranche, Descartes's theory of colors made it impossible for rays of light of different colors to cross without losing their color, because the same hard sphere of subtle matter could not carry different amounts of rotation. In contrast, Malebranche judged that his tiny vortices, being deformable, could simultaneously carry the deformations induced by crossing rays.[77]

Malebranche noted that the elasticity of the vortices seemed to imply a finite speed for the propagation of light, just as the elasticity of the air does for the propagation of sound. He believed, however, that the existence of very rigid bodies such as diamond implied that the pressure of the subtle matter on them was quasi-infinite, in conformity with "the perfection of the Creator." As he also believed that the velocity of sound increased with atmospheric pressure, this implied that the velocity of light should be infinite by analogy. He was apparently unaware of Newton's theory of sound propagation and of contemporary measurements of the velocity of sound, which indicated that the propagation velocity did not depend on pressure. And he did not believe in Rømer's determination of the velocity of light, which was still controversial.[78]

Malebranche's light therefore consisted of the quasi-instantaneous and rectilinear transmission of pressure variations through rows of tiny vortices. He assumed the periodic character of these vibrations, and related color to the "promptitude" or frequency of the "vibrations of pressure." As a "proof" of this view, he offered the analogy with sound:

> Light and the various colors are like sound and the various tones. The *amplitude* of *sound* comes from the higher or lesser *force* of the vibrations of gross air, and the *diversity of tones* from the higher or lesser *promptitude* of these vibrations, as everybody knows. Consequently, the *force* or brilliance of colors also comes from the higher or lesser *force* of the vibrations, not of the air but of the subtle matter, and the *various species of colors* of the higher or lesser *promptitude* of these vibrations.

1967. As Pierre Duhem (1896) noted, there is some similarity between Malebranche's subtle matter and some of Kelvin's models of the ether in the nineteenth century.

[77]Malebranche 1699; Fontenelle 1699. Cf. Duhem 1896.

[78]Malebranche 1699, pp. 27–8; 1712, pp. 260–1.

In favor of this analogy, Malebranche argued that both the auditory nerve and the optical nerve could only respond to the strength and promptitude of the impressions made by sound and light. He also evoked the colored after-images perceived after seeing a bright object and closing one's eyes. He believed these colors to be successively white, yellow, red, and blue, as befits a decreasing promptitude of the induced vibrations of the retina. For this reason, or perhaps because he knew that a painter could make all needed colors from these four colors, he assumed four primaries.[79]

Malebranche is not likely to have read Newton's optical letters in the *Philosophical Transactions*, which contained the first suggestion to identify color with frequency in the wave theory of light. Like most non-British philosophers, he became aware of Newton's work in optics through the *Optice* of 1706. In 1707, this reading prompted him to repeat Newton's basic experiments about the separation of simple colors. In a contemporary letter to a friend he reported:

> Although Mr. Newton is not a physicist, his book is very curious and very useful to those who have good principles in physics. Besides, he is an excellent *géomètre*. Everything I think about the properties of light can be adjusted to his experiments.

Newton indeed was not a physicist in the Cartesian sense. Malebranche only retained Newton's facts: the heterogeneous character of white light and the stability of its simple-color components; and he maintained that light was a periodic vibration whose frequency defined color:

> I say that whiteness is the most composite of all [colors] because it is made of the gathering of the vibrations, varying in promptitude, which each part of the flame produces in the subtle matter. As everything is full and infinitely compressed, every ray conserves through its entire length the same promptitude of vibration as the small part of the flame that produces it. And because the parts of the flame have a varied motion, the rays of colors necessarily have different vibrations and refractions. But on this subject one should consult the excellent work of Mr. Newton.

That the frequency of vibrations shares the immutability of Newton's simple colors is an important additional argument for identifying color with frequency. Malebranche naïvely justified this invariability by instantaneous propagation.[80]

Malebranche gave up the four primary colors of his previous theory, though not to adopt Newton's continuum of simple colors. He misread Newton as asserting the existence of seven simple colors only, and cited Newton's analogy between the diatonic scale and the spectrum of white light as a confirmation of this view:

> When an octave is harmonically divided, that is to say, in such a manner that the various tones contained in this octave be commensurable or that the aerial vibrations that cause them coincide and start together again as frequently as possible without

[79]Malebranche 1699, p. 32. Malebranche plausibly drew on Ango/Pardies for the analogy between sound and light, for the explanation of rectilinear propagation (balancing of the pressures of lateral vortices), and for making frequency a parameter of color; and on Huygens for the dynamic elasticity of the balls of the second element. Cf. Costabel 1964.

[80]Malebranche to Berrand (undated), quoted in Robinet 1970, p. 300; Malebranche 1712, p. 258.

> destroying each other, there can only be a definite number of tones; similarly, there can only be a definite number of simple rays. Moreover, in the experiment that Mr. Newton has done ... in order to exactly determine the specific quantity of refraction of each simple color, he has found that the simple colors were harmonically divided.

This misconstruction of Newton's musical spectrum increased the analogy between light and sound, since the seven colors of the rainbow now directly corresponded to the tones of the diatonic scale. Malebranche explained the genesis of the seven colors through a harmonization of the vibrations of the parts of the sun:

> It seems likely that next to the surface, the little vortices are constrained to agree to do their vibrations in mutually commensurable times, even though they are caused by the irregular movements of the parts of the sun.

He probably had in mind a process akin to acoustic resonance, in which the enhancement of an oscillation by another was often believed to require the commensurability of frequencies.[81]

Like Newton, Malebranche assumed that the least refracted rays, the red ones, were also the strongest. Whereas Newton associated the strength of the rays with the size of the corpuscles of light, Malebranche associated it with the period of vibrations:

> Hence [from Newton's experiments] one may conclude, with much plausibility, that the red ray, which is the strongest one, since it is less refracted than the other rays, is not as promptly repelled or repeats its vibrations less frequently than the following rays; and that the violet ray, which is the last and the most feeble is the one whose vibrations are the smallest and the promptest, or repeat themselves most frequently.

Having assumed a quasi-instantaneous propagation of light, Malebranche could adopt neither Newton's nor Huygens's (or Pardies's) derivation of the law of refraction. His own derivation was based on his theory of matter, according to which the centrifugal pressure of the tiny vortices of subtle matter was smaller in the denser medium than in the rarer, because of the lesser porosity of the former. As an oblique ray enters the denser medium, its angle with the normal has to decrease because the pressure acting from neighboring vortices on the vortices traversed by the ray is stronger in the rarer medium. As the perpendicular component of this pressure is proportional to the sine of the angle of incidence, Malebranche believed that the sine law of refraction followed from this mechanism. He also deduced partial reflection as a kind of reaction to the refractive action.[82]

This reasoning of Malebranche and more generally his mechanics of subtle matter betrays his poor understanding of the laws of mechanics. His belief that Newton only had seven simple colors shows how superficially he had read the *Optice* and how rough his replication of Newton's experiments must have been. When he published his improved

[81]Malebranche 1712, pp. 301–2.

[82]Malebranche 1712, pp. 294–8 (refraction), 302 (citation). Newton had associated a wavelength with the force of rays in his own version of the wave theory in 1672: see chap. 2, p. 87 (the wavelength is ill defined in Malebranche's theory, since the velocity of light is infinite). Malebranche (1712, p. 302) briefly mentioned the colors of thin plates, with no theory.

theory of light as an appendix to the 1712 edition of his *Recherche de la vérité*, this weakness was imperceptible to most of the French savants. His authority facilitated both the French reception of Newton's optics and the continuation of the Cartesian tradition. His French readers adopted a few components of his theory: the Cartesian reduction to contact action, the Newtonian concept of simple color, and the relation between color and frequency. Malebranche could content himself with this filtering reception, as he himself regarded his theory as a set of "conjectures or general views proven insufficiently."[83]

Mairan's optics

In 1717, a former student of Malebranche, the young and ambitious Jean-Jacques Dortous de Mairan, won a prize of the Academy of Bordeaux for a dissertation on what we would now call luminescence. In this writing Mairan distinguished two conceptions of light, without naming their inventors. The first made light "vibrations of pressure" transmitted through subtle matter, "as the trembling of a sonorous body only reaches the ear because it has excited a similar movement in the air." The other system made light a flux of corpuscles emitted from the source, in analogy with smells. After praising the first system, Mairan rejected it for a reason he probably found in Newton's *Principia*: by analogy with hydrostatic pressure, the vibrations of pressure had to be transmitted through the whole mass of subtle matter behind screening objects, so that there could be neither shadows nor nights. Mairan next brushed away evident objections to the emission theory, such as the resistance offered by the ether to the traveling corpuscles of light or the rarefaction of these corpuscles at large distances from the source. Unlike Malebranche, he accepted the finite velocity of light as definitely proven and used it as a further argument in favor of the emission theory (since Malebranche's and Descartes's theories assumed instantaneous propagation). He identified the matter of light with the "very subtle and very agitated" chemical principle then called sulfur, and explained the cold light of glow-worms, fireflies, will-o'-the-wisp, rotting wood, rubbed diamond, the Bologna stone, etc., by an excess of this sulfur liberated by chemical alteration, friction, or former illumination.[84]

On colors, Mairan endorsed Newton's theory, with the comment that "the ingenious experiments which he used to confirm it could by themselves immortalize a less famous name than his." While Mairan adopted the corpuscules of light, he remained a Cartesian who excluded void, reduced all interactions to contact, and built ordinary matter from particles interspersed within subtle matter or ether. In 1722, 1723, and 1738, he published three lengthy memoirs the main purpose of which was to give a neo-Cartesian derivation of the laws of reflection and refraction. Although he did not deny the expediency of Newton's reasoning with distance forces, he believed the essence of reflection and

[83]Malebranche 1712, p. 302.

[84]Mairan 1717, pp. 2 (citation), 2–7 (vibrations), 12–13 (emission), 22 (sulfur), 53–4 (luminescence). Cf. Kleinbaum 1970. Mairan's "phosphores" and "noctiluques" enjoyed various forms of chemically or electrically induced luminescence, as well as phosphorescence in the case of the Bologna stone (a calcined and sulfurated form of barium sulfate, invented in the early seventeenth century). On the history of luminescence and phosphorescence, cf. Harvey 1957; Hakfoort 1995, pp. 151–61. The orthograph of Mairan's name varies: d'Ortous instead of Dortous, Mayran instead of Mairan.

refraction to be more akin to the impact of an elastic ball on a solid plane that somehow represented a varnish of subtle matter on the bumpy interface between the two media.[85]

This plane had infinite inertia in the case of reflection, and a mass equal to a fraction of the mass of the ball in the case of refraction. As Mairan only considered the momentum balance, he did not see the incompatibility of this picture with the sine law of refraction. He did see that the velocity of the ball could only decrease when hitting the mobile plane. He therefore argued that the true process depended on the different "resistances" of the interstitial ether to the motion of the corpuscles of light in the two media. This resistance had to increase with the porosity of the medium in order to have the refracted ray closer to the normal in a denser medium. It implied a mass displacement proportional to the volume of the incoming corpuscles of light, so that corpuscles of different mass and equal velocities were equally deflected. This is why Mairan made velocity the parameter of color.[86]

By 1723, Mairan had become more tolerant of the vibrational theory of light. On the basis of Descartes's idea that the tendencies of the corpuscles of the subtle matter behaved like the actual motion of isolated balls, he argued that his theory of refraction was equally compatible with Malebranche's and Newton's theory. He nevertheless judged the emission theory "more convenient for the clarity of demonstrations, because it awakens in the imagination of the reader things that happen everyday under his eyes." He rejected all theories of refraction that assumed a smaller velocity in denser media: Fermat's and Leibniz's because they carried remnants of the "old philosophy" of "natural antipathies," Maignan's and Barrow's because they assumed an implausible prismatic form of the corpuscles of light, and all of them because they made "a moving body turn in the direction in which it moves with more difficulty."[87]

The ether only played a passive role in Mairan's neo-Cartesian formulation of the emission theory. In his explanation of partial reflection, Mairan imagined a partial blocking of the ether by the hard particles of gross matter. In his brief mention of Newton's investigation of thin plates, he reduced the phenomenon to "reflections or refractions of light diversely complicated by the various thicknesses and the various angles of the air blades left between the two glasses after pressing one on the other," and he compared the repetition of colors in Newton's rings to "the duplication of the spectrum by means of different prisms adjusted one above the other, or below the other."[88]

As was mentioned, in 1720 Mairan had sketched a theory of sound partly inspired by analogy with Newton's theory of colors:

> In Newton's system, what causes the various colors and their different degrees of *refrangibility* is particles or, if one wishes, globules of the ether which because of

[85]Mairan 1717, pp. 48 (citation), 50–2 (refraction); 1722, pp. 50–1 (varnish); 1723, 1738. Mairan had repeated Newton's experiments in Béziers.

[86]Mairan 1723 (refraction), 1738 (color).

[87]Mairan 1723, pp. 366, 371. The justification of the equivalence of the systems of vibrations and emission with respect to refraction is in Mairan 1737, p. 24: "One knows that the laws for the direction of bodies *solicited* to move are the same in respect to mere tendency as they are in respect to their actual motion or transport."

[88]Mairan 1723, p. 375; 1737, p. 43 (citation). The purpose of Mairan's remark on thin plates was to deny the existence of optical octaves.

> their different consistence or of their different size move or tremble in a different manner, with different speeds. Similarly, there should be in the air particles of different springs, which therefore make the same number of vibrations in a different amount of time.

In this report on Mairan's views, Fontenelle alluded to two possible models for the propagation of light: an emissionist one in which particles of light "move" with a translational velocity defining color, and a vibrational one in which light is conveyed by the "trembling" of the globules of the ether. In both cases, Mairan required different particles or globules for the propagation of different simple colors.[89]

Mairan published his full theory of sound in 1737, and the last installment of his theory of refraction in 1738. In both memoirs he gave more prominence to the vibrational theory of light than he had done in the past. He repeated that his mechanism of reflection and refraction was equally compatible with vibrational and emissionist theories: "I do not want here to exclude any system unless it goes outside mechanism or is manifestly contrary to experiments." He gave two interpretations of color, one in terms of the translational velocity of corpuscles of light, the other in terms of the frequency of vibrating globules of the ether. In conformity with his hints of 1720, he required that different sorts of globules, with different proper frequencies, should be responsible for the conveying of each simple color: "The vibrations-of-pressure system must impute to the different elasticities of the luminous globules, or of their lines, what the emission system will explain by their different masses and sizes." This version of the vibrational theory of light was as remote from a genuine wave theory as his theory of sound already was. In particular, Mairan denied that water waves were a proper analogy for the propagation of sound, as they did not involve any elasticity. This explains why he and later theorists of light spoke of "the system of vibrations" instead of the "wave theory" of light.[90]

Mairan enjoyed a high reputation at the French Academy of Sciences, of which he was the *Secrétaire perpétuel* for three years after Fontenelle's retirement from this function in 1741. His memoirs on reflection fell into sympathetic ears, because they conciliated a variety of neo-Cartesianism with Newton's optics at a time when a persistent faith in Descartes's philosophy and a rising interest in Newton's coexisted. In the second half of the century, two byproducts of Mairan's approach survived in some French quarters: the choice of velocity as the parameter of color, and the interpretation of diffraction as refraction by atmospheres of subtle matter. The latter consideration, which may be regarded as a variation on one of Newton's queries of 1718, inspired later studies of diffraction by Abbot de Molières in 1740 and by Etienne François du Tour in 1760.[91]

[89]Fontenelle 1720, p. 11.

[90]Mairan 1737, pp. 45–8 (sound and waves); 1738, pp. 32–4 (citations). Johann II Bernoulli's prize memoir on light, which had similarities with Mairan's views, probably decided Mairan to no longer delay the publication of his theories of sound and colors. Mairan (1747, pp. 429–35) rejected Euler's theory of light because he believed it did not provide the radiation pressure that he and Euler believed to be responsible of the curving of the tails of comets: see above pp. 128, 129n.

[91]Mairan 1738, pp. 53–61: "De la diffraction"; Fontenelle 1740 [after Molières]; Du Tour 1768–1776 [1760]. Fontenelle (1738, pp. 88–9) praised Mairan but preferred "the system of pressure." In order to explain the order of

However, the new generation of French Newtonians disliked Mairan's mechanistic approach. They took distance forces as their starting point, and they preferred mathematical concision and precision to lengthy mechanical illustrations. Clairaut's memoir of 1739 on the Newtonian theory of refraction was plausibly aimed against Mairan, despite a few words of conventional praise. So too was Clairaut's later proof, in 1761, that velocity could not be the parameter of color. Du Tour's contemporary reactivation of Mairan's atmospheres prompted the following comment from Abbot Nollet:[92]

> It seems to me that you often call on the configurations of the ultimate parts of bodies, on the arrangement of their pores ... , on an unknown matter to which you assign a large role, etc. I ought not hide from you that the Academy is getting more and more difficult about this way of philosophizing.

To summarize, Mairan gave a Cartesian reinterpretation of Newton's system in which the interaction between the corpuscles of light and matter was mediated by a mechanism involving the globules of subtle matter. After the decline of Cartesian philosophy and the rise of French Newtonianism, the main results of Mairan's efforts were the tilting of French opinions toward Newton's emissionist theory of light and colors, and the spreading of Malebranche's (and Newton's) idea that frequency should be the parameter of color in a wave theory of light.

Johann II Bernoulli's prize memoir

In 1736 the French Academy of Sciences offered a prize on the question: "How is light propagated?" The winner was Daniel Bernoulli's younger brother, Johann II Bernoulli. This prize memoir is a third example, after Malebranche and Mairan, of a neo-Cartesian theory of light integrating important elements of Newton's theory. Johann II's starting point was Malebranche's ether, with the tiny vortices that explain elasticity through centrifugal forces. Johann II nonetheless judged that a pure ether theory would imply more bending of light into the shadow than truly observed. Huygens's explanation of rectilinear propagation did not satisfy him, as he judged that it only proved a higher intensity on the envelope of secondary waves than elsewhere, not the strict vanishing of light in the shadow.[93]

Johann II's own explanation of ray propagation relied on the insertion of numerous hard corpuscles of various sizes in the ether. A shift of one of these corpuscles in a given direction implied a compression of the vortices contained between this corpuscle and the next corpuscle found in the same direction. This compression caused a shift of the latter corpuscle, and so forth, thus engendering a "luminous fiber" of aligned corpuscles and intercalated compressed vortices. Owing to the elasticity of the vortices, any shifted corpuscle was drawn to its original position by a force proportional to the shift and therefore behaved like a resonator with a specific frequency depending on its mass. Johann II believed that all the corpuscles on a given luminous fiber should have the same mass, because the vibration of the first shifted corpuscle

colors in the diffraction fringes, du Tour included a reflection on the surface of the diffracting needle, besides the two refractions at the limit of the atmosphere. He observed more diffraction fringes than Newton.

[92]Clairaut 1739, 1761 (see above, pp. 124–5); Nollet to du Tour, 13 March 1769, quoted in Heilbron 1982, pp. 68–9.

[93]J. II Bernoulli 1736, pp. 8–13. Cf. Hakfoort 1995, pp. 60–5.

(next to the source) could only induce vibrations of corpuscles of the same mass, the other corpuscles being ejected from the fiber.[94]

As many had done before him, Johann II illustrated resonance ("sympathetic vibration") with vibrating strings. In the same acoustic register, he assimilated each of his fibers with a vibrating string performing longitudinal vibrations instead of the transverse ones occurring in musical instruments. He propounded that the propagation of sound in air involved similar "sonorous" fibers:

> The propagation of light and that of sound have so much affinity, as I already said, that one can very conveniently and usefully handle the two matters at the same time.

The sole difference, Johann II went on, was that the air only agitated its own parts, whereas the ether agitated the immersed hard corpuscles. The compressible portions of air that replaced the corpuscles on a sonorous fiber acquired an oblate form during the vibration of the fiber, thus causing a small transverse shift of the air. Hence the propagation of sound was not as rectilinear as that of light.[95]

Johann II modeled a luminous or sonorous fiber through a succession of equal masses linked by massless springs, as his father Johann I had done a few years earlier for the transverse vibrations of a musical string. In analogy with Taylor's treatment of the continuous string, Johann I and II computed the force acting on one of the masses as the vector sum of the actions of the two attached springs, and they assumed the proportionality of this force to the displacement of the mass. In the limit of infinitely close masses, this procedure implies that the second derivative of the displacement with respect to the abscissa (along the fiber) should be proportional to the displacement. Hence the displacement should be a sine function of the abscissa. Its amplitude should also be a sine function of time in order that the acceleration should be proportional to the force.[96]

In modern notation, the motion obtained by Taylor and the Bernoullis has the form

$$y = \sin(\pi x/l)\cos 2\pi \nu t.$$

The frequency ν is given by

$$\nu = (1/2l)\sqrt{F/\sigma},$$

where l is the length of the string, σ its mass per unit length, and F the compressing force. For the purpose of sound or light propagation, this may be rewritten as

$$\nu = (1/2l)\sqrt{P/\rho},$$

[94] J. II Bernoulli 1736, pp. 11, 14–15. Mairan perceived some similarity between Johann II's luminous fibers and his own idea of propagation through resonance in a succession of similar globules, which brought him to write: "I acceded to his triumph with even greater pleasure, as I found a number of my ideas, written and printed under his authorship. It is true, however, that he did not give me the honor of citing me" (Mairan to Bouillet, 22 January 1737, quoted in Kleinbaum 1970, p. 186).

[95] J. II Bernoulli 1736, pp. 24–5.

[96] Ibid., pp. 16–17, 25–36. Cf. Truesdell 1955, pp. XXX–XXXI; 1960, pp. 129–36.

where P is the equilibrium pressure of the air or ether, and ρ its density (more exactly the contribution of the corpuscles that have the same mass as those of the fiber).

Johann II assumed that a ray of light or sound was analogous to a chain of fibers oscillating like vibrating strings with fixed ends, and that the time taken to add one fiber to the chain was the half-period of oscillation of one fiber. This yields the propagation velocity

$$c = 2\nu l = \sqrt{P/\rho},$$

in conformity with Newton's earlier result. Johann II used this formula to estimate the pressure ("elastic force") of the ether, which he found to be enormous compared with the atmospheric pressure (implicitly assuming the density to be the same).[97]

Johann II next explained the reflection of light on a mirror by repeated reflections of the corpuscle of the incoming fiber that is closest to the surface of the mirror. For the purpose of refraction, he imagined that the smaller size of the pores of a denser medium implied a denser packing of the vortices and thus a higher pressure (whereas Malebranche had assumed a smaller pressure in a denser medium). Again borrowing from his father, he derived the sine law of refraction by the condition that for a corpuscle constrained to move on the interface of the two media, the resultant of the forces acting on this corpuscle should be normal to this interface. In the fiber picture, these forces result from the pressures along the incident and the refracted fiber. The pressure being higher in the denser medium, the ray is closer to the normal on the side of the denser medium. As velocity increases with pressure, it has to be higher in glass than in air, just as in Newton's theory.[98]

By more obscure reasoning, Johann II found that the refraction of a ray increased for a higher mass of the corpuscles contained in the associated fiber, or, equivalently, for a slower oscillation of this fiber. He thus integrated Newton's notion of rays of different refrangibility in his theory, although his understanding of refraction widely differed from Newton's. Since in his theory the velocity of light depended on the mass of the corpuscles in a fiber, it also depended on color. Red light, which is the least refracted, corresponded to the smallest corpuscles and to the largest velocity of propagation. Johann II considered the color of the satellites of Jupiter at emersion as a possible test, and anticipated Mairan's remark that the test was impracticable.[99]

Lastly, Johann II praised Newton's fits of easy reflection or transmission, and proposed to explain them by the vibrations of the corpuscles of his luminous fibers. As he built rays from chains of vibrating fibers, the amplitude of the vibrations varied periodically along a ray. He tacitly assumed that the ray's ability to cross an interface between two transparent media depended on the value of the amplitude at this interface. He emphasized that this picture naturally explained partial reflection, because it yielded the periodic fits on any ray, whatever be its origin. In order to explain thin plates, he only had to assume that the

[97] J. II Bernoulli 1736, p. 38. [98] Ibid., pp. 44–9. [99] Ibid., pp. 51–61.

crossing of an interface by the ray always produced a new series of fibers with a node at the interface.[100]

To sum up, Johann II Bernoulli was the first of the neo-Cartesians to address what Newton believed to be an essential asymmetry in the behavior of light and sound: rectilinear versus curved propagation. He did this by loading the ether with hard corpuscles while preserving the continuity of the air. The resulting theory implied an imaginative combination of corpuscular and ether-based processes. It associated colors both with the mass of the corpuscles and with the frequency of their vibrations. Like Malebranche and Mairan, Johann II lacked the means to investigate his model mathematically. His arguments for the formation of fibers, for their reflection and refraction, and for the role of the mass of the corpuscular loads were mostly intuitive and often obscured by an ambiguous concept of force. For instance, in his derivations of the sine law and the mass-dependence of refraction he seems to have confusedly used "force" to refer to the elastic constant of the ether, to the Newtonian force acting on a corpuscle, and even to the velocity of the vibrations.

Such shortcomings did not bother the Academicians who recompensed Johann II Bernoulli's work. At that time, Cartesian mechanism was still welcome in France and in Switzerland. Ingeniousness, inclusiveness, and a seeming respect for the laws of mechanics were still the only criteria for accepting a given mechanism. In 1737, Johann I Bernoulli sent a copy of his son's dissertation to Euler and asked for his opinion. Euler replied:

> I thank You very deeply, most Famous Man, for sending me, as a token of Your love for me, the prize-winning dissertation on light of Your most Brilliant Son Johann, in which I admire the exquisite ingeniousness of the author in physical things of this kind. In the first place, I liked the explanation of the diversity of the rays of light, which Newton so well observed but no one tried to explain by physical principles; I did not either.

Apparently, Euler saw nothing wrong in the luminous fibers. He was more critical toward Johann II's derivation of the velocity of sound, which reproduced Newton's formula and contradicted his own. More rigorous mechanical reasoning would have disproved the luminous fibers and confirmed Newton's formula, as Euler later realized. This letter shows how little Euler understood the mechanics of continua in 1737, and how much sympathy he still had for Cartesian "physical" explanations. He did not settle for Johann II's theory, however. Much later, in the *Encyclopédie*, d'Alembert denied that Johann II's loaded ether behaved differently from the usual elastic ether with respect to propagation around obstacles.[101]

A couple of French treatises

In 1739, the physician Claude Nicolas Le Cat published his widely read *Traité des sens*. The chapter on sound reproduced Mairan's idea of propagation by resonance of similar elastic particles of the air. The chapter on light seems to have been largely inspired by

[100]Ibid., pp. 62, 66. Another neo-Cartesian, Jean Banières, similarly identified Newton's fits with vibrations of the subtle elastic fluid of light: Banières 1739, p. 295.

[101]Euler to J. Bernoulli I, 27 August 1737, in Eneström 1904, p. 256; D'Alembert 1765a, p. 722.

Malebranche's system, although Malebranche's name is not to be found there. Le Cat was also aware of Mairan's theory, but cited it only to disagree. He does not seem to have read Johann II Bernoulli. Like Malebranche, Le Cat justified the vibration theory of light by analogy with sound:

> Although the matter of light pervades all space, it is not always sensible, at least to ordinary eyes; it surely has motion, as any subtle fluid does, but this motion is too weak to impress our eyes, or, rather, the motion that light has as a fluid is not yet the one that it must have as the object of vision. The air is also constantly moving as a fluid; but in order to produce sound, another motion of vibration is needed, or of undulation which it receives from sonorous bodies; similarly, the matter of light, besides its motion of fluidity, needs the vibrations excited by the sun, by fire, or by any luminous body. These vibrations mainly take place along straight lines.

Unlike Malebranche, Le Cat cared to explain why the propagation of light was more clearly rectilinear than that of sound. He argued that the fluid of light being more subtle than air, its vibrations were less likely to spread laterally. He also believed that the lack of lateral spread explained the higher velocity of light.[102]

Lacking mathematical skills, Le Cat frequently appealed to such vague intuitions. This sometimes led him to crude nonsense, as in his concluding that a mirror reflects a ray of light in every direction from the fact that several people can see the same object from the same mirror. A good Cartesian, Le Cat agreed with Malebranche and Mairan that reflection and refraction should be understood by means of "impressions" rather than "attractions." But he rejected the ethereal varnish that Mairan imagined on reflecting surfaces to prevent diffuse reflection, since he believed this kind of reflection occurred even for well-polished mirrors. In his explanation of refraction, Le Cat followed Malebranche in imagining an intimate motion of the subtle matter that had more intensity in a more porous media. The resulting pressure gradient (to put it in modern terms), implied a force on any particle situated at the interface between two media of different density. This force, being directed from the rarer to the denser medium, caused rays to curve toward the perpendicular when entering the denser medium.[103]

On colors, Le Cat followed Malebranche's misreading of Newton, according to which the spectrum of light is a discrete spectrum with seven colors only. As he had difficulty confirming this result, he judged that Newton's theory of colors was not sufficiently established and he refrained from any precise interpretation. He only expressed the vague notion that the strongest rays (the red ones) where the least refracted. Unlike Malebranche, he did not explicitly correlate this strength with frequency. He called for the "great masters of experimental physics, such as Abbot Nollet" to decide the matter. Nollet read the call and kindly notified Le Cat that he had misread Newton on colors. Le Cat candidly admitted his mistake in the second edition of his treatise.[104]

[102]Le Cat 1739, pp. 306 (citation), 308 (lateral spread).

[103]Ibid., pp. 326–9, 335. The intimate motion imagined by Le Cat was more akin to Daniel Bernoulli's kinetic gas picture than to Malebranche's tiny vortices.

[104]Ibid., p. 361.

Le Cat cited Newton's fits of easy reflection and transmission as a further proof of the vibrational nature of light. After describing multiple reflections within a crystal he wrote:

> The fluid that receives these shocks from light and that returns reciprocal impulsions to it is elastic; these alternatives jets of light must therefore take place by fits, by vibrations, as Newton as observed. Besides, nearly all physicists think that light consists in the vibrations of luminous matter, as sound is formed by the vibration of air. Thus, Newton's observation only serves to confirm the most received system.

The judgment that the system of vibration was the most commonly admitted is not without interest, since Le Cat was perfectly aware of Mairan's and Voltaire's writings on light.[105]

Abbot Jean-Antoine Nollet, whom Le Cat counted among the "great masters of experimental physics," was another devoted Cartesian eager to reduce phenomena to the contact of globules of various sizes. Mairan, Johann II Bernoulli, and Le Cat (his physiology of the eye) were the main sources of his lectures on optics, first published in 1755. His definition of light rested on Descartes's authority combined with acoustic analogy:

> According to the thought of Descartes and those who exactly follow his doctrine, the proper matter of light is an immense fluid whose parts, smaller than can be said and rounded in the shape of globules, fill up the entire sphere of our universe uniformly and uninterruptedly: the sun which occupies the center, the fixed stars which are like the limits, and all bodies that burn on earth and elsewhere impel on this matter a motion which does not carry it from one place to another but stirs it into a kind of shivering somewhat similar to that occurring for sound in air; so that the star or the flaming body becomes the center of a luminous sphere in about the same way as a bell, or any other sonorous body causes the surrounding mass of air to resonate far away and in every direction.

That Descartes never asserted any analogy between sound and light did not bother Nollet or any of the neo-Cartesians. Against the emission theory, Nollet cited the then classical objections that the rays would be too far apart at large distance, that crossing rays would disturb each other, that the sun would exhaust its matter, and that collisions with the particles of matter would prevent transparency.[106]

Like Huygens and Malebranche, Nollet explained the finite speed of light by the elasticity of the globules of the ether. He admitted that rectilinear propagation was not easy to understand in the Cartesian theory, and referred to Johann II Bernoulli for a possible solution of this difficulty. For the specular reflection of light, he relied on Mairan's varnish of subtle matter. For refraction, he accepted that light was accelerated when entering a denser medium, although he found difficulties both in Newtonian and in Cartesian explanations of this oddity. He did not mention the alternative derivations by Fermat, Pardies, and Huygens.[107]

[105]Ibid., p. 333.

[106]Nollet 1755, pp. 7 (citation), 11–13.

[107]Ibid., pp. 49 (elastic globules), 56 (about Johann II), 151 (reflection), 255–65 (refraction).

On colors, Nollet drew on Mairan's version of the vibrational theory of light. In particular, he adopted Mairan's ideas that globules of different mass or different spring were needed to propagate different colors:

> While acknowledging several species of light [Newton's simple colors], cannot we suppose that what makes their difference is certain combination of motions, of which such and such order of globule is able by reason of its having more or less mass or spring; as it is likely that in the same volume of air there are coarser and less elastic particles through which the grave sounds are heard, and others whose different qualities enable to transmit sharper sounds?

In order to explain the colors of bodies, Nollet assumed that the pores of a colored body contained a higher proportion of the globules corresponding to their color. Thus, the incoming vibrations could only be reflected or transmitted by the body if their frequency was tuned to the characteristic frequency of the globules. He also mentioned Newton's explanation of colored bodies based on the colors of thin plates. His account of the latter colors was purely descriptive and did not include Newton's fits.[108]

All in all, Le Cat and Nollet did not add much to the neo-Cartesian theories on which they freely drew. They were less inclined to (or able of) geometrical reasoning than Malebranche, Mairan, and Johann II Bernoulli. Their discussions were qualitative and verbose, though illustrated by precise experiments in Nollet's case. Their easy style, but also their impressive erudition and the quality of some of their researches, contributed to the huge success of their treatises. As a result, most French-reading students of physics towards the mid-eighteenth century must have been aware that Newton's concept of simple colors could be reconciled with a vibrational theory of light that imitated the received theory of sound. The Cartesian analysis of the subtle matter that carried the vibrations was loose enough to allow escapes from Newton's well-known objections. Being focused on corpuscular processes, this analysis ignored much of Pardies's and Huygens's wave-theoretical reasoning and retained features of the emissionist theory, such as the higher velocity of light in denser media. Diffraction was usually ignored, save for Mairan's attempt to reduce it to atmospheric refraction. The colors of thin plates received little attention, save for vague vibrational interpretations by Johann II Bernoulli and Le Cat.

In the second half of the century, the neo-Cartesian style fell into disgrace. As the French versions of the vibrational theory of light were all neo-Cartesian, they were gradually forgotten. Newton's corpuscles and their attractions gained considerable ground. The only vibrational theory of light that received some attention was Euler's theory of 1746.

4.4 Euler's theory of light

The great Swiss mathematician Leonhard Euler authored the most precise and detailed theory of light as a vibration propagated through a medium in the eighteenth century. Although this theory severally drew on French neo-Cartesian optics, it does not truly

[108]Ibid., pp. 337–8 (citation), 411–12 (body colors), 425 (Newton's theory), 426 (Newton's rings). Malebranche and Mairan did not discuss the colors of bodies. Nollet did not refer to Euler's explanation of these colors, which is similar to his. He may have been inspired by Banières's explanation: see below, p. 158, note 124.

belong to this category: its main driving force was not Cartesian reduction to contact action but analogy with the phenomena of sound propagation.

On sound

Euler gave much thought to sound and music all through his life. One of his earliest writings was a dissertation on sound, which he published in 1727, at age 20. There he rejected the idea that sound might be propagated through the intrinsic vibrations of elastic globules (he perhaps meant to refute Mairan's views); he gave a formula for the speed of sound differing from Newton's by a factor $4 / \pi$; and he derived the pitch of an open pipe by analogy with a vibrating string. His *Tentamen novae theoriae musicae* of 1739 included these results, as well as a description of Sauveur's higher modes of vibration, both for strings and for flutes. He maintained his theoretical value for the velocity of sound, and still rejected Newton's theory.[109]

Euler's next contribution to the theory of sound, a reinterpretation of Newton's theory of compression waves, appeared in 1746 in his *Nova theoria lucis and colorum*. This is no coincidence: as Euler based his new theory of light on the strongest analogy yet imagined between light and sound, he felt compelled to clarify the propagation of sound. He may also have been challenged by Johann II Bernoulli's mass–spring model, which confirmed Newton's formula.[110]

In his *Principia* Newton considered the one-dimensional propagation of a periodic series of pulses in a homogeneous elastic medium whose "elastic force" (pressure) was proportional to the compression. Without any justification, he assumed that when a single pulse reached an undisturbed particle (or thin slice) of the medium, this particle performed a pendulous oscillation of the form

$$\delta x = \alpha[1 - \cos m(t - \tau)],$$

beginning at time τ and finishing at time $\tau + 2\pi/m$. Note that in Newton's intuition of a pulse this displacement is always positive (if α is so): the particle is pushed away from its equilibrium position and returns to it after the time $2\,\pi/m$ has elapsed.[111] Newton further assumed that the pulse traveled at the constant speed V, so that $\tau = ct$. Then he showed that under these assumptions and for small displacements the force acting on any thin slice of the medium owing to the different compressions of the two contiguous slices was equal to the acceleration of this slice multiplied by its mass if and only if the propagation velocity was given by $V = \sqrt{P/\rho}$, where P is the pressure ("elastic force") and ρ the density.[112]

It is not clear whether Newton meant to represent a periodic succession of pulses by the same formula $\delta x = \alpha[1 - \cos m(t - x/V)]$ at any time t and at any abscissa x, or he meant the pulses to be separated by finite intervals of undisturbed air. His calculation is

[109]Euler 1727, 1739a. Cf. Truesdell 1955, pp. XXIV–XXIX.

[110]Euler 1746a. Cf. Truesdell 1955, pp. XXXII–XXXIII.

[111]Euler explains this by assuming that the aerial motion is caused by a musical string pulled from its equilibrium position and released in calm air.

[112]Newton 1687, book 2, props. 48–9.

compatible with both options, because the dynamical equilibrium holds for each pulse individually. Euler decided in favor of the second alternative. More exactly, he interpreted Newton's reasoning as giving the approximate solution for the propagation of a single (plane) pulse in an elastic medium. He then associated a musical sound or tone with a periodic succession of pulses propagating independently of each other at the same constant speed *c*. He emphasized that the pulses should not overlap in order that the velocity of sound be independent of pitch.[113]

The *Nova theoria*

Euler's new theory of light and colors began with the assertion that light was more analogous to sound than to smell, and went on with a refutation of two of Newton's arguments against the medium theory: that the medium would slow down planetary motion, and that light would curve into the shadow. Euler argues that the first difficulty does not occur if the ether is subtle enough, whereas the unceasing emission of huge quantities of corpuscles of light in Newton's theory is more likely to interfere with planetary motion. As for the curving of light into the shadow, it should not occur because the similar phenomenon does not occur for sound. Euler believed that the contrary opinion derived from the unavailability of perfect acoustic screens: a non-negligible fraction of the sound traverses the screen, thus giving the illusion that it has traveled around it. Elsewhere in his memoir, Euler argued that Newton's own explanation of propagation implied that a pulse entering a portion of space induced a parallel pulse in the next portion of space.[114]

Euler dwelt on three outstanding difficulties of Newton's theory: the loss of mass of the luminous source, the mutual perturbation of crossing rays, and the difficulty in imagining the alignment of pores in transparent bodies. These objections were not new and they had received reasonable answers from the Newtonians. Euler's answers to Newton's objections against the wave theory were not any newer, and they rested only on uncertain observations and vague theory. Euler nonetheless judged them sufficient to justify the conclusion:[115]

> Thus, above all, I lay down that light is propagated by means of pulses through a certain elastic medium in a manner similar to sound; and just as sound usually spreads through air, I assume that light propagates through another elastic medium of some kind, which fills not only our atmosphere but all the universal space of the world that separates us from the most distant fixed stars.

The second chapter of Euler's memoir is devoted to the formation and propagation of pulses. Euler first rejects Descartes's model of hard spheres as incompatible with rectilinear propagation. Then he explains how the vibration of a musical string induces compressions in the nearby air, which in turn pushes the next layer of air, and so forth. Euler does not refer to any source, although similar ideas could be found in a number of earlier treatises since Ango's. Lastly, Euler gives the aforementioned reinterpretation of Newton's theory of propagation, together with upper and lower bounds for the values of the density

[113]Euler 1746a, chap. 2, chap. 3, pp. 199–201 (no overlap).

[114]Ibid., pp. 171–6, 198 (elsewhere). The most authoritative discussion of Euler's *Nova theoria* is in Hakfoort 1995, chap. 4.

[115]Euler 1746a, pp. 178–81, citation from p. 181.

and the elasticity of the ether compatible with unimpeded planetary motion and with the cohesion of matter (seen as resulting from compression by the ether).[116]

In his third chapter on "successions of pulses and rays of light," Euler first represents musical sounds by a periodic succession of non-overlapping pulses.[117] He goes on with the description of the light emitted by a periodic point-source, which involves concentric spherical pulses and the normal rays that define the direction of motion. The eye detects the direction of the pulses and "the number of percussions of the eye," in analogy with the eardrum as a counter of pulses. Euler explains the invariance of frequency more clearly than his forerunners: "It is evident that, wherever the eye be placed, the number of blows [*ictus*] that reach it is equal to the number of vibrations done by the body in A in the same lapse of time." Euler next introduces the notion of "compound rays" in which the time between two successive pulses varies, whereas this time is a constant for "simple rays." As he explains, the reason for this denomination is the connection with Newton's theory of colors: the frequency of simple rays has the immutability of simple colors. Refraction splits compound rays into simple rays, although Euler's compounds differ from the mixtures intended by Newton.[118]

This last point is made in Euler's fourth section, in which he discusses the reflection and the refraction of rays. The laws of reflection derive from the assumption that the agitation of the particles of the medium is reflected by a mirror as a ball would be by a hard wall. For refraction, Euler is the first eighteenth-century medium-theorist who does not admit a larger velocity of light in denser media. Without naming Pardies, he adopts his derivation of the sine law (see Fig. 4.6), which makes the refraction index inversely proportional to the velocity of propagation. He notes, as Pardies and Huygens had done before him, that this rule implies the validity of Fermat's principle of least time, which "satisfies a most important law of nature."[119]

In order to explain dispersion, Euler argues that the successive pulses of light overlap to an extent increasing with their frequency. As was mentioned earlier, in his and Newton's understanding of propagation, a pulse corresponds to a local shift of the particles of the medium in the direction of propagation. This picture justifies Euler's intuition that the trailing shift behind a given pulse eases the propagation of the next pulse. More precisely, Euler assumes that in a first approximation the velocity V in a given medium increases linearly with the inverse of the wavelength λ:

$$V = \bar{V} + \frac{\alpha}{\lambda},$$

where α is a constant independent of the medium and $\bar{V}$ is the velocity of an isolated pulse. This is the first law of dispersion ever given in a wave theory. It implies that vacuum (pure

[116]Ibid., II: *De formatione ac propagatione pulsuum.*

[117]Ibid., III: *De pulsuum successione atque radiis lucis.*

[118]Ibid., § 65.

[119]Ibid., IV: *De reflexione et refractione radiorum*, §78.

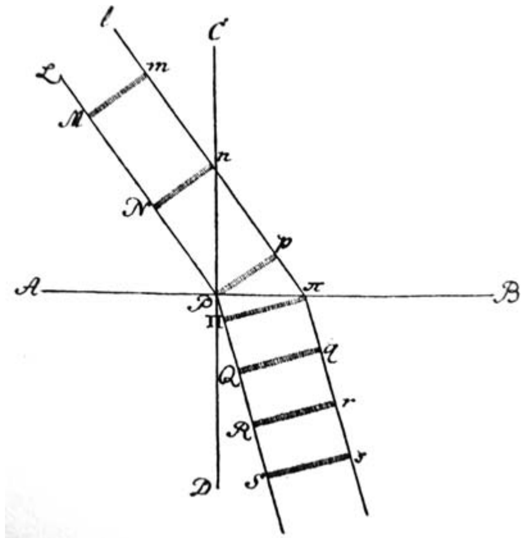


Fig. 4.6. Refraction according to Euler 1746a. The line AB separates the two media. The pulses Mm, Nn, and Pp travel at the velocity V_1 in the first medium; the pulses Ππ, Qq, Rr, and Ss at the velocity V_2 in the second medium. The sine law of refraction results from pπ/PΠ = V_1/V_2 and from the mutual perpendicularity of rays and pulses.

ether) is itself dispersive, and that red rays have the smallest wavelength since they are the least refracted.[120]

In order to explain the diverse refraction of sunlight, Euler assumes that the generating oscillations are not isochronous, that their frequency decreases with their amplitude as he believes is the case for vibrating strings. He rejects the alternative view that different harmonic oscillators are responsible for different frequencies, because it would require a uniform mixture of such oscillators at each point of the surface of the sun. Thus, white light according to Euler is a sequence of pulses with slowly decreasing frequency (see Fig. 4.7). This sequence can be decomposed into nearly periodic subsequences, refracted at specific angles and thus producing beams of well-defined color and frequency.[121]

Euler's fifth and last section is devoted to the distinction between four kinds of bodies: luminous, reflecting, refracting, and opaque. Luminous bodies are the primary sources of light; they oscillate as bells do to produce sound. Reflecting bodies involve a hard surface that reflects any incoming agitation and preserves its characteristics; they do not color light; they are similar to the walls that produce echoes.[122]

[120]Ibid., §§80–3. The resulting dispersion of refracted rays does not have the universality later required by Euler (cf. Euler 1750a).

[121]Ibid., §§87–93.

[122]Ibid., V: *De corporibus lucentibus, reflectentibus, refringentibus, et opacis.*

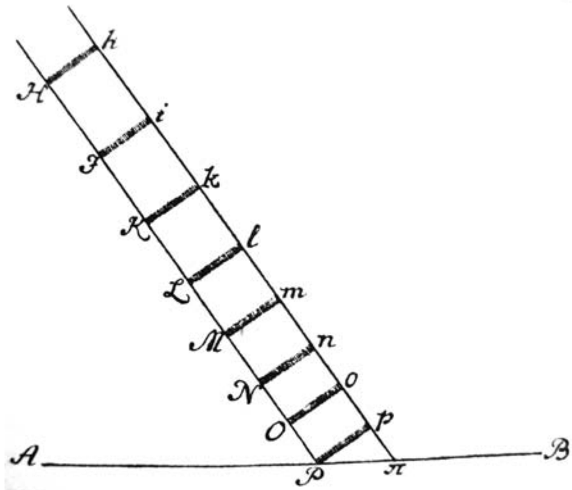


Fig. 4.7. White light according to Euler (1746a).

Refracting bodies allow the penetration and propagation of light, with a smaller velocity than in the ether. Such bodies, Euler argues, cannot be regarded as homogenous compressed ether, because their density would then require a huge amount of ether, and because the resulting value for the propagation velocity would be much too small (it would be comparable to the velocity of sound). Instead Euler adopts a Cartesian or Huygensian model in which the body is made of very hard contiguous particles, which transmit any applied pressure to the next ball in a quasi-instantaneous manner. He believes that proper choices of the hardness and proper connection of the particles should permit rectilinear propagation at a velocity of the same order as the velocity of light in the ether. Surprisingly, he does not try to combine the ether with particles of matter, as all Cartesians had done.[123]

Opaque bodies, Euler argues, cannot be identified with reflecting bodies because they color light and do not mirror the shape and color of the luminous bodies that surround them. The light they send back is not reflected light; it is light engendered by induced vibrations of their particles. These particles have their own oscillation frequency, and they only respond to incoming light that has the equal, multiple, or submultiple frequency, as happens for resonant strings:

> In the same way as a tense string is excited by a sound that is equal or consonant to the one that it produces, the smallest particles situated on the surface of an opaque body are able to oscillate under the effect of identical or similar rays and to produce pulses that spread in every direction. This is why the rays of [sun]light, as they involve pulses of every frequency, set all the particles of opaque bodies into motion; even if

[123]Ibid., §§101–2.

> the frequency of the pulses of these rays is not the same but is two or three times smaller or larger, it still induces vibrations though smaller ones.

The explanation of the colors of opaque bodies follows immediately: they correspond to the frequencies selected by the resonant oscillators of these bodies. Although Euler does not mention Mairan in this respect, he may well have been inspired by Mairan's emphasis on sympathetic vibrations. Euler briefly mentions Newton's theory of the colors of bodies, which he misrepresents as based on selective reflection and refraction of the incoming rays; and he dismisses it for implying the dependence of colors on the direction of observation.[124]

Had Euler used his earlier analysis of a harmonically driven harmonic oscillator instead of the vibrating-string analogy, he would have made the identity of frequencies the true condition of resonance. This is only one example of Euler's general tendency to reason through acoustic analogies rather than through a mathematical model of the luminous vibrations. An interesting consequence of the vibrating-string analogy is Euler's concept of optical octaves. In analogy with the similarity of the impressions produced by a tone and its octaves, Euler assumes that the same color results from vibrations whose frequencies are multiple of each other. Sunlight only involves part of one octave, since its refraction would otherwise produce several refracted rays with the same color. In contrast, the light from opaque bodies might involve (visible) rays more refracted than the violet rays from the sun. Euler judges this question to be worth an experimental investigation.[125]

At the end of his memoir, Euler makes clear that the properties of luminosity, reflection, refraction, and opacity can coexist in the same body. For instance, the phosphorescence of the Bologna stone seems to imply both opacity and luminosity; or the blue tint of the sky requires some opacity besides refraction. However, Euler does not try to unify the mechanisms by which these processes occur. He is satisfied with separate vibration-based mechanisms in each case.[126]

To sum up, Euler's memoir contains three innovations: a primitive dispersion theory based on the overlap of successive pulses, an explanation of the colors of bodies by means of sympathetic vibrations of material particles on their surface, and the concept of chromatic octaves. With some nuances, all his other ideas and arguments existed in earlier literature, a good deal of which he had certainly read. Yet he was more consistent than earlier vibration theorists in his endeavor to provide vibration-based explanations of every optical phenomenon, including refraction and dispersion. His constant appeal to acoustic

[124]Ibid., §113 (citation), §108 (Newton). Euler may also have drawn on a theory of the neo-Cartesian Jean Banières (1837, pp. lviii–lxvi, 88, 303; 1839, pp. xv–xx), in which the simple colors correspond to different elasticities ("springs" or "tensions") of the particles of the subtle matter that convey luminous disturbances and in which colored bodies "engage" more particles of the corresponding elasticity. Banières compared the selective reflection of the light of a given color to the resonance of two musical strings, although he rejected Malebranche's relation between simple color and frequency. Much earlier, in his reply to Hooke, Newton (1672e, p. 5091) had compared a colored body with an acoustic device capable of reflecting one tone only: see above, chap. 3, p. 88.

[125]Euler 1746a, §118. Euler 1756 contains an experimental suggestion for detecting octaves in the red from opaque bodies.

[126]Euler 1746a, §§122–5.

analogies somehow unified his theory. In the lack of a sufficiently developed theory of propagation in elastic media, he relied on dubious intuitions such as the overlap of pulses. He admitted a pre-established harmony between different mechanisms of propagation in ether and matter.

On the one hand, Euler was less eager than the French neo-Cartesians to exhibit contact-based, corpuscular mechanisms for every process in nature, and more willing to save phenomena with a geometrized continuum as Newton did in his theory of sound propagation. On the other hand, Euler did not hesitate to enter mechanistic speculation when nothing better was at hand, and he agreed with the Cartesian idea that any interaction should at least in principle be reducible to contact action.

Thin plates, dispersion, and phosphorescence

There are two striking omissions in Euler's memoir: diffraction and thin plates. That he never dealt with diffraction, even in later writings, is a bit surprising since this phenomenon had been discussed in writings of Newton and Mairan that Euler certainly read. We only know that Euler denied the existence of any diffraction for sound, against Newton's, Mairan's, and Johann II Bernoulli's opinion. He possibly agreed with Mairan's deflecting atmospheres, which are easily conciliated with the vibration theory of light. On thin plates, he broke his silence in 1752 with a memoir written in the wake of Mazeas's experiments. He rejected Newton's fits of easy transmission or reflection through the fallacious argument that the fringes of a thin plate, being localized on the plate, could not result from reflected light. According to his distinction between reflection and opacity, the colors of thin plates could only result from resonant scattering through particles of the thin plates. The only difference with usual opacity was the size and proper frequency of the particles, which now depended on the thickness of the thin plate. In this view, the frequency of the scattered light was divided by two when the thickness of the plate was doubled; the equivalence of optical octaves explained why the same color was perceived.[127]

Some flaws of this theory are so evident that Euler cannot possibly have given much thought to the problem. In the first place, this theory does not distinguish between reflected and transmitted colors, whereas experimentally the most reflected color at a given thickness is also the least transmitted. Secondly, this theory does not account for the colors' angular dependence, of which Newton had given a fairly accurate account. A third difficulty concerns the relation between thickness and frequency. In order to retrieve the periodicity of Newton's rings, Euler had to compare each resonant particle of the thin plate to a tense string connecting the two sides of the plate (with a tension independent of the thickness). In 1754, he acknowledged the precariousness of this assumption and replaced it with the analogy of a thin metal plate of variable thickness hit softly at different places. As the pitch of the resulting sound increases with the thickness at the hitting spot, he now believed that the frequency of the light scattered by thin plates increased with the thickness.[128]

[127] Euler 1752. Cf. Shapiro 1993, pp. 235–7.

[128] Euler 1754, p. 221.

As we saw earlier in this chapter, Euler was particularly interested in optical dispersion, with the building of achromatic lenses in view. He reached his logarithmic law of dispersion through universality considerations that did not involve the vibrational theory of light. He nonetheless tried to specify how refraction should depend on frequency. Universality in Euler's sense implies that the velocity in the medium i should have the form $p_i^{f(\nu)}$, where p_i is a characteristic constant of the medium and $f(\nu)$ a universal function of the frequency ν. Euler's guesses on the form of this function depended on his intuition of the propagation of pulses, on his understanding of thin plates, on his requirement that the solar spectrum should be within one optical octave, and on the non-dispersive character of the ether. His original idea of overlapping pulses and his second theory of thin plates implied that red had the highest frequency. His first theory of thin plates implied the opposite. In the relevant memoirs published between 1750 and 1754, Euler wavered between the two possibilities. In later writings on dispersion, he no longer used frequency as a parameter of color, presumably because he had become aware that the velocity of waves in a linear elastic medium did not depend on the shape or frequency of the waves.[129]

This does not mean, however, that Euler's faith in the wave theory of light had declined. In 1777, at age seventy, he welcomed Benjamin Wilson's description of phosphorescent substances that emitted light of a color different from the color of the exciting light, as further evidence for the superiority of the wave theory. In Euler's opinion, this fact and phosphorescence in general contradicted the emission theory, according to which the colors of bodies results from selective reflection. In reality, the emissionists had no difficulty in accounting for phosphorescence, either by a sponge model in which the received light was temporarily absorbed, or in a chemical model in which this light triggered a slow combustion process; Wilson's discovery only eliminated the first alternative. In his *Nova theoria*, Euler had traced phosphorescence to a delayed resonance of the particles of opaque bodies. Confronted with Wilson's results, he now admitted the possibility that the exciting light induced a mere tension of the particles that was later released through their vibrations. It is not clear how Euler imagined the details of this mechanism. He could hardly rely on acoustic analogy, since there is no acoustic counterpart to phosphorescence.[130]

Yet the old and blind Euler did not fail to remind his reader how the analogy between light and sound had defined his theory:[131]

> The foundation of this theory consists in assuming for light an origin similar to that of sound; namely, just as a sound is produced by a motion of vibration transmitted through air, light is caused by a similar motion of vibration transmitted through the ether; the elasticity of this ether being thousand times higher than that of the air while it is also several thousand times subtler, I have felicitously explained the incredibly

[129]Euler 1748 (highest frequency for red), 1750a (least frequency), 1752 (least frequency), 1754 (highest frequency), 1768a (no frequencies). Cf. Speiser 1962; Pedersen 2008.

[130]Euler 1777a; Euler 1746a, §122. Cf. Hakfoort 1995, pp. 151–61. In 1652, the Italian astronomer Niccolò Zucchi had already reported that phosphorescence did not depend on the color of the exciting light. Francesco Zanotti approved him in 1748; Giambattista Beccaria contradicted him in 1771. On the history of luminescence, cf. Harvey 1957.

[131]Euler 1777a, pp. 353–4.

high velocity with which the rays of light from the sun reach us. Then, I have very evidently shown that the different colors result from different degrees of rapidity of the motion of vibration, in the same manner as each of the sounds of an octave in music answers to a certain number of vibrations made in a second.

An aside on Daniel Bernoulli's harmonic analysis of light

Daniel Bernoulli shared Euler's interest in music and in the theory of vibrating bodies. As was mentioned, in the 1730s and 1740s he introduced the concept of simple modes of vibration in which the motion of every part of the body is a sine function of time and the restoring force is therefore proportional to the displacement of the part, and he figured that the most general motion of a linearly elastic body could be obtained by superposing such modes. In the case of musical instruments, for instance vibrating strings, he identified these modes with Sauveur's harmonics. Bernoulli insisted on the physical character of the harmonic components, because he could hear them separately and because he traced them to an observable fine structure of the motion of the string, as he explained in two long memoirs of 1753.[132]

Bernoulli applied this sort of analysis to "all small reciprocal motions" in nature, and thus accorded a privileged status to harmonic vibrations. Around 1750 he wrote to Johann III Bernoulli:

> I admire … the hidden physical treasure that natural motions which seem subject to no law may be reduced to the simple isochronous [harmonic] motions which it seems to me Nature uses in most of its operations. I am even convinced that the inequalities in the motions of the heavenly bodies consist in two, three, or more simple reciprocal motions.

In 1753 Daniel Bernoulli included light in this cosmic vision. After explaining what we now call the principle of superposition, he noted:

> This consideration will be of great help to conceive why it is that an infinite number of rays can pass through a small aperture and cross each other in a dark room without troubling each other: indeed a mass of luminous matter is a system made of an infinite number of parts, or globules, and each globule can be subjected at the same time to an infinite number of simple, isochronous vibrations that never fuse together nor trouble each other. One can thus conceive that the same ray of light may primitively include all possible colors; because the different colors probably are nothing but different perceptions in the organ of sight, caused by the different simple vibrations of the celestial [i.e., the ether's] globules. It is certain that in the same mass of air a great number of vibrations can be formed at the same time, very different from each other, each of which separately causes a different sound in the organ of hearing. This idea seems to me very fit to explain the different refractions, the different vivacities, and all other phenomena indicated by Mr. Newton on primitive colors. But this is so rich a matter that it can only be treated in the occasion of another theory.

In this extract, Bernoulli begins with the crossing of rays, in which context Huygens and Malebranche had earlier required the elasticity of the elementary parts of the ether. At first

[132] D. Bernoulli 1753a, 1753b. Cf. Darrigol 2007, pp. 352–60.

glance, Bernoulli's considerations seem reminiscent of Mairan's theory of sound or of Mairan's version of the vibrational theory of light, since Bernoulli's globules undergo harmonic oscillations like Mairan's. Yet there is an important difference: whereas Mairan's globules vibrate at a frequency determined by their size and elasticity, Bernoulli's vibrate at the frequency of the proper modes of a global system that includes a large number of interacting globules and also the parts of the vibrating source. As the number of proper modes is equal to the number of degrees of freedom, the frequency of the modes can vary in a quasi-continuous manner and thus be the parameter of color. The superposition of modes then explains the non-interference of rays of different color.[133]

At the end of the extract, Bernoulli promises a vibrational theory of light, which he never gave. Some clues can be found in a summary of his views on vibrating strings that he later wrote at Clairaut's request. There he recounted how, after deriving the (mutually dissonant) modes of a vibrating blade, he "then imagined the means of separating these tones, as Mr. Newton separates the primitive rays of different colors." He emphasized the "principle of the coexistence of vibrations," without which one would not be able to distinguish the various voices of a concert. And he described the propagation of sound through successive "concamerations" analogous to the loops of modern stationary waves. At that point, he mentioned his preference for a similar theory of the propagation of light:[134]

> I admit, *Monsieur*, that despite the authority of Mr. Newton, I am not quite convinced that there is not a perfect analogy of the propagation of sound in air with that of light in the ether, and that colors do not differ from each other as tones do.

Bernoulli went on to compare the sun to a sonorous body and to describe the coexistence of harmonic vibrations in the surrounding ether, each of them corresponding to Newton's simple colors. In order to explain the colors of thin plates, he assumed that "a ray could not pass from one medium to another unless the surface of this medium precisely correspond[ed] with a node of some undulation." Thus, he imagined transmission to be bound to the formation of (what we would now interpret as) a stationary wave within the plate, which implies that the thickness of the plate should be a multiple of the wavelength. He concluded this optical diversion with an apology: "You see, *Monsieur*, how far the pleasure of conversing with you carries me."[135]

Bernoulli made no mention of Euler's earlier theory of light, although he most likely was aware of it. The two mathematicians' habit of developing similar theories without much referring to each other may account for this silence. Another explanation is that Euler's and Bernoulli's concepts of light propagation differed significantly: not because they disagreed on the importance of the analogy with sound, but because they had different conceptions of the nature and propagation of sound. Whereas Euler tended to think in terms of sequences of pulses of arbitrary shape and discussed propagation in terms of individual pulses, Bernoulli privileged harmonic oscillations and analogy with the simple

[133]D. Bernoulli to J. III Bernoulli, undated, cited in Truesdell 1960, pp. 257–8; D. Bernoulli 1753b, pp. 188–9.

[134]D. Bernoulli 1758, pp. 158–60.

[135]Ibid., p. 160.

modes of oscillation of vibrating strings or organ pipes. In Bernoulli's view, white light was truly heterogeneous and involved a mixture of harmonic components that had individual reality, just as the harmonic components of a sound preexisted their hearing. In contrast, Euler conceived white light as a series of pulses with slowly decreasing frequency and thus departed from Newton's concept of heterogeneity. Whereas Euler explained the colors of thin plates by multiple resonance, Bernoulli evoked harmonic stationary waves satisfying boundary conditions on the sides of the plate.

A mixed reception

Although Euler and Daniel Bernoulli were closer to a genuine wave theory of light than the French neo-Cartesians were, the lack of originality of their most solid claims and the lack of solidity of their most original arguments prevented an easy victory over emissionism. Most of the British and the French writers on light politely ignored Euler's theory. A majority of Germans approved it, but only in a superficial manner. They were concerned only with Euler's broader considerations on the nature of light. None of them tried to develop the most innovative part of Euler's theory, the understanding of the colors of bodies and thin plates by resonance. Euler himself died without having tried to apply to light the partial differential equations that he and Lagrange had devised for the propagation of sound.[136]

Soon after Euler's death, the growing chemical evidence in favor of Newton's corpuscles of light discouraged the German supporters of Euler's theory. As was mentioned in the first section of this chapter, Senebier's and Berthollet's works of the 1880s on photochemical reactions strengthened the chemical theory of the colors of bodies as well as the concept of light as a substance with variable chemical affinity. Moreover, the growing prestige of chemistry and its promotion in German universities made physicists more receptive to chemical arguments. In 1789 the Bavarian Academy of Sciences offered a prize for a memoir deciding between Newton's and Euler's theories of light. A Benedictine monk of Regensburg, Placidus Heinrich, won by arguing that the chemical affinities of light could be explained only in Newton's system. Influential textbook writers such as Johann Samuel Traugott Gehler, Friedrich Albert Carl Gren, and Georg Christoph Lichtenberg soon expressed the same opinion.[137]

In France, Euler's *Nova theoria* never caught on, for its publication occurred at a time of rising Newtonianism. The only important adept of Euler's system, Bouguer, kept his convictions for private letters to Euler. In Britain, the only notable exception to the general indifference to Euler's theory is Bryan Higgins's voluminous essay on light, published in 1776. This London-based chemist discussed light as a prolegomenon for a chemical doctrine in which phlogiston, light, and their combination into heat played a central role. In his definition, light is an imponderable substance pervading all bodies and spaces and repelling all bodies except the phlogiston. Light in the ordinary sense is a vibration of this substance induced by matter in motion and communicated to the eye:

[136]Cf. Hakfoort 1995, pp. 129–46. [137]Cf. Hakfoort 1995, pp. 161–75.

> Illumination (commonly called Light) and darkness, are with respect to Light, what sound and stillness are with respect to air … Light affects the eye by communicated vibratory impulse; as air affects the ear by the like impulse, and not by progressive motion.

Higgins compared the reflection of light to echoes, and colors to musical tones: “The seven prismatic colours are, with respect to Light, what the seven tones are with respect to air.” He insisted that the transmission of light and sound did not involve any progressive motion of the implied substance, as this would imply their quick dissipation by “eddies and devious motion” in analogy with the air projected by bellows. He praised Huygens, Le Cat, and “especially” Euler for rejecting the emission theory, and reproduced Euler’s objections.[138]

4.5 Conclusions

The eighteenth century produced a large amount of literature on diverse aspects of optics. Much of it was devoted to a more phenomenological optics, based on ray propagation and concerned with the improvement of optical instruments. This ray optics saw important innovations in photometry, achromatic instrumentation, and the theory of systems of lenses. It implied a fruitful interplay between theory and experiments, although instrument makers ended up ignoring the most refined theories of Clairaut, d’Alembert, and Euler.

Eighteenth-century natural philosophers also addressed the more fundamental question of the nature of light. Their most frequent motivation was the integration of optics in a broader philosophy implying neo-Cartesian impressions, Newtonian attractions, or analogy with the theory of sound. The medium-theorists improved on their seventeenth-century forerunners by identifying frequency as the parameter of color and by relating the colors of bodies to resonance. They otherwise remained tributary of the persisting weakness and incompleteness of the acoustic theories they tried to imitate. The Newtonians purified Newton’s system by tracing every optical phenomenon to corpuscular attractions, and they imagined and tested a few odd consequences of this system implying the gravitation of light or effects of the motion of sources and instruments.

Except for the latter development, the eighteenth-century philosophizing on the nature of light hardly involved any experimentation. The phenomena later regarded as touchstones for competing theories, namely, diffraction, double refraction, and the colors of thin plates, were either ignored or forced into a preexisting explanatory framework. Newton’s and Huygens’s fine measurements of them largely remained unmatched. In general, precision measurement remained confined to astronomy and derived optical questions. On the whole, qualitative experimentation or observation played a more important role in guiding one’s choice between the emissionist and vibrationist concepts of light.

[138]Bouguer to Euler, 23 November 1752 and 8 February 1753, in Lamontagne 1966, pp. 232–6; Higgins 1776, pp. xii–xiv (chemical motivation), xlviii–xlix (citation), li (colors), 255 (bellow), 256 (praises). Cf. Cantor 1983, pp. 122–4. In a letter read at the Royal Society in 1756, Benjamin Franklin (letter to Cadwallader Colden, 23 April 1752, in Franklin 1905, pp. 82–6) attacked the emission theory and defended the vibration system (independently of Euler): cf. Cantor 1983, pp. 50, 53–6.

With the exception of Higgins, eighteenth-century British writers on optical questions were generally Newtonian. In France, neo-Cartesian views dominated the first half of the century, although there already were a few Newtonians. In the second half, the growing success of Newton's celestial mechanics made the idea of direct action at a distance more acceptable; Cartesian mechanisms went out of fashion; and the balance tilted in favor of the Newtonians, although the encyclopedists regarded the question of the nature of light as not yet quite settled. Elsewhere in Europe, the two views coexisted in the first half of the century; Euler's wave theory, which may be regarded as loosely neo-Cartesian, became popular some time after its publication in 1746, until in the last third of century photochemical considerations favored Newtonian theories.

5

INTERFERENCE, POLARIZATION, AND WAVES IN THE EARLY NINETEENTH CENTURY

In the early nineteenth century, two important discoveries revived the competition between the mediumnist and corpuscular concepts of light. The first of these discoveries was the principle of interference, according to which the intensity of two superposed lights originating from the same source is a periodic function of their path difference. The second was the relation between the polarization of light and its reflection or refraction at the interface between two transparent media. The first discovery favored the wave theory of light by making interference depend on phase differences, while the second favored the corpuscular theory by tracing polarization to the polarity of the light corpuscles. They were both exploited in a refined experimental physics involving creative variations of the setups and painstaking attention to quantitative details. And they were integrated in broader theoretical programs in which the earlier neglected phenomena of diffraction and anisotropic propagation became centers of attention. This evolution permitted sharper arguments for and against Newtonian optics, leading to the ultimate victory of the wave theory in the 1830s and 1840s.

The first section of this chapter is about Thomas Young's discovery and exploitation of the principle of interference, which remained a mostly individual research program until Fresnel's involvement. The second section recounts Louis Etienne Malus's and Jean-Baptiste Biot's works on polarization within the neo-Newtonian program of the Marquis de Laplace. For a while, their findings were more efficient in promoting Newtonian optics than Young's discovery of interference had been in demoting it. The third and last section is devoted to Augustin Fresnel's decisive investigations of diffraction, polarization, and crystal optics. With Arago's support, Fresnel revived Young's program and brought much stronger proofs of the wave theory.

5.1 Thomas Young on sound and light

The budding polymath

Born in 1773 in Somersetshire from a Quaker family, Thomas Young early showed intense curiosity and astonishing prowess in many fields including mathematics, botany, chemistry, ancient languages, and music. In 1792 he began a peregrinating study of medicine that took him to the universities of London, Edinburgh, Göttingen, Cambridge, and London again. He made it his duty to explore any science related to his future profession, in any of the forms he encountered on his way. This partly explains his later fertility in most diverse

fields of inquiry including optics, the theory of colors, mechanics, the theory of tides, and the deciphering of hieroglyphs.[1]

Young once noted that "his pursuits, diversified as they were, had all originated in the first instance from the study of physic [medicine]: the eye and the ear led him to the consideration of sound and of light." In 1893, at age twenty, he impressed the Royal Society with "Observations on vision" that made the muscular nature of the crystalline lens responsible for its deformation in the process of accommodation. Although both the novelty and the correctness of this assumption were soon challenged, Young later confirmed them beyond any reasonable doubt and made this theory the topic of his first Bakerian lecture.[2]

Two years later in Göttingen, Young investigated human voice for the *lectio cursoria* of his Göttingen thesis. The only extant document about this lecture is a universal alphabet of forty-seven letters designed to express all the sounds that the human voice can produce. This project implied competence in three fields in which Young was bound to make major contributions: anatomy, acoustics, and languages. In the anatomic and acoustic registers, we know from later reminiscences that he adopted the theory of Denis Dodart according to which the human voice results from vibrations of the glottis in tune with the cavity of the mouth. As Young liked to say, this mechanism is similar to that of the organ pipes called *Voix humaine* in France, since these pipes involve both a vibrating blade and a resonating tube.[3]

Young's desire to confirm this theory of the human voice led him to study Daniel Bernoulli's, d'Alembert's, and Euler's acoustic writings and to experiment on the production of sound. In the course of these experiments, Young "was forcibly impressed with the resemblance" between the sounds of stopped organ pipes and the colors of thin plates. He reasoned that white light was akin to the continuous air draft from the bellows of the organ, and the colored light transmitted by a plate of a given thickness to the tone produced by a pipe whose ends corresponded to the two surface of the plate. On thin plates, he had read Newton's *Opticks* a few years earlier. On the wave theory of light, he was aware of Euler's theory, of which he could have heard from his Göttingen professors or from Bryan Higgins whose lectures he had attended in London.[4]

Contemporary documents attest that in his college room at Emmanuel College in Cambridge Young was blowing smoke through tubes, and that his Cambridge fellows asked him to answer objections to "Huygens's theory of light," which he favored over Newton's. Hence we may surmise that Young's acoustic experiments were in good part designed to prepare a better analogy with the theory of light. This can be verified by

[1]On Young's biography, cf. Peacock 1855; A. Wood 1954; Robinson 2006.

[2]Young [1826–1827], p. 253; 1793; 1801a.

[3]Young 1796; 1804b, pp. 199–200. Cf. Robinson 2006, p. 51.

[4]Cf. Young 1804b, pp. 199–200. On Higgins, see above, chap. 4, pp. 163–4. The Göttingen physics Professor Georg Christoff Lichtenberg, whose lectures Young attended, opposed Euler's theory of light for being based on unwarranted hypotheses and for being unable to explain the chemical effects of light. Lichtenberg otherwise based his lectures on Erxleben 1787, who favored Euler's theory over Newton's. Cf. Lichtenberg to Georg Friedrich Werner, 29 November 1788, in Joost and Schöne 1983–2004, vol. 3, p. 599, in which Lichtenberg criticizes Werner's ether and Euler's "hair-raising" speculations, with the comment: "*Einer* der grösten *Mathematicker*, die je gelebt haben, und gewiss der gröste Calcultateur, der je gelebt hat; aber ein Physiker war [Euler] nicht."

examining the substance of the letter he sent on 8 July 1799 to the *Transactions* of the Royal Society.[5]

A letter on sound and light

Young first wanted to answer Newton's main objection against the wave theory of light: the necessary divergence of the light passing through an aperture. Having read Robison's encyclopedia article on the speaking trumpet, Young was familiar with Lambert's assumption of the ray propagation of sound in this device. He also approved Euler's argument that the hearing of sound behind screens probably resulted from the permeability of the screens. His own refutation of Newton's objection relied on the analogy that Newton saw between the divergence of sound and the divergence of a continuous stream of fluid into a stagnant portion of the fluid. Against this view, he determined that air blown through a tube formed a sharply delimited jet of a length that increased when the velocity of the jet diminished. He believed that sound, which implies very small velocities of the particles of air, similarly formed beams even though he conceded that sound beams differed from jets in their implying non-synchronous motions of the particles of air.[6]

Young's letter successively deals with jet formation, the nature of sound, the analogy between sound and light, and the production of sound. The section on the nature of sound is meant to fill a few persistent gaps in the foundations of acoustics. In particular, Young offers a new explanation of the sound of organ pipes. For the stopped diapason pipes, he traces the sound to the multiple reflection of a blast in a cavity with parallel walls. For reed pipes, he evokes the coincidence between the successive pulses of the vibrating reed and the pulses doubly reflected on the walls of the pipe. Young thus related the "appropriate sound" of a cavity with the propagation of pulses, whereas Daniel Bernoulli and Lagrange had based their theories of organ pipes on standing waves.[7]

Young emphasized the importance of two of Lagrange's results in his own theory of acoustic cavities: the demonstration that the velocity of propagation of a pulse did not depend on its shape, and the laws for the reflection of pulses on walls. He was less inclined to believe in another of Lagrange's results: the decrease of the amplitude of a spherical wave as the inverse of the distance from its center. In the lack of a proper distinction between amplitude and intensity, he worried that this result contradicted the quadratic law carefully established by Bouguer and Lambert in the case of light: "Should the result favour the conclusions from that calculation, it would establish a marked difference between the propagation of sound and of light." Young hoped to solve the conflict by some mutual compensation between the positive and negative crests of the wave, based on the relation $\frac{1}{r} - \frac{1}{r+\varepsilon} \sim \frac{\varepsilon}{r^2}$. This speculation betrays his lack of understanding of the mathematics of wave propagation. But it also gives a first illustration of his tendency to regard

[5]Young 1800a. Cf. Robinson 2006, pp. 62–3.

[6]Young 1800a, p. 64–9 (smoke jets), 74 (Lambert), 74–5 (sound beams), 80 (Euler on screens). Cf. Latchford 1974, pp. 119–22. The first experiment of Young's letter, in which he proved that air escaped from a punctured bladder with a velocity varying as the square of the pressure, provided the rationale for a velocity gauge he used to explore the velocity field within the jet issuing from a tube.

[7]Young 1800a, pp. 71–3. Young first explained standing waves by the superposition of a direct and a reflected wave in his *Lectures* (Young 1807, vol. 1, pp. 288–9): cf. Kipnis 1991, p. 49.

waves as a succession of positive and negative disturbances and to imagine the mutual destruction of the positive and the negative.[8]

In his paragraph on "the analogy between light and sound," Young judged Euler's objections to Newton's theory of light "not sufficiently powerful to justify the dogmatical reprobation with which he treats them." He added two objections of his own: the "Newtonian theory" made the uniformity of the velocity of the particles of light very unlikely; and it did not account for partial reflection. In contrast, the "Huygenian theory" provided a well-defined velocity as a characteristic of the medium, and it implied partial reflection in analogy with the collision between two balls of different sizes: when a lighter ball hits a larger ball initially at rest, the former ball is reflected while the second moves forward at a smaller speed.[9]

Young was familiar with Newton's queries and with neo-Cartesian writings in which the ether or subtle matter served to unify various kinds of phenomena, including light, heat, electricity, magnetism, and gravitation. In his letter he regarded electric phenomena as an "undeniable proof" of the existence of an ether, which he hoped to be the same as the ether required by the analogy between light and sound: "Whether the electric ether is to be considered as the same with the luminous ether, if such a fluid exists, may perhaps at some future time be discovered by experiment." The kind of experiment he had in mind concerned the effect of electricity on the refractive power of a fluid. Thus, for Young, better agreement with optical facts was not the only advantage of a wave theory of light. It also offered a better hope for a unified physics.[10]

Having earlier argued that sound had very little tendency to diverge, Young easily invalidated the main objection against the wave theory of light: that it did not permit ray propagation. Like Huygens, he believed that the much higher elasticity of the ether implied an even smaller divergence for light. He admitted that the light seen in every direction from the edge of a knife might imply divergence, although he favored a different view at that time. As we saw, some of Newton's queries and Mairan's optical memoirs contained explanations of refraction, reflection, and inflection (diffraction) by a variable density of the ether or subtle matter in various media. Young exploited the same idea, simply replacing the curving of the path of the light corpuscles in the variable ether with the deviation of waves. Like Huygens and Euler (and unlike Newton and Mairan), he assumed a higher density of the ether in a denser medium, which implied a smaller velocity. In conformity with this assumption he adopted Euler's derivation of the sine law of refraction which, as we saw, was a remote descendent of the derivations by Hobbes, Barrow, and Pardies. For inflection, Young used Mairan's atmospheres of variable density, thanks to which the curving of rays was reduced to refraction.[11]

Although Young borrowed his derivation of the law of refraction from Euler, he ridiculed Euler's explanation of the transmission of light in a refracting medium by an agitation of the particles of the medium itself. Ignoring Euler's specific mechanism for this

[8]Young 1800a, pp. 72 (Lagrange), 75–6 (decay of sound).

[9]Ibid., pp. 78–83, citation from p. 78.

[10]Ibid., p. 79. Faraday later had the same idea. Young also approved William Herschel's identification of radiated heat with infrared light (see above, chap. 4, note 56).

[11]Ibid., pp. 79–81.

transmission, he argued that the very high density of material media compared with that of the ether would then imply an exceedingly high refraction. This objection, and also the dogmatism he saw in Euler's rejection of Newton's system are mild compared with his later judgment of Euler's contribution: "By inaccurate and injudicious reasoning, [Euler] has done a real injury to the cause which he endeavoured to support."[12]

In addition, Young reproached Euler with postulating the relation between color and frequency rather than proving it. He ignored Euler's connection between the frequency and the thickness of thin plates of the corresponding color, either because he had not yet read Euler's relevant memoir or because this connection depended on Euler's controversial picture of the propagation of light in matter. Instead Young offered his own theory of the colors of thin plates based on analogy with organ pipes. As was already mentioned, in this occasion he represented white light as a continuous influx of ether, analogous to the air entering an organ pipe; and he related the observed colors to periodic vibrations of the ether, akin to the oscillations of the air in the pipe. He explained the repetition of the same color for an arithmetic progression of thicknesses by the similarity of the sounds emitted by pipes of proportional lengths, which means that he implicitly accepted Euler's optical octaves. He believed in a similar explanation of the colored fringes of inflected light, based on multiple reflections on the limits of the atmosphere and on the material surface.[13]

Young did not see only advantages in this theory: "The greatest difficulty in this system is, to explain the different degree of refraction of differently coloured light, and the separation of white light in refraction." He hoped that a more perfect theory of elastic fluids would help elucidate this matter. But he rejected any dogmatism on the nature of light: he offered new arguments "without pretending to decide positively on the controversy."[14]

In the rest of his letter, Young addressed acoustic phenomena whose relevance to the wave theory of light was not yet clear. First, he discussed the "coalescence of musical sounds," namely: the beat phenomenon and Tartini's third sound (combination tones). Following Lagrange, Young explained the latter phenomenon as beats produced by the superposition of two sounds of commensurable frequencies. Yet he departed from Lagrange in the explanation of the beats themselves. Since Sauveur, it was usually assumed that beats corresponded to the coincidence of the pulses of the two superposed tones. Indeed the frequency of these coincidences is equal to the difference of the frequencies of the two tones, as the beating frequency should be.[15]

Young replaced this picture with a more realistic representation of the aerial vibration associated with each tone: a continuous alternation of positive and negative displacements of the particles of air. Whereas his predecessors reasoned in terms of a combined effect on the ear, Thomas Young directly superposed the aerial vibrations corresponding to the two tones. The legitimacy of this superposition resulted from Daniel Bernoulli's, Lagrange's, and Euler's theories of the propagation of sound, but contradicted Robert Smith's (and

[12]Ibid., p. 80; Young 1807, vol. 1 (2nd ed.), p. 380. [13]Young 1800a, pp. 81–2.
[14]Ibid., pp. 82, 79. [15]Ibid., pp. 83–5.

Mairan's) assertion that different sounds required different particles for their transmission. Young brushed the latter view away:

> It is surprising that so great a mathematician as Dr SMITH could have entertained for a moment, an idea that the vibrations constituting different sounds should be able to cross each other in all directions, without affecting the same individual particles of air by their joint forces: undoubtedly they cross, without disturbing each other's progress; but this can be no otherwise effected than by each particle's partaking of both motions.

Young regarded the beat phenomenon and the Tartini tones as the best proof of the superposition principle.[16]

For the sake of simplicity, Young assumed the variation of the displacement to be piecewise linear, which yields a saw-shaped graph. He nonetheless recognized that the precise shape depended on the mode of production of the sound, and that it was more likely to have rounded angles. At that time, he did not give special importance to the sine shape, although he noted that it yielded more pronounced beats than the saw shape. By graphical means, he showed that the intensity of the superposition of two tones of equal intensity implied a modulation of the oscillations at the beating frequency. He also showed that in the case of saw-shaped vibrations, there was a "recurrence of a similar state of joint motion" at another frequency, in conformity with the occasional hearing of two combination tones instead of one. For instance the combination of two tones separated by a major third (4: 5) yields the double octave below the lower tone, but also the fourth below this tone, so that one hears four tones in the ratio (1: 3: 4: 5).[17]

Young understood the importance of the sign of the aerial displacements in his reasoning. He saw that the superposition of two displacements of opposite signs yielded a vanishing displacement in the middle of the interval between two successive beats:

> The strength of the joint sound is double that of the simple sound only at the middle of the beat, but not throughout its duration; and it may be inferred, that the strength of sound in a concert will not be in exact proportion to the number of instruments composing it. Could any method be devised for ascertaining this by experiment, it would assist in the comparison of sound with light.

Young here ignored phase relations and confused amplitude and intensity. He nonetheless imagined what we would now call the interference of acoustic waves, and he raised the question of a similar behavior of light.[18]

In his discussion of vibrating strings, Young showed his familiarity with the construction of Euler and d'Alembert, and also with Daniel Bernoulli's harmonic analysis of the motion. He was torn between two sides of the quarrel of vibrating strings, one in which harmonic analysis was essential, another in which d'Alembert's solution was

[16]Ibid., p. 83. Cf. Kipnis 1991, pp. 26–30, 38–9. M. Young 1784, which T. Young read, already emphasized the sign of wave pulses: cf. Darrigol 2009, pp. 134–5.

[17]Young 1800a, p. 84. Although there is an error in Young's reasoning, his result is correct (tone corresponding to the difference between the frequency of the harmonic 2 of the lower tone and the frequency of the higher tone).

[18]Ibid., p. 84. Robinson had already considered the destructive interference of sound in his encyclopedia article on the speaking trumpet: cf. Darrigol 2009, pp. 135–6.

preponderant. His position in 1799 seems to have been close to Euler's final position, according to which harmonic components truly existed but did not necessarily have a sinusoidal shape. This interpretation of vibrating strings, together with Lagrange's and Euler's proofs that waves of any shape conserved their shape during propagation, explain why his discussion of "coalescence" favored saw-shaped waves over sine-shaped ones.[19]

In the same section, Young reported an old observation by John Wallis and used this opportunity to introduce the idea of a mutual destruction of waves along tense strings:

> If the string of a violin be struck in the middle, or at any other aliquot part, it will give either no sound at all, or a very obscure one. This is true, not of inflection, but of the motion communicated by a bow; and may be explained from the circumstance of the successive impulses, reflected from the fixed points at each end, destroying each other: an explanation nearly analogous to some observations of Dr MATTHEW YOUNG on the motion of chords.

In his acoustic treatise, Matthew Young had related the production of the higher modes of vibration of a string to the mutual reinforcement of the disturbances traveling from the bowing point and reflected by the extremities of the string; this reinforcement could only occur if the plucking or bowing point divided the string into two parts, one of which was a multiple of the other. Thomas Young's explanation of Wallis's observation was of the same kind. It provided a second sort of interference, and confirmed his tendency to relate standing waves to progressive waves.[20]

Close to the end of his letter, Young gave the views on human voice that he had first expressed in the Göttingen *lectio cursoria*. He also discussed the various musical temperaments, with a further insult to Robert Smith's memory: "Dr SMITH has written a large and obscure volume, which, for every purpose but for the use of an impracticable instrument, leaves the whole subject precisely where he found it." Young agreed with Sauveur that dissonance should be appreciated through beats, and recommended the tolerance of imperfect concords as long as the most frequently used keys remained the most harmonious. He represented old and new temperaments on a disk, the logarithmic divisions of the octave being represented on a different circle for each temperament.[21]

Young's letter was read to the Royal Society on 16 January 1800, and published under the title "Outlines of experiments and inquiries respecting sound and light." This title indicates that Young supported his arguments by many experiments; that his work, though mainly about acoustics, in part aimed at preparing a new wave optics by analogy with acoustics, that he did not pretend to definitely settle the issues. The conclusion of his letter was modest: "Thus, Sir, I have endeavoured to advance a few steps only, in the investigation of some very obscure but interesting subjects."[22]

Young asserted the novelty of his observations, but left the reader unclear about the originality of his more theoretical suggestions. One has to be familiar with the earlier

[19] Young 1800a, pp. 85–91. [20] Ibid., p. 90.
[21] Ibid., p. 93. [22] Ibid., p. 96.

history of acoustics in order to realize that Young's insistence on the mutual destruction of the positive displacements of a wave with the negative displacements of another was essentially new. This destruction occurred in an entirely new theory of beats, in a remark about the non-additivity of intensities, and in the discussion of vibrating strings. In modern terms, Young inaugurated both temporal interference (temporal modulation of intensity) and spatial interference (permanent cancellation at some place), although he did not yet describe patterns of spatial interference.[23]

In optics, Young clearly regarded his arguments as better than Euler's. A well-informed, critical reader could yet have doubted this superiority. Young's alleged proof of the non-divergence of sound rested on an unwarranted analogy between continuous flow and propagated vibrations. His two main objections against the Newtonian theory, the uniform velocity of the light particles from various sources and partial reflection, had already been addressed by Newtonian or neo-Cartesian emissionists. Young did perceive a genuine weakness of Euler's theory: the reliance on different mechanisms for the propagation of light in ether and matter. But he did not see one of its strengths: a plausible explanation of the colors of bodies by resonance. His own theory of the colors of thin plates rested on a very loose analogy with organ pipes and implied a concept of white light incompatible with the analogy with sound waves. His theory of inflection hardly improved on Mairan's and du Tour's earlier theories.

Enduring characteristics of Young's style can be appreciated from this letter of July 1799. He avoided mathematical developments as much as possible, and favored geometrical, diagrammatic reasoning over an algebraic one. Although he recognized the importance of the results obtained by Euler, Lagrange, and Laplace, he criticized their fondness for algebraic methods and took the pain to translate their results and demonstrations in a more geometrical and more intuitive language:

> The strong inclination which has been shown, especially on the Continent, to prefer the algebraical to the geometrical form of representation, is a sufficient proof, that, instead of endeavouring to strengthen and enlighten the reasoning faculties, by accustoming them to such a consecutive train of argument as can be fully conceived in the mind, and represented with all its links by the recollection, they have only been desirous of sparing themselves as much as possible the pains of thoughts, and labour by a kind of mathematical abridgment, which, at best, only serves the office of a book of tables in facilitating computations, but which very often fails even to this end, and is at the same time the most circuitous and the least intelligible.

Young compared the algebraic thinker to a night traveler unable to appreciate the scenery, and preached a return to good old Newtonian methods. He went so far as stating: "It seems, indeed, as if mathematical learning were the *euthanasia* of physical talent."[24]

Young's favorite method was analogy, complemented with experiment and geometrical reasoning: "Where the analogy is sufficiently close, it is a most satisfactory ground of physical inference." In many cases, he used analogy not only to illustrate otherwise understood phenomena, but to suggest new concepts and new phenomena. In the

[23] Cf. Broeckhoven 1971; Latchford 1974, p. 32; Kipnis 1991, p. 26.

[24] Young 1800b, p. 100; "Life of Lagrange," *YMW* 2, pp. 557–82, on p. 581.

introduction to his *Lectures on natural philosophy* (1807), he characteristically stated: "This combination of experimental with analogical arguments is the principal merit of natural philosophy."[25]

In the early years of the century, Young's hostility to dry mathematics and his predilection for experimentally inspired analogy reflected the dominant practice of British physics. They became more singular when, in the 1820s and 1830s, British natural philosophers massively adopted and adapted methods of continental physics. Other characteristics of Young's style were unusual from the beginning: the extreme concision of his writings, his ebullient, at times insolent tone, his ability to distance himself from Newton's towering figure, the liberty with which he imagined analogies between widely different fields, his willingness to take risks and his candid admission of past errors. These qualities did not necessarily please his contemporaries.

The principle of interference

On 27 July 1801, Young wrote to a friend:

> I am at present employed in some further optical investigations, which, I imagine, will be considered as more important than any of my former attempts, as I think they will establish almost incontrovertibly the undulatory system of light, and extend it to the explanation of an immense variety of phenomena.

Young's breakthrough, which had occurred a month earlier, was a new understanding of the colors of thin plates. In his previous analogy with organ pipes, the incoming white light corresponded to the air from the bellows, and the color to the tone emitted by the pipe. Young had at least two reasons to be dissatisfied with this analogy: it made white light a continuous stream of ether, against the theory of propagation he otherwise adopted; and he had no adequate explanation for the production of sound in an organ pipe, other than saying that it was "somewhat analogous" to the production of a tone by a blast between two parallel walls. Plausibly, he thought of replacing the organ pipe with a reed pipe, in which case the sound produced implies the resonance of a cavity subjected to an external vibration. In his letter of July 1799 to the Royal Society, Young had explained this resonance by the coincidence of the incoming pulses of the external vibration with the pulses reflected by the walls; he ignored the signs of the aerial displacements, which played an important role in his subsequent discussion of "coalescence." At some point, he must have realized that the coalescence of undulations rather than the coincidence of pulses was the key to cavity resonance, and also, by analogy, the key to the colors of thin plates.[26]

Young understood that the light sent back by a thin plate resulted from reflection either by the first interface or by the second interface (he ignored multiple reflections). In the latter case the light follows a longer path, part of which is included in the medium of the plate. Consequently, the undulations are shifted by an amount determined by the difference in traveling time. When the time difference is a whole number of periods, the intensity

[25] Young 1804b, p. 211; Young 1807, p. 5.

[26] Young to Dalzel, 27 July 1801, in Dalzel 1862, pp. 206–7. See also Young to Dalzel, 29 March 1802 (ibid., p. 212): "The theory of light and colours, though it did not occupy a large portion of time, I conceive to be of more importance than all that I have ever done, or ever shall do besides."

of the combined light is a maximum; when the time difference differs by half a period from a whole number of periods, the intensity of the combined light is a minimum because the undulations have opposite signs at any given point (it would vanish if the amplitudes of the two combined waves were equal). Young expressed this interference through the following "universal law":

> When two portions of the same light arrive at the eye by two different routes, either exactly or very nearly in the same direction, the appearance or disappearance of various colours is determined by the greater or less difference in the lengths of the paths: the same colour recurring, when the intervals are multiples of a length, which, in the same medium, is constant, but in different mediums, varies directly as the sine of refraction.

Young emphasized the analogy with beats in acoustics:[27]

> The general law by which all theses appearances are governed, may be very easily deduced from the interference of two coincident undulations, which either cooperate, or destroy each other, in the same manner as two musical notes produce an alternate intension and remission, in the beating of an imperfect unison.

In the case of thin plates, Young calculated the path difference from Fig. 5.1, in which the sun and the eye are very far from the plate, so that the rays issuing from them are very nearly parallel. The segments AB and CD being perpendicular to the incoming and refracted ray respectively, the Hobbesian construction of refracted rays implies that the

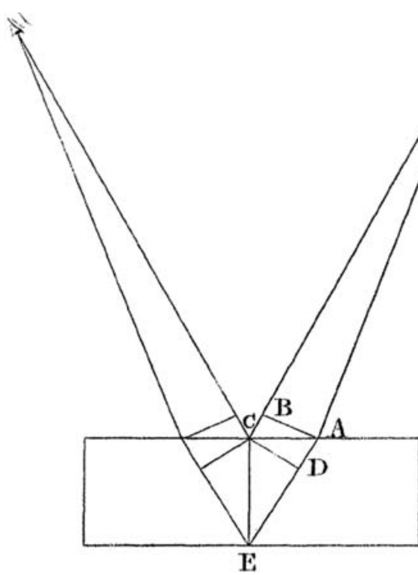


Fig. 5.1. Young's drawing for interference in the light from a thin plate. From Young 1802b, plate.

[27]Young 1802a, pp. 114, 118. On Young's concept of optical interference, cf. Kipnis 1991, chap. 4.

light takes the same time to travel the distance BC in the first medium as it does to travel the distance AD in the second medium. Consequently, the effective path difference is twice the length DE, which is the thickness CE multiplied by the cosine of the refraction angle. Young concluded that the same color should recur whenever this product is the same constant, in fair agreement with Newton's observations.[28]

Young had more difficulty explaining why the maximal reflection of a given color corresponded to the minimal transmission through the plate. For this purpose, he argued that next to the first interface the combined undulation implied a displacement of matter that made it fit for reflection and unfit for transmission, or vice versa. So to speak, he moved Newton's fits from the second interface to the first, and applied them to traveling waves instead of rays.[29]

Young soon imagined other situations in which his "universal law" would apply. The simplest one was the interference of the lights scattered by two parallel lines drawn on a glass surface. Young thus interpreted the colors of striated surfaces that Boyle, Mazeas, and Brougham had observed. He verified the theoretical interference pattern by his own experiment on a grating. He did not fail to discuss an acoustic counterpart: "There is striking analogy between this separation of colours, and the production of a musical note by successive echoes from equidistant palisades." Young also gave a basically correct theory of thick plates involving the interference of two rays: the first ray enters the plate perpendicularly, bounces back on the silvered surface, and it is scattered by dust on the front surface; the second ray is scattered by dust when entering the plate, then reflected by the silvered surface, and refracted by the front surface. As Young commented, Newton had erred in applying a direct analogy with thin plates to this problem. Lastly, Young gave a new theory of the colored fringes resulting from diffraction by an edge. This theory implied the interference of two rays, one inflected by the atmosphere around the edge, another reflected by the surface of the edge (see Fig. 5.2).[30]

Despite the qualitative success of this explanation of diffraction fringes, Young was beginning to question the atmospheres and to open his mind to the idea that diffraction was a sui generis wave process: "I do not consider it as quite certain, until further experiments have been made on the inflecting power of different substances, that Dr HOOKE's explanation of inflection, by the tendency of light to diverge, may not have some pretensions to truth." The probable reason for this doubt was Young's new reflections on the rectilinear propagation of light, prompted by his reading or rereading of Newton's and Huygens's texts.[31]

Young wondered why wave motion proceeded rectilinearly from its source and was not lost into lateral motions. He noted Huygens's explanation:

> The theory of HUYGENS indeed explains the circumstance in a manner tolerably satisfactory: he supposes every particle of the medium to propagate a distinct

[28] Young 1802b, pp. 160–1. Cf. Kipnis 1991, pp. 92–5.

[29] Young 1802b, p. 160.

[30] Ibid., pp. 158–60 (striated surfaces), 163 (thick plates), 164–6 (inflection). Cf. Kipnis 1991, pp. 102–5; Mollon 2002.

[31] Young 1802b, p. 165.

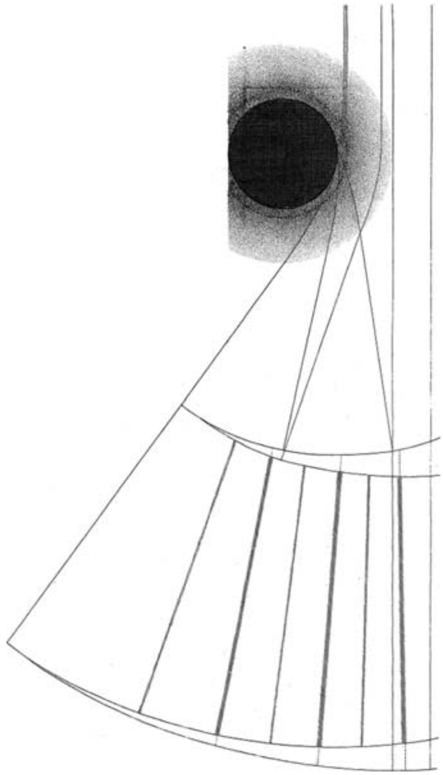


Fig. 5.2. Young's explanation of colored fringes caused by a diffracting edge (black disk). Interference occurs between a ray inflected by the atmosphere of the edge (in gray) and a ray reflected by the edge. From Young 1802b, plate.

> undulation in all directions; and that the general effect is only perceptible where a portion of each undulation conspires in direction at the same instant.

Young then recalled a classical objection to Huygens's secondary waves:

> But, upon this supposition, it seems to follow, that a greater quantity of force must be lost by the divergence of the partial undulations, than appears to be consistent with the propagation of the effect to any considerable distance.

Young nevertheless adopted Huygens's secondary waves in his subsequent discussion of the light traveling from a point source through an aperture.[32]

About the undulations drawn on Fig. 5.3, Young wrote:

> The principal undulations will proceed in a rectilinear direction towards GH, and the faint radiations on each side will diverge from B and C as centres, without receiving

[32]Ibid., p. 150.

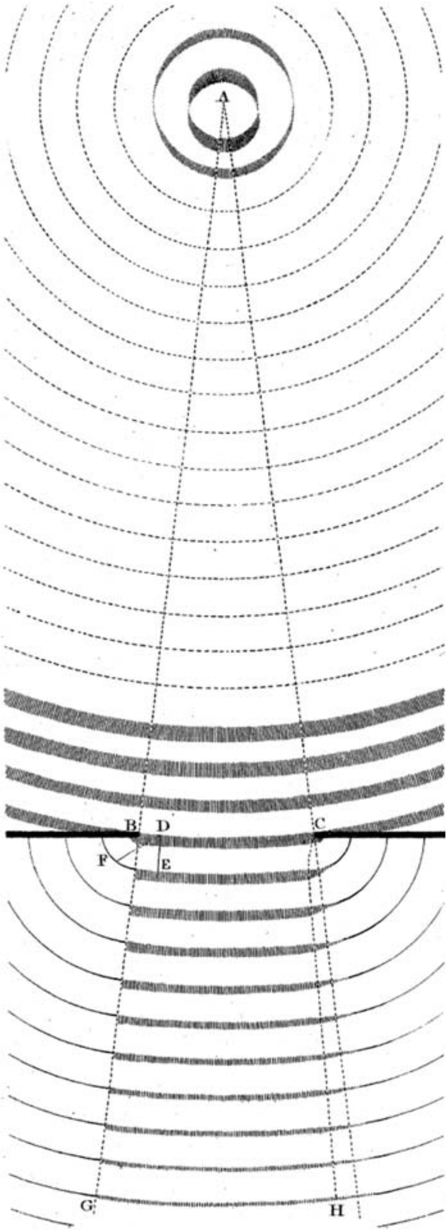


Fig. 5.3. Young's drawing of a wave passing an aperture on a screen. From Young 1802b, plate.

> any additional force from any intermediate point D of the undulation, on account of the inequality of the lines DE and DF.

This means that Young imagined Huygensian wavelets issuing from the aperture, but failing to reinforce each other in the shadow of the screen. The invoked cause of this failure, lack of timing, is not entirely clear. Young did not simply mean that narrow pulses starting from B and D could not reach F at the same time without violating the constancy of the velocity of light. That he meant more can be inferred from the remark:

> If … the aperture bears but a small proportion to the breadth of an undulation, the newly generated undulation may nearly absorb the whole force of the portion admitted; and this is the case considered by NEWTON in the Principia.

This statement is important in itself, for it is the first suggestion that diffraction essentially depends on wavelength. In addition, it shows that Young was again thinking in terms of the overlap of undulations, not in terms of the coincidence of pulses: as long as the aperture is much smaller than the wavelength, the wavelets issuing from any D overlap in the F point. Perhaps Young also meant that they interfere destructively in the contrary case, as Fresnel later argued in his explanation of linear propagation.[33]

Young announced his new findings in his second Bakerian lecture, read on 12 November 12, 1801 to the Royal Society. He summarized them in the syllabus of his lectures at the Royal Institution, written in this period and published in the following year. In the Bakerian lecture he supported "undulations" against emission and also against vibrations:

> I use the word undulation, in preference to vibration, because vibration is generally understood as implying a motion which is continued alternately backwards and forwards, by a combination of the momentum of the body with an accelerating force, and which is naturally more or less permanent; but an undulation is supposed to consist in a vibratory motion, transmitted successively through different parts of a medium, without any tendency in each particle to continue its motion, except in consequence of the transmission of succeeding undulations, from a distinct vibrating body; as, in the air, the vibrations of a chord produce the undulations constituting sound.

Young thus departed from the late neo-Cartesian usage, which favored the name "system of vibrations" and carefully avoided analogy with water waves. He also meant to replace the idealized pulses of older acoustics and of Huygens's theory of light with "undulations" that implied an alternation of positive and negative displacements.[34]

Careful word choice was only one of Young's rhetoric devices. Another was his recourse to the Newtonian mode of exposition: numbered hypotheses and propositions, with scholia and corollaries. Most astutely, he brought the "stamp of Newtonian approbation" by citing extracts of Newton's writings in support of three of his four basic hypotheses: the existence of a pervading elastic ether, the undulations caused by luminous bodies, and the relation between color and frequency. He did not say when Newton was discussing the theory of his opponent instead of his own, and he neglected to mention that Newton's

[33] Ibid., p. 151. [34] Ibid., p. 143.

ether waves resulted from the impact of the corpuscles of light. It would be excessive, however, to reduce Young's reference to Newton to shrewd rhetoric. He is likely to have been impressed by Newton's extensive use of acoustic analogies in optics, in spite of Newton's predilection for light corpuscles. And the wave-theoretical elements of Newton's theory plausibly inspired him.[35]

Newton was not the only authority that Young evoked in support of importing acoustic concepts in the theory of light. In a letter to Marc Auguste Pictet written on 21 August 1801 and published in French in the same year, he argued that Aristotle's texts, if properly translated, contained the analogy between pitch and color as well as their correspondence with frequency. In the *De audibilibus* he found the idea that the pitch of a sound corresponded to the frequency of the responsible vibration, as well as the idea that the consonance of octaves corresponded to the coincidence of pulses. In the *De anima*, he found the analogy between color and pitch in the following extract (translated literally from Young's French translation of the Greek):

> The difference between sonorous bodies is felt by the very act of their sounding; indeed, just as we do not perceive colors without light, the sounds, sharp or grave, are not heard without the act that brings the sounding into play.

Aristotle only meant to emphasize the need of a third entity (sunlight or the hammer of a bell) to actualize both sounds and colors, regarded as the potential objects of auditory and visual perception. Young's reading only shows his ignorance of the elements of Aristotle's theory of perception, as well as a misinterpretation of Aristotle's *chroma*, which implied a mixture of white and black in various proportions.[36]

Young's exploitation of Newton's writings was more inspired than his reading of Aristotle. A remarkable example is the scholium that Young added to one of the Newtonian queries cited in his Bakerian lecture. In this query, Newton argued from the persistence of retinal impressions that the motion of the retina had to be a vibration whose frequency depended on the bigness of the impacting rays. Young accepted Newton's suggestion that the retina vibrated. In addition, his undulatory concept of light compelled him to regard retinal excitation as a resonance phenomenon involving the frequency of the incoming light and the characteristic frequency of retinal oscillators. This understanding, together with his probable awareness that any color could be obtained by mixing pigments of three primary colors, led him to reason as follows:

> Since, for the reason here assigned by NEWTON, it is probable that the motion of the retina is rather of a vibratory than of an undulatory nature, the frequency of the vibrations must be dependent on the constitution of this substance. Now, as it is almost impossible to conceive each sensitive point of the retina to contain an infinite number of particles, each capable of vibrating in perfect unison with every possible undulation, it becomes necessary to suppose the number limited, for instance, to the

[35] Ibid., p. 141.

[36] Young 1801b. Compare with chap. 1, above, p. 6. J. A. Smith's translation (Ross 1908–1952, vol. 3, 420a) reads: "The distinctions between different sounding bodies show themselves only in actual sound; as without the help of light colors remain invisible, so without the help of actual sound the distinctions between acute and grave sounds remain inaudible."

> three principal colours, red, yellow, and blue, of which the undulations are related in magnitude nearly as the numbers 8, 7, and 6; and that each of the particles is capable of being put in motion less or more forcibly, by undulations differing less or more from a perfect unison; for instance, the undulations of green light being nearly in the ratio of 6½, will affect equally the particles in unison with yellow and blue, and produce the same effect as a light composed of those two species: and each sensitive filament of the nerve may consist of three portions, one for each principal colour.

This is the basis of the three-receptor theory of the perception of colors. Young did not develop it any further. He only commented that this theory contradicted Newton's optical analogues of musical concords, and remarked that "in hearing, there seems to be no permanent vibration of any part of the organ." Thus, there were some limits to the analogy between sound and light.[37]

In Young's opinion, his most important discovery was the principle of interference (not yet named so) and its applications to striated surfaces, thin plates, thick plates, blackness, and inflection; which together provided "a very strong confirmation" of the undulatory theory. This principle was expressed in Young's proposition VIII:

> *When two Undulations, from different Origins, coincide either perfectly or very nearly in Direction, their joint effect is a combination of the Motions belonging to each.*

To compensate the vagueness of this statement, Young explained how undulations of different signs could cancel each other. The applications made clear that he only meant to apply the principle to undulations originating from the same source and then following different paths.[38]

Another novelty of Young's lecture was the treatment of dispersion sketched in his proposition VII:

> *If equidistant Undulations be supposed to pass through a Medium, of which the Parts are susceptible of permanent Vibrations somewhat slower than the Undulations, their Velocity will be somewhat lessened by this vibratory Tendency; and, in the same Medium, the more, as the Undulations are more frequent.*—For, as often as the state of the undulation requires a change in the actual motion of the particle which transmits it, that change will be retarded by the propensity of the particle to continue its motion somewhat longer: and this retardation will be more frequent, and more considerable, as the difference between the periods of the undulation and of the natural vibration is greater.

Young thus anticipated the modern theory of dispersion, developed in the 1870s. He may have been inspired by Euler's theory of the color of bodies, which implied particles of matter behaving like oscillators. The similarity is only partial: whereas Euler made the

[37]Young 1802b, pp. 146–7. Cf. Hargrave 1973. After Wollaston's improved description of the spectrum of white light, Young changed the three principal colors to red, green, and violet (*YMW* 1, p. 176). On nineteenth-century elaborations of Young's theory of color vision, cf. Sherman 1981.

[38]Young 1802b, p. 157. Young explained blackness by destructive interference of the light reflected by an uneven surface (ibid., pp. 163–4).

particles of matter the sole conveyers of light waves, Young imagined a coupling between the undulation of the ether and the vibrations of bound material oscillators.[39]

A few months later, Young clearly separated "the general law of interference" from his commitment to the wave theory of light:

> Whatever opinion may be entertained of the theory of light and colours which I have lately had the honour of submitting to the Royal Society, it must at any rate be allowed that it has given birth to the discovery of a simple and general law, capable of explaining a number of the phenomena of coloured light, which, without this law, would remain insulated and unintelligible. The law is, that 'wherever two portions of the same light arrive at the eye by different routes, either exactly or very nearly in the same direction, the light becomes most intense when the difference of the routes is any multiple of a certain length, and least intense in the intermediate state of the interfering portions; and this length is different for light of different colours.'

The formulation is more precise than the earlier ones, as a consequence of Young's desire to make the law applicable without reference to the wave theory. Young's strategy here resembles Newton's enunciation of the concept of fits of easy reflection or transmission without reference to ether waves. For all that, Young's confidence in the superiority of the wave theory did not abate. He emphasized that interference phenomena required a smaller velocity of light in denser media, in contradiction with the emissionist explanations of refraction and inflection.[40]

In the rest of his communication, Young described a few new applications of the law of interference to the colored fringes produces by a hair, to atmospheric halos, and to the colored rings of mixed plates. In the case of the hair, Young imagined interference between a ray reflected on one side of the hair and a ray inflected on the other. He similarly explained atmospheric halos by the interference caused by mist made of droplets of equal size. In the case of mixed plates, he conceived interference between a ray passing a thin layer of water (trapped between two glass plates) and a ray passing through a hole in this layer. He also made the important remark that the dark central spot of light reflected by bubbles (the thickness of which is much smaller than the wavelength), required that monochromatic light should be retarded by half a wavelength when reflected at the surface of a denser medium. He verified that, in agreement with this rule, the central spot became bright when a fluid of intermediate optical index was pressed between a plate and a convex lens of unequal indices.[41]

Harmonic waves

By analogy with mixed plates, Young imagined a new wave theory of dispersion based on the following assumption:

[39]Ibid., pp. 156–7. At the end of his memoir (pp. 166–9), Young addressed the few objections to the wave theory of light that he had not yet refuted: the difficulty of explaining the "sides" of the rays emerging from the Iceland spar, the momentum of light allegedly detected by John Mitchell, and phosphorescence.

[40]Young 1802c, pp. 170 (citation), 174 ("law of interference," wave theory supported).

[41]Ibid., pp. 171–2 (hair), 172–3 (halos), 173–4 (mixed plates), 174–5 (central spot). Cf. Kipnis 1991, pp. 98–100, 105–11.

> We may suppose that every refractive medium transmits the undulations constituting light in two separate portions, one passing through its ultimate particles, and the other through its pores; and that these portions re-unite continually, after each successive separation, the one having preceded the other by a very minute but constant interval, depending on the regular arrangement of the particles of a homogeneous medium.

In modern terms, the superposition of two waves of different phases produces a wave whose phase has an intermediate value depending on the phase difference and on the relative amplitude of the two component waves. Young thus explained why the phase of the combined waves in the refractive medium globally traveled at a velocity smaller than the velocity of light in the ether, to an extent depending on the porosity of the medium.[42]

Young realized that a quantitative version of this argument required a specific assumption for the form of the undulations. "Assuming the law of the harmonic curve for the motions of the particles, we might without much difficulty reduce this conjecture to a comparison with experiment." Very likely, he performed the simple phase-shift calculation in this case, although he did not include the result in his communication. The important point is that he now favoured harmonic curves to represent simple colors whereas in his memoir of 1800 on sound and light he regarded saw-shaped undulations as simpler and insisted on the arbitrariness of the form of the undulation.[43]

This evolution in Young's concept of simple vibration is not specific to optics. It appears in an acoustic context in the syllabus of his lectures:

> In order to examine the effect of a combination of different sounds, we must assume some law for the motions of the particles; and none is so simple as that of the cycloidal pendulum, or of the harmonic curve, which seems indeed to have some natural claim for preference, for the ear generally analyses more complicated vibrations into such subordinate ones as may be derived from this form.

Young knew that in the case of a plucked string the motion was saw-shaped and yet the ear perceived a few harmonics of the fundamental sound. He now regarded this phenomenon as a confirmation of the ear's ability to perform harmonic analysis, although he had earlier traced it to some heterogeneity of the string and to its rotational motion.[44]

Young's conversion to the harmonic curve plausibly resulted from his increasing awareness of its mathematical simplicity and from its pervasiveness in the study of small oscillations of any kind. In the section of the syllabus devoted to the theory of tides, he described two cases of interference between sinusoidal motions:

> The solar tide is about two fifths of the lunar tide, and the joint effect of both is to produce a periodical increase and diminution, in the same manner as the undulations of sound are combined with each other, and also according to the same law as in the simplest cases of sound ... In the same manner as a solar and lunar tide are combined in different proportions, so it frequently happens that the same tide arrives at a given place by two different courses, and the joint effect is increased, diminished, or

[42] Young 1802c, p. 175.

[43] Ibid., p. 176.

[44] Young 1802a, p. 91; Young 1807, vol. 1 (2nd ed.), p. 301.

destroyed, according to the difference of the times occupied in the routes of the different portions.

The latter case of interference is a probable allusion to Newton's account of the tides in the Tonkinese port of Batsha, which Young later cited as an anticipation of the law of interference.[45]

In 1802, Young advertised the "harmonic slider" that he had invented to mechanically superpose two harmonic curves (Fig. 5.4). On this occasion he judged the harmonic curve as "by far the most natural as well as the most convenient to be assumed, as representing the state of an undulation in general." He envisioned applications of his slider to musical consonance and dissonance, to tides, and to optics. He opined that the "purest and most homogeneous sounds" should be represented by harmonic curves. His theory of tides of 1813 was essentially based on analogy with the resonance of a pendulum subjected to a sinusoidal excitation. In 1717 he wrote: "In the undulatory theory, the analogy between the law of interference, and the phenomenon of tides, and the effects of the combination of musical sounds is very direct and striking."[46]

Even so, the sinusoidal form of optical undulations played little role in Young's optics. His discussions of interference only required the consideration of the minima and maxima of illumination, for at least two reasons: he never measured the intensity of the illumination; and he only considered two-ray interference, in which case the interference pattern can be derived without knowing the precise shape of the superposed undulations. He never

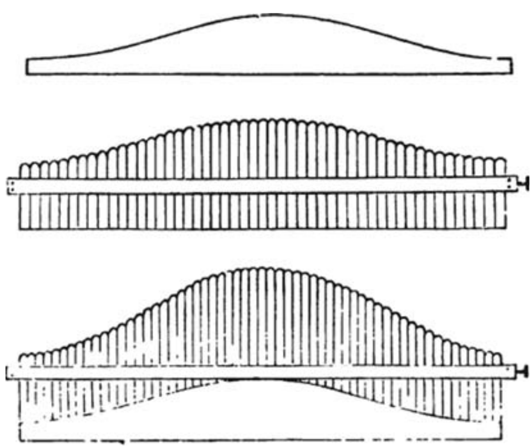


Fig. 5.4. Young's harmonic sliders. Sliders of a length proportionate to a sine curve are placed above a board whose profile is another sine curve. From Young 1802d, plate.

[45]Young 1802a, pp. 143–4; Young 1804b, p. 203. On the Batsha tides, cf. Cartwright 2003. On Young's reference to Newton's explanation, cf. Kipnis 1991, pp. 49–52. The modern explanation of the diurnal (instead of semidiurnal) character of these tides is based on standing waves in the Gulf of Tonkin rather than on interference.

[46]Young 1802d; Young 1817, p. 329.

used the algebraic identity for the sum of two sine functions, in conformity with his general preference for geometrical methods.[47]

Crucial experiments

In 1803, Young gave his third Bakerian lecture, the second of those devoted to physical optics. His main new result was "an experimental demonstration of the general law of the interference of light." Although "the ingenious and accurate Grimaldi" had long ago observed fringes in the shadow of a narrow rectangular blade, this phenomenon had generally been overlooked owing to Newton's failure to notice it. Young not only studied these fringes with unprecedented accuracy, but he realized that they disappeared when the light passing on one side of the blade was blocked. In his opinion, this observation provided a direct proof of interference since it demonstrated the necessity of light traveling through two different paths. The Newtonian explanation by differential inflection near the edges of the blade failed, since the inflection on one side of the blade could not plausibly depend on the other side. In contrast, the law of interference immediately explained the equal spacing of the fringes in the plane of observation as well as their hyperbolic trajectory when the distance of this plane from the blade grew.[48]

Whereas Young had earlier justified interference by acoustic analogy, he now regarded the law of interference as establishing the analogy between light and sound:

> For, hitherto, I have advanced in this Paper no general hypothesis whatever. But, since we know that sound diverges in concentric superficies, and that musical sounds consist of opposite qualities, capable of neutralising each other, and succeeding at certain equal intervals, which are different according to the difference of the note, we are fully authorised to conclude, that there must be some strong resemblance between the nature of sound and that of light.

Moreover, Young abandoned the Newtonian concept of inflection that he had used in his earlier accounts of diffraction. He now declared:

> I have not, in the course of these investigations, found any reason to suppose the presence of such an inflecting medium in the neighbourhood of dense substances as I was formerly inclined to attribute to them; and, upon considering the phenomena of the aberration of the stars, I am disposed to believe, that the luminiferous ether pervades the substance of all material bodies with little or no resistance, as freely perhaps as the wind passes through a grove of trees.

According to this poetical metaphor, the ether could not accumulate in the vicinity of material bodies, so that inflecting atmospheres à la Mairan could not exist.[49]

The famous experiment of Young's holes or slits appears in Young's *Lectures* of 1807. It provides the simplest illustration of the principle of interference, since the two holes

[47]In an article on tides, Young (1823a, p. 325) gave the identity $\sin a + \sin b = 2\sin[(a+b)/2]\cos[(a-b)/2]$; but he did not use it to discuss tidal interference.

[48]Young 1804a, pp. 179–81. Two earlier exceptions to the general neglect of internal fringes were du Tour 1768–1774 [1760], and Jordan 1799.

[49]Young 1804a, p. 188.

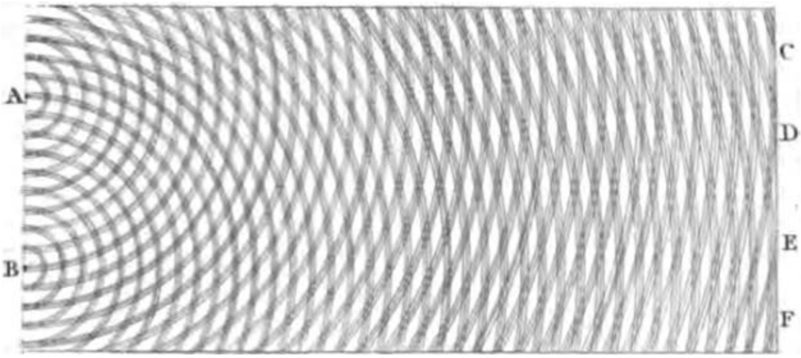


Fig. 5.5. Water wave interference according to Young. The points A and B are the centers of circular waves. The points C, D, E, F belong to hyperbolic lines of destructive interference. From Young 1807, plate XX, fig. 267.

materially determine the paths of the interfering portions of light. It is also the optical counterpart of a case of water-wave interference described in the *Lectures* (Fig. 5.5). In conformity with his use of the phrase "undulatory theory of light," Young now made water waves the central metaphor for any wave process:

> The subject of waves is of less immediate importance for any practical application than some other parts of hydraulics; but besides that it is intimately connected with the phenomena of the tides, it affords an elegant employment for speculative investigation, and furnishes us with a sensible and undeniable evidence of the truth of some facts, which are capable of being applied to the explanation of some of the most interesting phenomena of acustics [sic] and optics.

Water waves were also important in Young's latest approach to dispersion, which relied on analogy with the well-known dispersion of deep-water waves. Young clearly had trouble with optical dispersion. He gave no less than three theories in the span of five years. In retrospect, the first was the best; and the last was the worst since it erroneously related dispersion to the imperfect elasticity of the medium (even for water waves!).[50]

A disturbing novelty

To summarize, the analogy between thin plates and organ pipes prompted Young's interest in the wave theory of light, as well as acoustic studies in which he developed the concept of interference. In May 1801, he explained the colors of thin plates by interference. In the following two years, he discussed other cases of interference with impressive experimental and theoretical acumen. These included striated surfaces, diffraction fringes,

[50]Young 1807, vol. 1 (2nd ed.), pp. 220 (water-wave interference, citation), 364–5 (holes), 364 (dispersion, water-wave interference). Cf. Kipnis 1991, pp. 118–24.

and supernumerary rainbows.[51] Although Young soon separated the "law of interference" from its wave-theoretical justification, he regarded the numerous successful applications of this law as evidence in favor of the analogy between light and sound. He gradually purged his undulatory theory from remnants of the emissionist or neo-Cartesian viewpoints such as the inflection of rays and the atmospheres. He gave wave-theoretical explanations of all major optical phenomena, including rectilinear propagation, reflection, refraction, diffraction, and dispersion (but not yet the double refraction of Iceland spar). These explanations relied more on physical intuition and analogy than on a rigorous mechanics of continuous media. They derived from earlier reasoning by Huygens, Euler, and Hooke, except for dispersion, in which case Young imagined three different theories of his own.

Young's early works in optics were well received, as may be judged from their being the subject of two Bakerian lectures and also from the favorable reviews of these lectures. Lord Brougham's famous attacks were violent but exceptional. Yet no one before Fresnel joined Young in the study of interference phenomena or in the pursuit of undulatory optics. Many reasons have been evoked to explain this lack of interest, including Young's terse style, his spare use of mathematical reasoning, the weakness of the wave-theoretical explanations of rectilinear propagation and refraction, and the availability of emissionist explanations for the phenomena that Young explained by interference. Related reproaches can indeed be found among Young's contemporary readers, both in Britain and on the continent. There is one more reason for the contemporary neglect of Young's optics: his contemporaries ignored acoustic interference, on which he crucially drew to introduce optical interference.[52]

5.2 Laplacian optics

Laplace's discovery of Malus

While Young was developing his principle of interference, the French astronomer Pierre Simon de Laplace was trying to reduce all natural phenomena to central forces acting between molecules. In this grand neo-Newtonian scheme, the molecules of light interacted with the molecules of matter through short-range forces whose precise form was irrelevant to observable phenomena. Laplace expounded this view in the theory of atmospheric refraction found in the fourth volume of his *Mécanique céleste*, published in 1805. His theory of refraction completed Newton's by making the deflecting forces the sum of the actions of the individual molecules of the transparent media.[53]

In the simple case when the molecules of light cross the plane surface of a homogenous transparent medium, Laplace's theory implies that the force acting on a light molecule has

[51]Young 1804a, p. 185. There are two different ray paths for the diffuse light in a given direction within the primary rainbow (for a given color). Young traced the supernumerary rainbows to interference between theses two paths. Cf. Kipnis 1991, pp. 100–2; Hulst 1981, §13.2; Cowley 1998.

[52]Cf. Kipnis 1991, chap. 6. An example of emissionist interpretations of interference is Biot's suggestion that internal diffraction fringes would result from a sharpening of du Tour's atmospheric model: cf. Kipnis 1991, p. 255. On the contemporary ignorance of acoustic interference, see also Darrigol 2009, pp. 131–6.

[53]Laplace 1805, pp. 233–77. Cf. Grattan-guinness 1990, vol. 1, pp. 470–2; Heilbron 1993, pp. 150–4. On Laplacian physics, cf. Crosland 1967; Fox 1974.

the same intensity and the same direction at two points that are the mirror images of each other with respect to the surface. The reason is that, within the medium, the action of the layer of material molecules contained between the surface of the medium and a plane passing through the molecule of light is exactly balanced by the action of an equally thick layer on the other side of the molecule. Consequently, the variation of the squared velocity of the light molecule when it travels from infinity to the limiting surface is half the variation of the same quantity when the molecule crosses the whole deflecting zone (in modern terms, the separating surface is at the middle of a symmetric potential barrier).[54]

As Laplace saw, this result implies that the condition of total reflection within a dense transparent medium is not the usual one when the external medium is opaque. In the usual case, the external medium is transparent and total reflection occurs when the angle of incidence is superior to $\bar{i}_1 = \arcsin(n_2 / n_1)$, where n_1 and n_2 denote the optical indices of the two media. At this angle, the molecules of light slightly penetrate the external medium before returning to the denser medium. When the external medium is opaque, Laplace reasoned, penetrating rays are absorbed and reflection is possible only if the molecules of light turn round before reaching the interface. This begins to occur for a decrease of squared velocity that is half the decrease required for total reflection in the normal case. Since only the normal component of velocity is altered, the corresponding angle of incidence is $\hat{i}_1$ such that $\cos^2\hat{i}_1 = (1/2)\cos^2\bar{i}_1$.[55]

Although Laplace did not write so, he probably meant to refute William Hyde Wollaston's astute determination of the optical index of some opaque substances. This retired physician had measured the limiting angle of total reflection at the interface between glass and the opaque medium, and inferred the index from the relation $\bar{i}_1 = \arcsin(n_2 / n_1)$. In 1807, a brilliant newcomer in Laplace's circle, Etienne Louis Malus, examined the question experimentally. With measurements on beeswax (which can be made both transparent and opaque) and pioneering attention to causes of error, he confirmed the predictions of Laplace's theory.[56]

Malus was a military engineer trained in the early Ecole Polytechnique. He had privately reflected on the nature of light during his traumatic participation in Napoleon's Egyptian campaign. He then believed light to be a compound of caloric and oxygen, in agreement with the fact that light usually occurs during the combustion of bodies. A few years later, in Paris, he wrote an optical treatise in which the nature of light no longer mattered. The purpose of this treatise was to found the theory of optical instruments on a general theory of systems of rays in the spirit of the new *géométrie analytique* of his mentor and friend Gaspard Monge. Its core was the proof that a continuous family of rays depending on two parameters (what we now call a congruence) could always be construed as resulting from the intersections of two families of developable surfaces, whose cuspidal edges describe the two surfaces on which consecutive rays intersect. Malus used this result to determine caustics, to construct reflected and refracted beams, and to determine their intensities.

[54]Laplace 1805, pp. 233–43.

[55]Ibid., p. 243. See also the more lucid account in Malus 1808b, pp. 78–9. Cf. Chappert 1977, pp. 58–9; Buchwald 1989, pp. 28–31.

[56]Wollaston 1802a; Malus 1808b. Malus also verified the density dependence that was predicted by Laplace's theory.

He proved that after a first reflection or refraction of the rays issuing from a single point, the developable surfaces of the two generating families were mutually orthogonal, and that in this case the rays admitted a family of common orthogonal surfaces. The extension of the latter result to any number of reflections and refractions is usually called Malus's theorem, although Malus believed that the property no longer held after a second reflection or refraction. Malus did not connect these geometrical considerations with Huygens's waves.[57]

This treatise drew the attention of a few French mathematicians, and we will see that it later inspired some of William Hamilton's powerful theory of systems of rays. Laplace's appreciation probably determined Malus's involvement in optical questions of Laplacian interest. The first of these questions was the aforementioned determination of the optical index of opaque substances. The second was the verification of Huygens's law of extraordinary refraction.[58]

Extraordinary refraction

In the late eighteenth century, the crystallographer René Juste Haüy had dismissed Huygens's law of extraordinary refraction in favor of a simpler law of his own. In 1802, Wollaston claimed to have accurately verified Huygens's law with a method relying on indirect index measurement through complete reflection. He judged that his finding vindicated the undulatory theory, which Thomas Young was then pressing him to adopt. Laplace must then have worried that crystal optics was becoming a threat to molecular optics. On 4 December 1807, the Paris Academy offered a prize for the exact determination of double refraction. Malus won with a powerful memoir read on 4 June 1808. With measuring techniques and data analysis of unprecedented sophistication, he verified Huygens's law within a 1% margin of error. The challenge now was to derive these laws on a neo-Newtonian basis.[59]

Laplace and Malus both met this challenge by showing that Huygens's laws resulted from the principle of least action if the squared velocity of the light molecules in the crystal had the form

$$\upsilon^2 = \alpha^2 + \beta^2 \cos^2 \theta, \tag{1}$$

where θ is the angle that the velocity makes with the axis of the crystal. A straightforward but cumbersome way to prove this is to compare the analytical results of the variation

[57]Malus 1808a; 1811a (amplified version with corpuscular theory of refraction). On Malus's biography, cf. Arago 1855; Chappert 1977, chap. 1; Buchwald 1989, pp. 23–6. Malus silently borrowed the generating developable surfaces from Monge's theory of the optimal transport of earth for construction purposes: cf. Chappert 1977, pp. 114–15. Charles Dupin generalized Malus's theorem in 1817: cf. ibid., p. 135. This theorem is a direct consequence of Huygens's construction of reflected or refracted rays, since these rays are orthogonal to the envelope of the spherical wavelets emitted at the reflecting or refracting surface.

[58]On later developments of Malus's approach, cf. Atzema 1993a, 1993b. On Hamilton, see below, chap. 7, p. 264.

[59]Haüy 1788; Wollaston 1802b; Malus 1811b, chap. 1. Cf. Frankel 1974; Buchwald 1980a; 1989, pp. 12–19 (Haüy and flaws), 19–23 (Wollaston); 31–6 (Malus).

of the action with Malus's analytical expression of the consequences of Huygens's construction. Laplace and Malus both did this, in slightly different ways. Laplace also indicated a much simpler derivation.[60]

The first step of the latter derivation is the remark that Huygens's construction is equivalent to Fermat's principle of least time, if only the velocity is given by the radius of Huygens's ellipsoidal wavelets. In order to see this, consider Fig. 5.6 from Laplace. KB represents the trace of the surface of the crystal, OC the trace of a wave plane, EDA the trace of an ellipsoidal wavelet centered in C; OK the distance traveled by the wave to reach the surface of the crystal during the time that the wavelet has taken to develop. According to Huygens's construction, the extraordinary wave plane at the end of this time is obtained by drawing the plane tangent to the wavelet and passing through K; and the refracted ray is the line CI joining the center of the wavelet to the point of tangency. Now consider the ray *oci* refracted at *c* and the infinitely close ray $o'c'i'$. The length oc' differs from $o'c'$ by an infinitesimal of second order, since $o'c'$ is perpendicular to oo'. The lengths $c'i'$ and $c'i$ are in general unequal because the refracted rays are not perpendicular to the refracted waves. Yet the times that light takes to travel along these two lengths are the same (to first order) because by Huygens's construction *i* belongs to the tangent plane of the wavelet centered in c'. In addition, the construction implies that light takes the same time to travel *oci* and $o'c'i'$. Consequently, light takes the same time to travel the paths *oci* and $oc'i$. This is Fermat's principle of least time. The spheroidal shape of Huygens's wavelets implies that the velocity of light V in a direction that makes the angle θ with the optical axis verifies:

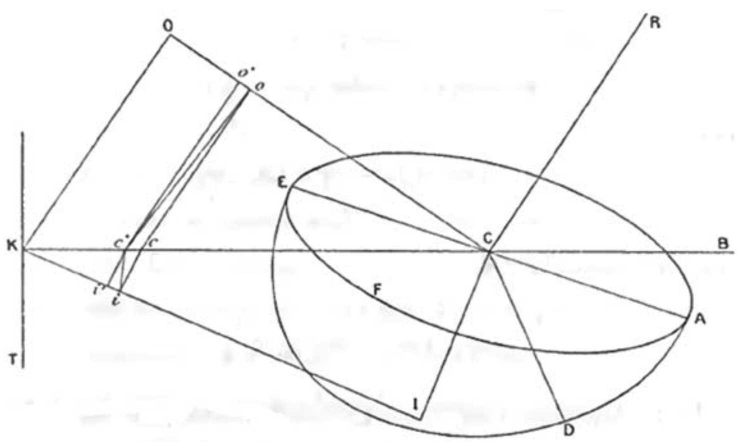


Fig. 5.6. Laplace's figure for proving the equivalence of Huygens's construction with Fermat's principle. From Laplace 1810, 281.

[60]Laplace 1809, 1810; Malus 1811b, chap. 2. Cf. Chappert 1977, pp. 62–6; Buchwald 1989, pp. 36–40.

$$\frac{V^2\cos^2\theta}{b^2}+\frac{V^2\sin^2\theta}{a^2}=1 \qquad \text{or} \qquad \frac{1}{V^2}=\frac{1}{a^2}+\left(\frac{1}{b^2}-\frac{1}{a^2}\right)\cos^2\theta. \tag{2}$$

The derivation ends with the obvious remark that Fermat's principle for light traveling at the velocity V is formally equivalent to Maupertuis's principle for particles traveling at the velocity $1/V$.[61]

Laplace announced a new victory for his neo-Newtonian program:

> I have found that the law of extraordinary refraction, laid down by Huygens, satisfies this condition [that the velocity of light only depends on the direction of the rays in the crystal], and agrees at the same time with the principle of least action; so that there is no reason to doubt that this law is derived from the operations of attractive and repulsive forces, of which the action is only sensible at insensible distances.

The claim prompted Thomas Young to write a lambasting review of Laplace's memoir in the *Quaterly review.* Young belittled Laplace's successful application of the principle of least action as a trivial consequence of the (to him) evident validity of Fermat's principle in Huygens's theory. He next denied any physical import of this success: Laplace had failed to identify the forces that could produce the angular dependence of the velocity of light molecules, whereas Huygens's spheroid of velocities could easily be justified by unequal compressibility of the medium in different directions. Lastly, Young reproached Laplace with not citing Wollaston's verification of Huygens's laws, although he added the following note in proof:[62]

> We must do Mr Laplace the justice to observe, that since this article was written, he has published, in the Memoirs of Arcueil, another paper on this subject, in which the name of Dr Wollaston is mentioned with due respect. The same volume contains also an account of some highly interesting and important experiments of Mr. Malus, on the apparent polarity of light, as exhibited by oblique reflexion, which present greater difficulties to the advocates of the undulatory theory than any other facts with which we are acquainted.

Polarization

In the course of his experiments on double refraction, Malus happened to observe the light reflected from the windows of the Luxembourg palace through a crystal of Iceland spar. To his surprise, the intensities of the two images varied when he rotated the crystal around the axis of vision. He suspected that the reflected rays of light acquired an asymmetry similar to that observed by Huygens for the ordinary and extraordinary rays issuing from a crystal of Iceland spar. He soon found that the analogy was complete when the reflection occurred at a specific incidence, 52° 45′ in the case of a water surface. For example, the reflected light suffered ordinary refraction only when it entered a crystal whose principal section (that is, the plane perpendicular to the face of the crystal and parallel to its axis)

[61] Laplace 1809, pp. 306–7; 1810, pp. 282–3.

[62] Laplace 1809, p. 305; Young 1809, pp. 282–3.

was parallel to the plane of reflection. Malus announced these results on 12 December 1808 at the Institut de France.[63]

Three months later, Malus reported that light reflected by a glass plate at the polarizing incidence (54° 35′) was no longer reflected by a second glass plate at the same incidence when the two planes of reflection were orthogonal. If the two planes of reflection made an angle θ (*ceteris paribus*), Malus added, the intensity of the reflected light varied as $\cos^2\theta$. In the lack of precise photometric measurements, Malus probably selected this law as the simplest trigonometric formula that yielded the correct value of the angles of minimum and maximum intensity.[64]

In Malus's opinion, the discovery of what he later called "polarization" by reflection decided the nature of light:

> All the ordinary phenomena of optics can be explained either by Huygens's hypothesis, which traces them to the vibrations of an ethereal fluid, or according to Newton's opinion, which traces them to the action of bodies on the luminous molecules considered as belonging to a substance subjected to the attractive and repulsive forces that serve to explain the other phenomena of physics. The laws concerning the progress of rays in double refraction can still be explained by either hypothesis. In contrast, as the above given observations prove that the phenomena of reflection differ for the same angle of incidence—which cannot happen under Huygens's hypothesis, the author concludes not only that light is a substance subjected to the forces that animate other bodies but also that the form and disposition of these molecules have a great influence on the phenomena.

Malus assumed like Newton that the molecules of light had a transverse asymmetry, which he described through the molecular axes *a, b, c*. He further assumed that the orientation of the transverse axes *b* and *c* depended on the direction of the repulsive forces exerted by matter:[65]

> If we consider, in the translation of luminous molecules, their movement around three principal axes *a, b, c*, the quantity of molecules whose *b* or *c* axis becomes perpendicular to the direction of the repulsive forces is always proportional to the square of the sine of the angle that these lines need to describe in their rotation around the axis *a* in order to assume this direction, and, reciprocally, the quantity of molecules whose *b* or *c* axes become the closest possible to the direction of the repulsive forces, is proportional to the square of the cosine of the angle that these lines need to describe in their rotation around the axis *a* in order to be included in the plane which contains this axis and the direction of the forces.

From this somewhat mysterious molecular principle, Malus hoped to derive a complete theory of partial reflection and of refraction in any pair of media and for any incidence. Partial reflection had been a stumbling block for the neo-Newtonian theorists of light, who

[63]Malus 1809a. The Luxembourg anecdote is from Arago 1855. Cf. Chappert 1977, pp. 59–66; Buchwald 1989, chap. 2.

[64]Malus 1809b. This memoir also contains a few results concerning metallic reflection.

[65]Ibid., pp. 343, 344. For the term *polarisation*, see Malus 1811c, pp. 106–7.

were little disposed to admit Newton's fits. Malus made polarization the key to this phenomenon, natural light being for him a mixture or rays polarized in all possible directions. Malus's last memoirs, read in 1811, contained observations regarding the amount of reflected light and the polarization of transmitted light for various orientations of the polarization of the incident light. He died a few months later.[66]

Malus's discovery opened an entire field of new researches in optics. The basic experiments on polarization were easy to replicate, and they could be improved upon. So did the Scottish experimenter David Brewster, who welcomed any confirmation of the Newtonian concept of light and excelled in designing optical instruments and apparatus. Whereas Malus had failed to find any regular relation between the polarizing angle i_B and the index of refraction, Brewster discovered the law $\tan i_B = n$, which now bears his name. As we will see in a moment, his numerous and precise experiments on polarization by refraction, on metallic reflection, and on the polarizing angle of crystals were important sources for the leaders of French optics. He was a major player in the study of chromatic polarization, the history of which we are about to encounter.[67]

Chromatic polarization

Before Malus read his two last memoirs, the young astronomer and freshly elected academician François Arago reported a series of experiments in which he studied the polarization of Newton's rings. Some of his findings contradicted received ideas. Most puzzlingly, he found that the light of the rings observed by transmission through the two compressed lenses was polarized in the same plane as the light of the rings observed by reflection. This contradicted Malus's principle that the production of polarized light by reflection always implied the production of an equal amount of transmitted light polarized in the perpendicular direction. In an attempt to decide which of the surfaces of the air gap controlled the formation of the rings, Arago replaced the second lens by a flat mirror. He found that the rings produced by reflection had the same polarization properties as if their light has been reflected at the interface between glass and air. This seemed to contradict Newton's opinion that the fits of easy reflection or transmission belonged to the other end of the air gap. As Arago's original manuscript is lost, it is not clear whether he seriously considered Young's theory at that time (he mentioned it in the version published in 1817). There is no doubt, however, that he framed his subsequent findings under the molecular concept of light.[68]

A few months later, Arago reported his discovery of the "depolarization" of light when crossing a plate of crystalline medium, a phenomenon later called "chromatic polarization." The starting point was his observation, through a double-refracting crystal, of a thin plate of mica with a clear sky behind it. He found that the two images of the plate were colored, although their overlap was white. The two colors changed when he rotated the

[66]Malus 1811c, 1811d. For a thorough analysis of these memoirs, cf. Buchwald 1989, pp. 54–64. Malus here introduced the concept of "depolarization" of polarized light reflected by a metal.

[67]Brewster 1818 (his law). Cf. Chen 2000, pp. 6–7.

[68]Arago 1817 [1811], pp. 5 (Young), 13–14 (mirror), 16–17 (transmission), 31 (Young). Arago was also puzzled by his observation of two systems of complementary rings when observing through a crystal the light reflected by the mirror–lens system at incidences more oblique than the polarizing incidence (for glass).

crystal around the axis of vision, but remained complementary. Arago soon found out that the phenomenon required that the light incident on the plate should be polarized (as the light from a clear sky partially is). He also understood that the mica of the plate had to be anisotropic, since a rotation of the plate in its own plane changed the intensity of the colors of the two images (there were four orientations for which the two images were white). Using light reflected at polarizing incidence as a source, he studied how the phenomenon varied with the inclination, thickness, and material of the plate. On a fairly thick (6 mm) plate of quartz cut perpendicularly to the optical axis, he again observed two complementary colors, but with no variation of their intensities during a rotation of the plate in its own plane. In both cases he regarded the plate as a "depolarizing" device, since it turned polarized light into light that could never be extinguished by a polarizing device. This light also differed from white light, since it lost its whiteness when seen through a polarizing device. Arago concluded:[69]

> It the properties of [polarized] rays depend, as has been assumed, of the particular disposition of the axes of the molecules with which they are formed, there will be, between a polarized ray … and the same ray after its emergence from a mica plate, the difference that in the first ray the axes of the molecules of diverse colors are parallel, whereas in the second ray there are molecules of diverse colors whose axes have diverse directions.

In his subsequent researches, Arago multiplied qualitative experiments on colors and polarization, partly intending to disprove Newton's theory of the colors of bodies. He also tried to relate the new phenomenon to double refraction, by assimilating a double-refracting crystal with a pile of depolarizing plates. He did not attempt to quantitatively determine the effects of thickness and orientation, be it by theory or by experiment. Laplace's protégé Jean-Baptiste Biot did just this, with a success that won him worldwide fame—and Arago's lasting rancor.[70]

With Laplacian meticulousness, Biot determined how Arago's phenomenon depended on the relevant angles and on the thickness of the plate. He first studied the case of the normal incidence of polarized rays on a gypsum lamina of given thickness, whose optical axis was in its plane (see Fig. 5.7). For the polarizer and analyzer he used polarizing reflection on glass plates, as Malus had done in his demonstration of Malus's law. Biot first made clear that for a given orientation of the polarizer and analyzer, a rotation of the lamina in its own plane altered only the intensity of the observed light, not its tint. From this he concluded that for a well-defined subset of simple colors (depending on the thickness of the plate) the incident polarized rays were unaffected by the lamina, while for the complementary subset the polarization of the rays was modified by the lamina. He first tried to determine the modification of the affected rays by Malus's law. Call i the angle that the axis of the plate makes with the original polarization. A fraction $\cos^2 i$ of the affected rays emerges with a polarization parallel to the axis, while the complementary fraction $\sin^2 i$ emerges with a polarization perpendicular to the axis. Biot then applied

[69]Arago 1812a, pp. 37–8 (mica and blue sky), 42 (citation), 45–7 (thickness), 47–8 (inclination), 49 (minimum thickness), 54–64 (quartz). Cf. Rosmorduc 1983; Buchwald 1989, chap. 3; Levitt 2009, pp. 35–8.

[70]Arago 1812b, 1812c, 1812d.

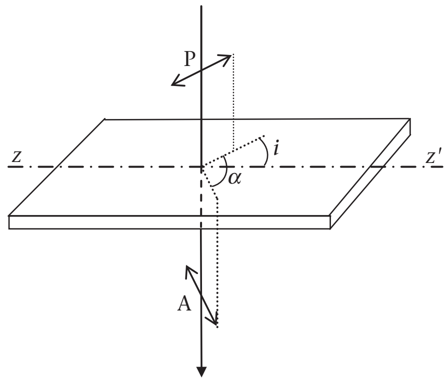


Fig. 5.7. Schematic view of Biot's chromatic polarization device. The vertical ray is polarized by the polarizer P, which makes the angle i with the axis zz' of the lamina. After crossing the lamina, it is observed through the analyzer A, which makes the angle α with the polarizer P.

Malus's law a second time, to the action of the analyzer on the unaffected (U) and affected (A) rays. Calling α the angle that the analyzer makes with the polarizer, this yields the intensity

$$I = U\cos^2\alpha + A\cos^2 i\cos^2(\alpha - i) + A\sin^2 i\sin^2(\alpha - i). \qquad (3)$$

This formula did not represent the observed variations of the outgoing light very well. By trial and error, Biot found that the simpler trigonometric formula

$$I = U\cos^2\alpha + A\cos^2(2i - \alpha) \qquad (4)$$

worked excellently.[71]

Biot then studied the effect of the thickness of the lamina and found that the affected rays were the same as the rays reflected by a thin (amorphous) plate of proportional thickness according to Newton. He also related the effect of the inclination of the crystal plate with the effect of the angle of incidence on Newton's rings. He thus believed he had reached "novel and intimate analogies between the yet unknown causes that produce the ordinary reflection of light and the causes which polarize light in crystallized bodies."[72]

In his first memoir, Biot refrained from speculating on the molecular process from which the empirical regularities might derive. A few months later, he believed he could safely identify this process. First, he interpreted the intensity formula (4) as the result of

[71]Biot 1812a, pp. 140–9. Cf. Frankel 1977; Buchwald 1989, chap. 4. In analogy with double refraction, Biot called the unaffected rays "ordinary" and the affected rays "extraordinary." In all rigor, I, U, and A should represent spectral densities.

[72]Biot 1812a, part 1 (thickness), part 2 (inclination), p. 136 (citation).

Malus's law applied to a mixture of two subsets of molecules. The molecules of the first subset, corresponding to the $\cos^2 \alpha$ factor, have the same transverse orientation as the molecules of the original polarized beam; the molecules of the second subset, corresponding to the $\cos^2(\alpha - 2i)$ factor, have the symmetrical orientation with respect to the axis of the lamina, at an angle $2i$ with respect to the original orientation. Comparing these two orientations with transmission and reflection in Newton's theory of fits, Biot had them recur periodically during the travel of the molecule across the lamina. In this picture, the axis of Biot's molecules of light oscillates around the optical axis of the lamina at a frequency depending on the simple color and proportional to the frequency of Newton's fits. Biot developed a similar picture in the case of quartz plates cut perpendicularly to the optical axis, simply replacing the oscillation of the light molecules with a continuous rotation of the molecules during their travel through the plate.[73]

Biot traced the oscillations of the molecules in a gypsum lamina to two forces acting between the axis of the molecules and the axes of the lamina in its plane. From the fact that the observed tint did not depend on a rotation of the lamina in its plane, he inferred that the frequency of the oscillations did not depend on their amplitude, which in turn implied their sinusoidal character. Yet Biot's intensity formula (4) and his use of Newton's construction of color as a function of thickness implied that after exiting the lamina the molecules could be found only in their original orientation or in the orientation symmetric with respect to the axis. Biot solved this contradiction by assuming an ad hoc transitory process that made the molecules reach one of these two orientations, with a probability depending on the phase of their oscillation just before exiting the lamina. One difficulty remained: as Biot carefully demonstrated, two successive laminas of thicknesses e and e' separated by an air gap behave like a single lamina of thickness $e + e'$ when their axes are parallel and like a single lamina of thickness $e - e'$ when their axes are perpendicular. This is only understandable if in the air gap the molecules retain the orientation that they had just before exiting the first lamina. Biot obscurely distinguished between the actual orientation in the air gap (which could only take two discrete values) and a tendency that was preserved in the gap.[74]

Although Biot's main findings are easily summarized in a few sentences, his attention to experimental details and his compulsion to explore every side of the subject brought him to fill some five hundred pages of the *Mémoires de l'Institut*. In contrast with this extreme prolixity, Young offered terse assessments of the new French optics. In 1810 he saluted Malus's discovery of polarization as "the most important that has been made in France, concerning the properties of light, at least since the time of Huygens." He deplored the lack of explanation in the system of undulations, but denied that the corpuscular theory could

[73]Biot 1812b, pp. 60–7 (oscillations), part 4 (quartz plate); Biot 1814a. In his treatise (Biot 1816, vol. 3, pp. 192, 195), Biot similarly interpreted Newton's fits as the consequence of a rotating polarity of the light molecules.

[74]Biot 1812b, pp. 75–88 (forces), 66–7 and 108 (transitory process), part 3 (successive laminas). In order to account for the effect of the inclination of the lamina (around an axis making a variable angle with the optical axis), Biot distinguished two forces acting on the molecular axes, one from the optical axis, the other from an axis perpendicular to the optical axis.

properly account for the role of polarization in reflection and refraction. He then decided to suspend his decision between the two systems:[75]

> We confess that we are compelled to remain for the present undecided, and we can only look forwards for further information to the discoveries which may result from future experiments.

Biot's memoirs on chromatic polarization brought some of the desired information. In 1814, Young interpreted this "intricate and laborious investigation" in his own way:

> We have no doubt that the surprise of these gentlemen [Biot and Brewster] will be as great as our own satisfaction in finding that they are perfectly reduceable, like all other cases of *recurrent* colours, to the general laws of the interference of light … and that all their apparent intricacies and capricious variations are only the necessary consequences of the simplest application of these laws.

Young went on to explain that the colors of crystalline laminas corresponded to the interference between light propagated at the velocity of the ordinary rays and light propagated at the velocity of extraordinary rays in the lamina. He had nothing to say on the role of polarization in this phenomenon. The following year he admitted to Brewster his growing perplexity about polarization:[76]

> With respect to my own fundamental hypotheses respecting the nature of light, I become less and less fond of dwelling on them, as I learn more and more facts like those Mr Malus has discovered: because, although they may not be incompatible with these facts, they certainly give us no assistance in explaining them. But this does not extend to my laws of interference, as explanatory of the phenomena of periodic colours.

Biot did not worry as much about interference as Young worried about polarization. In a contemporary letter to Brewster, he mentioned that the *Institut* had failed to get a copy of Young's *Lectures* and that "he did not know [Young's theory] at all." As Brewster had cited Young's authority against one of Biot's colored-rings observations, Biot replied:

> I very much esteem the merit and talent of this distinguished *savant*, but you will permit me to tell you that his witness has no more weight here than the authority of Aristotle against the observations of Galileo on gravity. Mr Young can do nothing against facts such as those I just mentioned.

In his treatise of 1816, Biot presented refraction and polarization as unshakable evidence for the emissionist view of light:[77]

[75]Young 1810, pp. 247–54.

[76]Young 1814, p. 269; Young to Brewster, 13 September 1815, *YMW* 1, pp. 360–4, on p. 361.

[77]Biot to Brewster, 20 February 1816, Bibliothèque de l'Institut de France, Paris, cote 4895 #70, quoted in Frankel 1976, p. 155; Biot 1816, vol. 3, p. 149.

> All these phenomena seem today to place the system of emanation beyond doubt, if we can call a system something that is so natural a consequence of the facts and serves to reproduce them so exactly and so easily.

5.3 Fresnel's optics

Early reveries

In May 1814, Augustin Fresnel wrote to his brother Léonor:

> I read in the *Moniteur*, a few months ago, that Biot had read at the *Institut* a very interesting memoir on the *polarization of light.* No matter how much I rack my brain, I have no idea what that means.

Fresnel was then an engineer of bridges and roads, with a strong interest in chemistry and a superficial knowledge of physics. During his studies at the Ecole Polytechnique, he had been exposed to the doctrine of the caloric and to analogies between heat and light as different states of this imponderable fluid. Whereas Malus had not ventured very far from this doctrine, the young Fresnel dared to reject the substantiality of heat. He believed, for instance, that the combustion of carbon could not possibly imply the emission of caloric since the global volume of matter increased during this reaction. Heat and light had to be vibrations of the caloric, not the caloric itself. Against the corpuscular theory of light, Fresnel argued that it would imply diverse velocities for the corpuscles emitted by the sun and other luminaries, and therefore a never observed reddening of the first rays emitted after an eclipse of the sun. He worried for a while that this theory was the only one that explained the aberration of fixed stars, but soon convinced himself (as Euler had done much earlier) that the vibration theory yielded the same result.[78]

Fresnel's wrote down his "rêveries" in a memoir on which he sought Ampère's opinion. After seeing this piece, Arago met Fresnel and directed him to Young's *Lectures* (which include most of his optical papers in an appendix). Fresnel's subsequent letter to Arago gives us a vague idea of the extent to which he penetrated Young's optics:

> As for Young's book, about which you had told me so much, I was very eager to read it. Since I do not know English, however, I could only understand it with the help of my brother, and after leaving him the book was again unintelligible to me.

Fresnel decided to study the formation of shadows, as he believed that this phenomenon best decided between Newton's system and the system of vibrations. He did so at his mother's house in Mathieu (near Caen), as a consequence of his involvement in the opposition to Napoleon's return from Elba. His ingenuity and the skills of a local locksmith compensated for the lack of proper equipment.[79]

[78]A. Fresnel to L. Fresnel, 15 May 1814, *FO* 2, p. 819; 5 July 1814, *FO* 2, pp. 820–4; 6 July 1814, *FO* 2, pp. 824–6. Fresnel ([1815a], p. 10) later argued that the molecules of light could not travel freely through the atmosphere as they would interact with the analogous molecules of the included caloric. Cf. Verdet, *FO* 1, pp. xxvii–xxix; Silliman 1975; Buchwald 1989, pp. 110–18.

[79]A. Fresnel to L. Fresnel, 3 November 1814, *FO* 2, pp. 829–39 (*rêveries*); Fresnel to Arago, 23 September 1815, *FO* 1, p. 7; Fresnel [1815a], p. 10. Cf. Verdet, *FO* 1, pp. xxx–xxxi.

Diffraction and interference

In the fall of 1815, Fresnel focused sunlight through a drop of honey acting as a highly convex lens and placed an iron wire or blade at some distance from the focus. He observed the resulting shadows and fringes on a white sheet of cardboard, or, more accurately, through a magnifying glass and a primitive double-thread micrometer at its focus (thus reaching a precision of 1/40 mm). He did this first for the external fringes, which are the easiest to observe. Then he turned his attention to the internal fringes. He realized, as Young had earlier done, that these fringes disappeared whenever the light was blocked on one side of the wire. He concluded that "the concourse of the rays from both sides [of the wire] was necessary to the production of theses fringes." He went on:

> [The fringes] cannot result from the mere mixing of these rays since each side of the wire separately throws a continuous light in the shadow. It is therefore the meeting, the crossing of these rays which produces these fringes. This consequence, which only is a sort of translation of the phenomenon, is quite opposed to Newton's hypothesis and confirms the theory of vibrations. One easily conceives that the vibrations of two rays that cross each other under a very small angle can contradict each other when the nodes [*noeuds*] of the first correspond to the antinodes [*ventres*] of the other.

Despite the confusing terminology, Fresnel clearly meant that two rays could interfere in Young's sense. Whether or not his perusal of Young's *Lectures* helped him draw this conclusion is anyone's guess. It may be noted, however, that former Newtonian observers of the internal fringes, including du Tour in 1760 and Jordan in 1799, did not judge them incompatible with the Newtonian system. Moreover, wave interference was almost completely unknown for any kind of wave before Young made it an essential part of optics and acoustics.[80]

Fresnel constructed the internal fringes by superposing circular waves emitted by the extremities of the wire or blade, and the internal fringes by superposing the waves emitted from the luminous point source with circular waves emitted by one extremity (see Fig. 5.8). In the ray language he usually favored, he superposed an inflected ray with another inflected ray or with a direct ray (see Fig. 5.9). Allowing a 180° phase shift for the reflected rays, he obtained a good match with the observed fringes. He emphasized the hyperbolic shape of the external fringes as the strongest proof of the falseness of Newton's theory. The resulting memoir, which he sent to the Academy on 15 October 1815, contained this result together with a few other objections to Newton's system and with a new derivation of the laws of reflection and refraction. This derivation is important as a first attempt to combine Huygens's principle with the principle of interference. If two parallel rays were reflected by

[80]Fresnel [1815a], p. 17. Fresnel later recognized Young's priority, e.g., in a letter to Léonor Fresnel (19 July 1816, *FO* 2, pp. 835–6): "If a revolution occurs in optics, as I hope, the self-esteem of the learned will be interested in giving me a more important share than I deserve because once they admit their passed error, they will probably say, as an excuse ... that nothing had earlier demonstrated the falseness of Newton's system. Yet, Doctor Young has proven long ago the influence that luminous rays exert on each other."

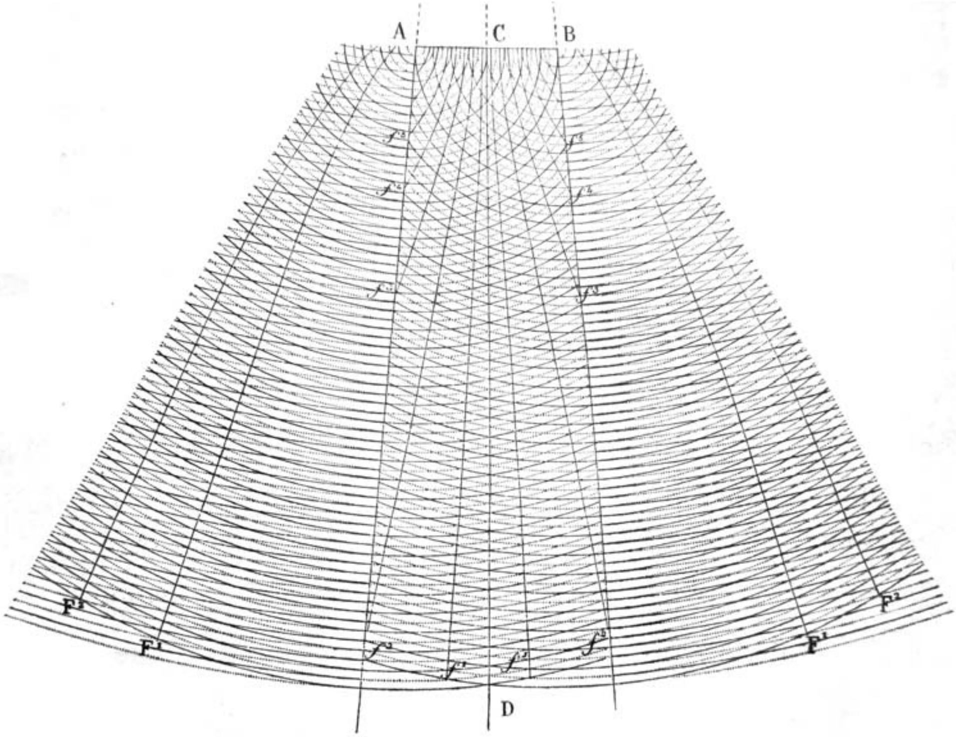


Fig. 5.8. Fresnel's construction of diffraction fringes behind a blade. The light from a distant point O (not represented, at the intersection of the dotted lines) is diffracted by the blade AB. The external fringes F^1 and F^2 pass through the consecutive intersections of two series of circular (traces of) waves emanating from O and from the edge A or B. The internal fringes f^1, f^2, f^3, f^4, f^5 pass through the consecutive intersections of waves emanating from the edges A and B. From Fresnel [1815a], p. 23.

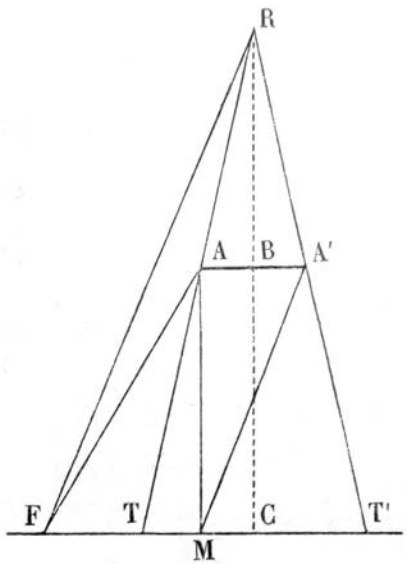


Fig. 5.9. Fresnel's ray diagram for diffraction by a blade (AA′). Interference between the direct ray RF and the reflected ray AF leads to the external fringes. Interference between the inflected rays AM and A′M leads to the internal fringes. From Fresnel 1819a, p. 270.

a plane surface at an angle different from the angle of incidence, Fresnel reasoned, the vibrations along the reflected rays would no longer have the same phase in a common perpendicular plane. In fact, for every reflected ray there would be a neighboring reflected ray in which the vibration has the opposite sign. Consequently, the total vibration would vanish. Fresnel offered a similar reasoning for refraction. A month later, he sent to the Academy a supplement including the phenomena of gratings and Newton's rings.[81]

Arago responded enthusiastically to Fresnel's memoir, although he knew that most of the results could be found in Young's earlier memoirs. He urged Fresnel to strengthen the determination of the hyperbolic shape of diffraction fringes, which he believed to be a new and devastating objection to Newton's system.[82] In early 1816, Fresnel did so in Paris with Arago's help and with a better micrometer. In this process, the two men discovered that the internal fringes were shifted when the rays on one side of the diffracting blade were made to pass through a glass lamina. Fresnel traced this observation to the phase shift caused by the slower velocity of light in glass. Arago used it to explain the colors in the scintillation of stars as the result of interference between light rays that had traveled through parts of the atmosphere subjected to different density fluctuations. In his praiseful report on Fresnel's memoir for the Academy, he tactfully avoided Laplacian discord by separating the experimental results from their hypothetical interpretation. An improved version appeared in March 1816 in the *Annales de chimie et de physique*. In the same month Arago reported Fresnel's mirror experiment, in which the rays reflected by two mirrors interfere (see Fig. 5.10). Fresnel regarded this result as the most effective proof of interference yet given, since it did not involve inflection by the edges of a screen.[83]

The prize-winning memoir on diffraction

As Fresnel acknowledged in his first letter to Young, there was little in his memoir on diffraction that Young had not anticipated. Fresnel's superior technique of measurement soon bore more original fruits. By the summer of 1816, attention to the precise interval of the internal fringes near the limit of the shadow and observations of diffraction by a slit brought him to assume that the interfering rays originated from points situated at some distance from the edges of the screen. He explained this feature through the Huygenian notion that the light from an interrupted wave front should be regarded as the resultant of vibrations originating from the various points of this front. In his reasoning based on Fig. 5.11, the portions AC, CC′, C′C″... of the interrupted wave front emit waves that reach the observation point F with alternating phases. Owing to this alternation, the contributions of CC′, C′C″... nearly cancel each other. There remains the contribution of AC, which Fresnel approximated by the contribution of the "efficacious ray" BF that is a

[81]Fresnel [1815a], [1815b]. The reasoning is rough, for it does not explain why the vibrations along neighboring rays should be superposed. Its connection with Huygens's secondary waves is more explicit in the published memoir: Fresnel 1816a, p. 118. Cf. Buchwald 1983; 1989, pp. 118–35.

[82]As Young later told Arago (letter of 12 January 1817, *FO* 2, pp. 742–4), Newton was quite aware of the curved propagation of external fringes. See above, chapter 3, pp. 101–2.

[83]Arago to Fresnel, 8 November 1815, *FO* 1, pp. 38–9; Arago 1816a (glass lamina), [1816b] (report), 1816c (scintillations); 1816d (mirror); Fresnel 1816a; Fresnel [1816b], pp. 150–5 (mirror).

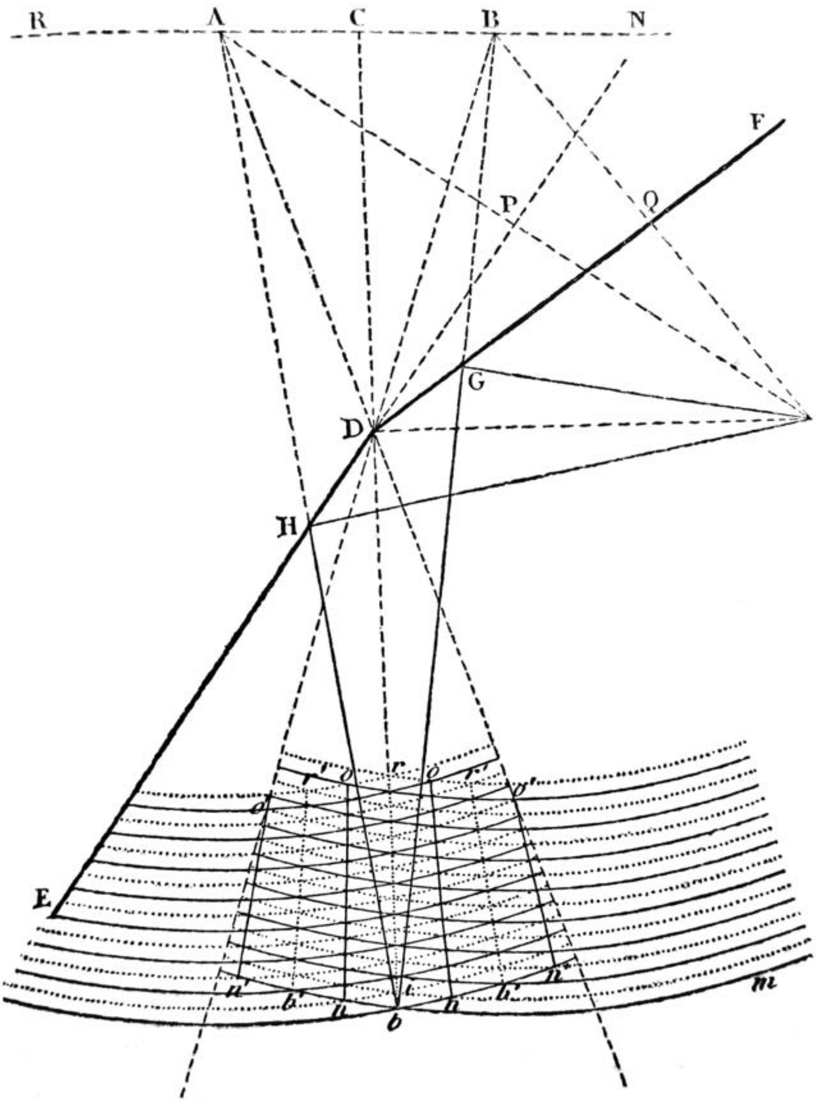


Fig. 5.10. Fresnel's mirror experiment. The circular waves centered on the virtual images A and B of the source S through the mirrors DE and DF interfere to form the bright fringes *on, o′n′* and the dark fringes *rb, r′b′*. From Fresnel 1822e, p. 55.

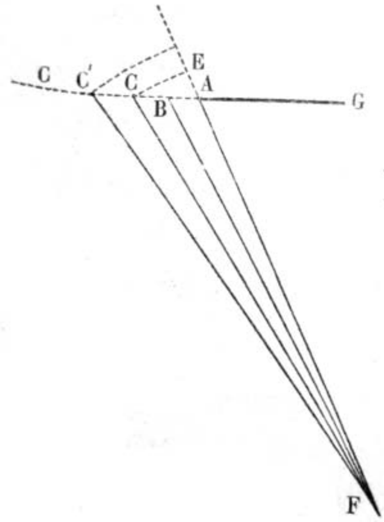


Fig. 5.11. Fresnel's method of zones. The screen AG intercepts light from above. The portions AC, CC′, C′C″… of the interrupted wave front have alternating phases. From Fresnel [1816], p. 161.

quarter-wavelength longer than the ray AF. This is the first intimation of what is now called the method of zones.[84]

This procedure only worked for the internal fringes. In the case of the external fringes, the superposition of the direct ray with the efficacious ray led to wrong results. For more than a year, Fresnel was busy working on another challenging topic, chromatic polarization. When he returned to diffraction in the winter of 1817, he suspected that the diffracted light should generally be computed by superposing wavelets emanating from the various points of the interrupted wave front. He initially doubted he could manage the resulting calculations. On 17 March 1818 the Academy opened a prize competition for a study of diffraction. The following month Fresnel mailed to the Academy a *pli cacheté* containing his completed theory.[85]

Fresnel considered the interrupted wave front AF of Fig. 5.12, and assumed that each of its elements mm', $m'm''$…, nn', $n'n''$… was the origin of a secondary vibration whose intensity varied little in directions close to the normal of the element. According to Fresnel's version of Huygens's principle, the actual vibration at the point of observation P is the superposition of these secondary vibrations. Fresnel next noticed that whenever the line mP significantly departed from the normal to the element mm', the neighboring elements interfered destructively. Consequently, the sum of the secondary vibrations only

[84]Fresnel to Young, 24 May 1816, *FO* 2, pp. 737–40; Fresnel [1816b]. Cf. Buchwald 1989, pp. 139–54.

[85]Fresnel [1818d].

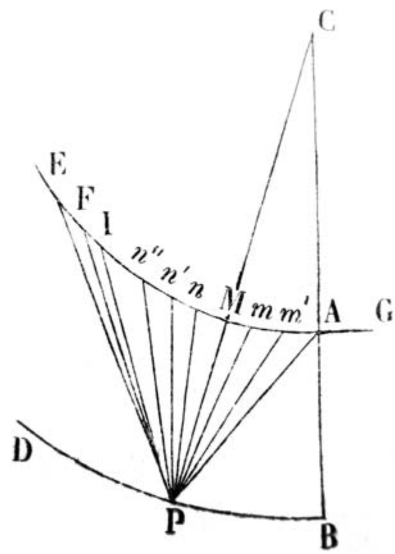


Fig. 5.12. Fresnel's diagram for the calculation of diffracted light. The wave front EA from the point source C is interrupted by the screen AG. The element *mm'*, ... *nn', n'n''* ... contribute to the total vibration at P with different phases. From Fresnel [1818d], p. 174.

involves elements close to the point M for which PM is perpendicular to the wave front. The intensity of these vibrations at point P is approximately constant. Their phase varies as the distance *m*P, which is very nearly proportional to the square of the distance *m*M = z. Lastly, Fresnel assumed that the vibrations were sinusoidal, as he had earlier done in his work on chromatic polarization. The resulting oscillation at P has the form $\int \mathrm{d}z \cos(\omega t + \alpha z^2)$. Its amplitude is proportional to $\sqrt{I_c^2 + I_s^2}$, with[86]

$$I_c = \int \mathrm{d}z \cos \alpha z^2, \qquad I_s = \int \mathrm{d}z \sin \alpha z^2. \tag{5}$$

These integrals, now called "the Fresnel integrals", are to be taken over the whole extent of the interrupted wave front. Fresnel evaluated them by approximation based on integration by parts, or, more intuitively, by a refinement of his earlier method of zones. He treated the main cases of interest in which the diffracting screen is a half-plane (see Fig. 5.13), a strip, a slit, and a double-slit, and he showed that the results agreed excellently with his measurements of diffraction fringes. In particular, he showed that in the case of a screen much larger than the wavelength, the intensity of diffracted light decreased very rapidly within the geometric shadow (see Fig. 5.13). This result amounts to

[86]Fresnel [1818d], pp. 174–7; Fresnel 1819a, pp. 293–7, 313–16. Ibid. on pp. 297–8, Fresnel provided the three-dimensional generalization of this reasoning. Cf. Buchwald 1989, chap. 6; Grattan-guinness 1990, vol. 2, pp. 855–70.

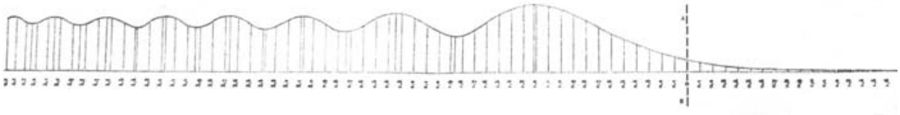


Fig. 5.13. Fresnel's intensity curve for the light behind a semi-infinite screen (in a plane parallel to the screen). The intensity oscillates outside the shadow of the screen, with a period decreasing with the distance from the geometrical shadow (marked by the vertical dotted line). It decreases rapidly within the shadow From *FO* 1, p. 383.

a proof of the rectilinear propagation of light in the wave theory, indeed the first proof that a modern physicist would still accept.[87]

With his impressively thorough and accurate considerations, Fresnel easily won the Academy prize. One of the commissaries, the very Laplacian Siméon Denis Poisson, noted that Fresnel's theory led to a simple analytical formula for diffraction by a disk or a circular hole when the source and the point of observation were on the axis of symmetry. The reason is that in this case the element of integration is the surface element $2\pi z\mathrm{d}z$ (z being reckoned from the axis), whose product by cos αz^2 or sin αz^2 is easily integrable. In the disk case, the intensity turns out to be the same as if there were no screen at all. With a disk of 2 mm diameter, Arago confirmed this strange prediction of Fresnel's theory. This episode and other proofs of the predictive power of Fresnel's theory did not suffice to convert Laplace and his disciples to the wave theory of light. As the wording of Arago's prize report suggested, it was still possible to regard the Huygens–Fresnel principle and the interference principle as rules for combining rays, independently of the deeper nature of light.[88]

There were still reasonable arguments against the wave theory. Young's and Fresnel's derivations of the laws of reflection and refraction, and Fresnel's derivation of the rectilinear propagation of light were only "*aperçus*" and not "geometrical demonstrations," as Laplace told Young in a letter of 1817. Fresnel's version of Huygens's principle lacked a solid foundation on wave dynamics, despite the retrospective soundness of Fresnel's intuitions. The situation slightly improved in 1823 when Poisson, through frightfully complicated mathematics, derived the laws of refraction from the wave equations in the two media with proper boundary conditions at the interface. As we will now see, this did not prevent Poisson from attacking Fresnel's application of Huygens's principle.[89]

Poisson agreed with Fresnel that as a consequence of the principle of superposition the motion of the elastic fluid at a given instant could be regarded as the sum of the motions obtained by superposing the motions that would result from initial conditions for which the fluid has its actual displacement and velocity within a given element of volume and is at rest everywhere else. Poisson judged, however, that each partial motion should proceed in the direction of the velocity of the element and in the opposite direction, whereas Fresnel

[87]Fresnel 1819a, pp. 299–312 (zones), 317–23 (numerical analysis), 340 (shadows).

[88]Arago 1819, p. 236; Fresnel 1819a, pp. 368–69.

[89]Laplace to Young, 6 October 1817, *YMW* 1, pp. 374–5; Poisson 1823a, 1831.

assumed a forward motion with a significant angular spread. In his reply, Fresnel referred to his earlier remark that the effects of the compression and the velocity of a given element on a neighboring element canceled each other when the latter element was in the rearward direction. He also defended the angular spread of the perturbation, both through the empirical fact of diffraction and through an intuitive dynamical argument. Poisson remained unconvinced. Many years elapsed before a more rigorous justification of Fresnel's method became available.[90]

Polarization

Another challenge for the wave theory was the integration of polarization phenomena. In the summer of 1816 Fresnel and Arago focused on this problem. By analogy with the mirror-interference experiment, Fresnel had tried to make the two beams from an Iceland spar interfere. He failed, although he carefully compensated for the large phase difference between the ordinary and extraordinary rays. Suspecting that the perpendicular polarizations of the two rays forbade their interference, he multiplied experiments in which two such rays were superposed. Arago imagined the most decisive one, in which the two slits in a double-slit device were covered with polarizers made of a pile of inclined glass lamellas. The interference pattern gradually disappeared and reappeared when one of the polarizers was rotated around the ray. At that time, Fresnel and his friend Ampère suspected that this phenomenon might have to do with the transverse character of at least part of the vibrations. As Fresnel did not know how to develop this assumption, he let it rest for a few years.[91]

Fresnel next applied his discovery of the polarization-dependence of interference to the theory of chromatic polarization. Like Young, he believed chromatic polarization to result from interference between the ordinary and extraordinary rays issuing from the crystal lamina. Whereas Young had ignored the role of polarization, Fresnel explained the whiteness of the two images seen without an analyzer as a consequence of the non-interference between two perpendicularly polarized rays. The analyzer permitted the interference by bringing part of the ordinary and extraordinary lights to a common polarization. The initial polarizer was necessary because the interference patterns caused by the various polarizations included in natural light would cancel out. There is only one difficulty with this theory: the color of the image is conserved when the analyzer is rotated by 90°, since this operation simply permutes the angles that the analyzer makes with the polarization planes of the ordinary and extraordinary rays. In order to retrieve the observed complementarity of the colors in these two situations, Fresnel assumed an additional phase shift of 180° in one of them. He judged that a deeper understanding of polarization would be necessary to solve this anomaly.[92]

[90]Poisson to Fresnel, 6 March 1823, *FO* 2, pp. 186–9; Poisson 1823b; Fresnel 1819a, p. 296n (earlier remark); 1823c. Cf. Buchwald 1989, pp. 188–98. As Kirchhoff and Poincaré later proved (see below chap. 7, p. 280), the absence of rearward radiation is only true for a surface portion much larger than the wavelength.

[91]Fresnel [1816c], pp. 386–9 (experiments), 394n (transverse vibrations); Arago and Fresnel 1819.

[92]Fresnel [1816c], pp. 394–403. Cf. Frankel 1976, pp. 163–5; Buchwald 1989, chap. 7.

Although this theory reproduced Biot's main observations, it flatly contradicted his theory of mobile polarization. Indeed Biot assumed the emerging light to be polarized either in the direction of the original polarization, or in the direction symmetrical with respect to the axis of the lamina. Fresnel noticed that this assumption contradicted the polarizing properties of doubly-refracting crystals. Biot had himself shown that the same crystalline material could be used to produce both chromatic polarization and double refraction (originally, mica, gypsum, and quartz were used for chromatic polarization only, and Iceland spar for double refraction only). In order to explain the transition between oscillating polarization and static polarization, he assumed the damping of the oscillations for large thicknesses and some mysteriously growing disposition to static polarization. Fresnel criticized the ad hoc character of these assumptions. As we saw, Biot's theory needed other ad hoc assumptions in order to reconcile the behavior of a succession of laminas with his assessment of the polarization of the emerging light. Fresnel did not fail to note this other weakness of Biot's theory.[93]

Most strikingly, Fresnel offered an experimental proof that the light from a crystal lamina was a superposition of ordinary light polarized in the direction of its axis and extraordinary light polarized in the perpendicular direction. For this purpose, he placed two laminas in front of the slits of a double-slit interference device, with the axis of one lamina parallel to the slit and the other perpendicular to the slit. He observed two systems of interference fringes: one shifted by the amount corresponding to the phase difference between extraordinary and ordinary propagation in the laminas, the other shifted by the opposite amount. When seen through a polarizer oriented in the direction of the slits, one system of fringes disappeared; when seen through a polarizer in the perpendicular direction, the other system disappeared. Fresnel regarded this result as a proof that the first system resulted from interference between the ordinary light from the first lamina with the extraordinary light from the second lamina, and the second system from interference between the extraordinary light from the first lamina with the ordinary light from the second lamina.[94]

About a year later, Fresnel investigated the depolarization of light by complete reflection on a transparent medium. By making the depolarized light pass through a chromatic polarization device, he established that this light was actually a mixture of lights polarized in two perpendicular directions, with a well-defined phase difference depending on the number of depolarizing reflections. In early 1817, these considerations prompted him to investigate more closely the rules of superposition of two polarized beams that have a well-defined phase difference. He assumed, as Young had earlier done, that the waves were sinusoidal, and that their phase depended on the velocity of light in the medium they had traversed. In addition, he assumed that a doubly-refracting device whose principal plane made the angle θ with the original polarization yielded an ordinary vibration of amplitude $a\cos\theta$ and an extraordinary vibration of amplitude $a\sin\theta$, in conformity with Malus's law for the intensities. Fresnel first justified this rule by analogy with vector composition, although he soon preferred to evoke the conservation of live forces. Combining this rule with the phase shifts in sinusoidal amplitudes, he determined the final

[93]Fresnel [1816c], pp. 403n (superposed laminas), 407–8 (mobile polarization); Biot 1813, 1814b.

[94]Fresnel [1816c], pp. 420–1.

intensity in particular cases involving initially polarized or depolarized light, crystal laminas, and an analyzer. In particular he showed that in the case of two successive laminas whose axes made an angle of 45°, the predictions of his theory departed from Biot's and agreed with experiment.[95]

In the case of chromatic polarization (see Fig. 5.7), Fresnel later gave the general intensity formula

$$I = \cos^2 i \cos^2(i - \alpha) + \sin^2 i \sin^2(i - \alpha) + 2\cos\phi \sin i \cos i \sin(i - \alpha)\cos(i - \alpha), \qquad (6)$$

where i is the angle between the original polarization and the axis of the lamina, α the angle between the polarizer and the analyzer, and ϕ the phase difference between the ordinary and extraordinary rays at a given wavelength (taking the original intensity as the unit of intensity). The derivation begins by noting that the lamina decomposes the original vibration into the ordinary and extraordinary amplitudes $\cos i \cos \omega t$ and $\sin i \cos(\omega t + \phi)$. The analyzer then generates from these the combined vibration $\cos i \cos(i-\alpha)\cos \omega t + \sin i \sin(i-\alpha)\cos(\omega t + \phi)$ whose intensity is given by Fresnel's intensity formula (6).[96]

On 4 June 1821 Arago reported to the Academy on Fresnel's polarization memoir and recent additions, with revengeful emphasis on the refutation of Biot's earlier results and theory. He repeated Fresnel's arguments leading to the conclusion that the light from the lamina was a superposition of vibrations polarized along and across the axis of the lamina. These two vibrations having the forms $\cos i \cos \omega t$ and $\sin i \cos(\omega t + \phi)$, their resultant is a rectilinear vibrations only if the phase difference ϕ is a multiple of π; it makes the angle i with the axis whenever ϕ is an even multiple of π, and the angle $-i$ whenever ϕ is an odd multiple of π. In intermediate cases, there is no definite polarization (in Fresnel's later terminology, the polarization is elliptical). For $\phi = \pi / 2 + 2n\pi$ (n = integer), the intensity of the light from the lamina does not at all depend on the orientation of the analyzer, as Fresnel verified through an experiment with monochromatic light. Fresnel and Arago considered this case as a flat contradiction of Biot's theory, which they took to imply complete polarization in this case. Arago also mentioned Fresnel's refutation of Biot's predictions and measurement in the case of two successive plates whose axes made an angle of 45°.[97]

Biot attempted to block Arago's report for procedural and for scientific reasons. On the scientific side, he first established the identity of his intensity formula (3) with Fresnel's intensity formula. The latter may indeed be rewritten as

$$I = \cos^2(\phi/2)\cos^2 \alpha + \sin^2(\phi/2)\cos^2(2i - \alpha), \qquad (7)$$

which agrees with Biot's formula if the spectral densities of the unaffected and affected rays are chosen to be

[95]Fresnel [1817], pp. 488–95 (interference of sine waves), 495–6 (vector composition), 498–49 (two laminas); [1818b], [1819b]; Biot 1816, vol. 4, p. 407.

[96]Fresnel 1821c, pp. 613–15.

[97]Arago 1821a, pp. 557 (polarization), 567 (two laminas). Fresnel 1821b (polarization). Cf. Frankel 1976, pp. 165–8; Buchwald 1989, chap. 9.

$$U = \cos^2(\phi/2) \text{ and } A = \sin^2(\phi/2). \tag{8}$$

With respect to the colors and intensities of chromatic polarization, the two theories differ only in the way the coefficients U and A are determined. For Biot, they should be taken from Newton's theory of thin plates. For Fresnel, they can be very simply computed from the velocities of ordinary and extraordinary rays in the crystal of the lamina.[98]

To the contention that Fresnel's and Biot's theories disagreed for the thicknesses corresponding to $\phi = \pi / 2 + 2n\pi$, Biot replied that it only applied to the picture given in 1816 in his *Traité* (and cited by Arago). It did not apply to a natural refinement of his theory in which the transition from complete polarization at the azimuth i to complete polarization at the azimuth $-i$ was smooth. Indeed, in his memoir of 1812, complete polarization only occurred for all thicknesses that were a multiple of a certain minimal thickness (also for thicknesses such that $\phi < \pi$); for intermediate thicknesses the light was made of a mixture of rays polarized at the angles i and $-i$, in a proportion depending on the phase of the oscillation of the axis of the light molecule near the exit of the lamina. Since Biot there assumed the oscillation to be sinusoidal, his prediction implicitly agreed with the expressions of U and A from Fresnel's theory. In 1821, he believed the truth to be intermediate between this prediction and the abrupt alternation of polarizations he had assumed in 1816. As for the case of successive laminas with axes making 45º, Biot had already conceded the truth of Fresnel's prediction but he now regarded it as a consequence of his theory if properly applied. Having thus refuted a good part of Arago's criticism, Biot reasserted his conviction that the *individual* rays from the lamina were polarized either at the angle i or at the angle $-i$. He regarded this feature and his intensity formula (3) as a necessary consequence of his observations, irrespective of any assertion about the deeper nature of light.[99]

Arago managed to get his report through, with slight alterations. In his reply to Biot's defense he blamed his adversary for having opportunistically altered his view to accommodate Fresnel's results and for having ignored Fresnel's double-slit experiment with laminas. Arago and Fresnel emerged as the clear victors of this polemic, although they failed to convert Biot and other Laplacian physicists to their views.[100]

Transverse waves

Arago did not mention that a few months earlier Fresnel had come to believe in the strictly transverse character of the luminous vibrations. The reasons for this theoretical move were multiple. If we believe Fresnel's account, he and Ampère had long known (since October

[98]Biot 1821, pp. 574–7. Cf. Levitt 2009, pp. 54–7.

[99]Biot 1816, vol. 4, p. 389; 1821, pp. 583–6 (alternating polarization), 586 (two laminas). As Buchwald remarks, this conviction depended on an unconscious residue of the molecular concept of light: the idea that it can be regarded as a mixture of individual rays with well-defined color and polarization. Biot (1821, pp. 577n–579n) contested that Fresnel's theory accurately represented the colors of chromatic polarization, to which Fresnel (1821d) replied that Biot overestimated the accuracy of Newton's observations of the colors of Newton's rings, on which Biot based his judgment.

[100]Arago 1821b.

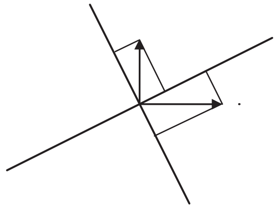


Fig. 5.14. Interference in a chromatic polarization device for two orthogonal orientations of the analyzer in a chromatic polarization device. The two arrows represent the vibrations of the ordinary and extraordinary rays at the exit of the lamina. If their projection on a line (representing the trace of the direction of the orientation of the analyzer) have the same sign, they have opposite signs on the perpendicular line.

1816) that this hypothesis would explain the lack of interference between perpendicularly polarized rays, the vector decomposition of polarized light by a doubly-refracting crystal, and even the mysterious phase shift of 180º when the analyzer is rotated by 90º in a chromatic polarization device (see Fig. 5.14). Fresnel nevertheless discarded the hypothesis, because he did not see how transverse vibrations could fail to be contaminated by longitudinal vibrations. Sometime in the winter of 1820–21, he decided that purely transverse vibrations were after all possible and that light was made entirely of such vibrations.[101]

Fresnel then imagined a model of the ether that allowed transverse vibrations and forbade longitudinal vibrations. This model consisted of a regular lattice of molecules held in equilibrium by distance forces. The forces could be such that a small transverse shift of a plane layer of molecules implied a proportional restoring force, whereas shifts by a multiple of half the intermolecular distance led to another configuration of equilibrium. The medium would then be rigid with respect to tiny vibrations, and fluid with respect to larger displacements. In addition, Fresnel believed that the intermolecular forces could be such that a very large force would be needed to change the distance between two successive layers of molecules. This assumption excluded longitudinal vibrations.[102]

Fresnel published these mechanical considerations in July 1821. In the same year Claude Louis Navier published a molecular theory of elasticity that implicitly permitted transverse vibrations. Félix Savart had recently shown the existence of such vibrations in elastic solids. These findings did not prevent Poisson to assert, as late as 1823, that the laws of wave propagation in an isotropic elastic solid were the same as in an elastic fluid. The confusion profited Fresnel, who could freely conceive a molecular compromise between rigidity and fluidity without contradicting established knowledge. As Fresnel knew, Thomas Young had repeatedly considered the possibility of partially transverse vibrations in the ether although he did not know how to reconcile this assumption with the fluidity of

[101]Fresnel [1821a], p. 529 (180º shift); 1821c, pp. 611 (180º shift), 629–30 (history). Cf. Buchwald 1989, chap. 8.
[102]Fresnel 1821c, pp. 631–4.

the ether. In a comment of 1823 on Fresnel's hypothesis, Young still objected that it would imply a rigid ether through which no matter could move:[103]

> This hypothesis of Mr Fresnel is at least very ingenious, and may lead to some satisfactory computations: but it is attended by one circumstance which is perfectly *appalling* in its consequences … [Namely,] the luminiferous ether pervading all space, and penetrating almost all substances, is not only highly elastic, but absolutely solid!!!

Despite the fragility of its underlying model, the hypothesis of transverse waves bore many fruits. Fresnel used it to elucidate the new kinds of polarization that Biot, Brewster, and he himself had discovered in reflected light and in light transmitted through crystal laminas. Consider, for instance, light polarized rectilinearly and passed through a "quarter-wave lamina" (namely, a crystal lamina cut with the optical axis in its plane and yielding a $\pi / 2$ phase shift between the ordinary and extraordinary rays) whose axis makes an angle of 45° with the initial polarization. When seen through an analyzer this light behaves as if it were completely depolarized: the intensity of the image does not depend on the azimuth of the analyzer. Yet this light differs from natural light, since it can be turned into rectilinearly polarized light by making is pass through another quarter-wave lamina. In Fresnel's picture of transverse vibrations, the motion of a point of the medium has the components $(a/\sqrt{2})\cos\omega t$ and $(a/\sqrt{2})\cos\omega t$ along the two axes of the lamina in its plane. The phase shift in the lamina yields the components $(a/\sqrt{2})\cos\omega t$ and $-(a/\sqrt{2})\sin\omega t$, which describe a circular motion in transverse planes. For this reason, Fresnel called this sort of light "circularly polarized." For phase shifts caused by other laminas or by total reflection, he spoke of "elliptical polarization" since the transverse motion of a particle of the medium is then elliptical. For the kind of chromatic polarization obtained by Biot and Arago with quartz plates cut perpendicularly to their axis or certain liquids such as turpentine, Fresnel explained that the initial (rectilinearly) polarized light was decomposed into two circularly polarized components that traveled at different velocities through the active substance. The resulting vibration at the exit of the lamina was again rectilinearly polarized at an angle equal to half the phase shift of the two components.[104]

Fresnel thus reduced a large set of intriguing observations to simple rules for composing transverse oscillating vectors. This was not enough to convince even his most sympathetic interlocutors, Arago and Young, of the soundness of the concept of transverse ethereal vibrations. However, Fresnel soon brought this concept to bear on two other important questions: the intensity of partially reflected light, and the general laws of double refraction.

[103]Poisson 1823a, p. 195; Young 1823b, p. 415. On contemporary theories of elasticity, cf. Darrigol 2000, pp. 109–15, 119–31. On Young's anticipations of transverse waves, cf. Verdet 1866–1870, p. 634.

[104]Fresnel 1822f, pp. 724 (circular polarization), 725–6 (optical rotation); [1822g], p. 744 (elliptic polarization). Fresnel had earlier shown ([1817], p. 460n; [1818c]) that optical rotation (as we now call it) could be imitated by replacing the active substance with a crystal lamina cut parallel to its axis and placed between two doubly reflecting glass parallelepipeds (which, as Fresnel later explained, turned rectilinearly polarized into circularly polarized light and vice versa).

Intensities

Fresnel gave the rules of intensity for partially reflected light at the interface between two transparent media in the summer of 1821, in an appendix to his memoir on chromatic polarization. As the incident wave can always be regarded as a superposition of a vibration in the plane of incidence and a vibration perpendicular to this plane, Fresnel focused on these two cases. In the latter case, he imitated an earlier reasoning by Young based on energy and momentum conservation and thus obtained the expression

$$\frac{I'}{I_0} = \left(\frac{\tan i - \tan r}{\tan i + \tan r}\right)^2 \tag{9}$$

for the intensity ratio of the reflected and incident lights. In the case of a vibration included in the plane of incidence, he gave the formula

$$\frac{I'}{I_0} = \left(\frac{\sin 2i - \sin 2r}{\sin 2i + \sin 2r}\right)^2 \tag{10}$$

without proof. Assuming that the vibrations were perpendicular to the conventional plane of polarization, he found these formulas to agree with intensity measurements by Arago, with Brewster's law for the polarizing incidence, and with his own measurements for the rotation of the plane of polarization during reflection.[105]

Two years later Fresnel provided a fundamental derivation of these two formulas, as well as their extension to depolarization by total reflection. He thereby assumed equal elastic constants for the two media, and he used two boundary conditions: the equality of the energy of the incoming wave to the sum of the energies of the reflected and refracted waves, and the equality of the parallel components of the total vibration on both sides of the interface. For the vibrational amplitude w the mechanical energy flux is $V\rho w^2$, wherein the propagation velocity V is inversely proportional to the optical index n and the density ρ is proportional to the square of the index (since Fresnel assumes the same elastic constant in both media). Energy conservation therefore reduces to

$$n_1(w_1^2 - w_1'^2)\cos i = n_2 w_2^2 \cos r, \tag{11}$$

where the indices 1 and 2 refer to the two media and the accent to the reflected wave.

In the case of a vibration perpendicular to the plane of incidence (see Fig. 5.15, left), the continuity of the parallel component of the vibration $\mathbf{w}$ gives

$$w_1 + w_1' = w_2. \tag{12}$$

[105]Fresnel 1821c, pp. 641–9; Young 1817, pp. 336–7 (normal incidence only). As Fresnel mentioned, Poisson (1819a, p. 372) had managed the case of normal incidence for compression waves in an acoustic context: cf. Grattan-guinness 1990, vol. 2, pp. 880–4.

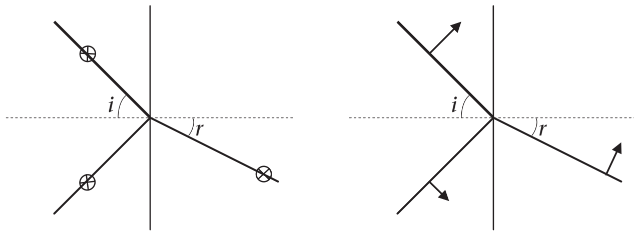


Fig. 5.15. Fresnel's two cases of partial reflection. The arrows indicate the direction of vibration.

In the case of a vibration in the plane of incidence (see Fig. 5.15, right), it gives

$$(w_1 - w_1{}')\cos i = w_2 \cos r. \tag{13}$$

Together with the law of refraction n_1 sin $i = n_2$ sin r and the expression (11) of energy conservation, these two conditions lead to "Fresnel's sine law"

$$\frac{w_1{}'}{w_1} = -\frac{\sin i - \sin r}{\sin i + \sin r} = -\frac{\sin(i - r)}{\sin(i + r)} \tag{14}$$

and to "Fresnel's tangent law"

$$\frac{w_1{}'}{w_1} = \frac{\tan(i - r)}{\tan(i + r)} \tag{15}$$

for the ratio of the amplitudes of reflected and incident waves in the two cases. These formulas agree with the intensity formulas (9) and (10).[106]

Fresnel extrapolated these laws to total reflection, in which case the angle r becomes imaginary and the complex amplitude ratios involve a phase shift. This phase shift is responsible for the elliptic polarization of the reflected light when the incident light is polarized at an azimuth intermediate between the two former cases. Fresnel verified the conformity of these predictions with his earlier observations on the properties of reflected light. The weak point in Fresnel's derivation is the discontinuity Fresnel tolerated in the normal component of the vibration. Intuitively, one would expect complete continuity of the total vibration in order to preserve the contiguity of the two media. Fresnel left the justification to a future study he never completed. For the moment, he simply suggested

[106]Fresnel 1823a, 1823b. Cf. Whittaker 1951, pp. 123–5; Buchwald 1989, appendix 17. Energetic considerations were growingly popular among French engineer–physicists. They were formulated in terms of "live force" and "work," the modern acceptance of the word "energy" being a much later innovation by William Thomson.

that in his model of the ether shearing motions and forces were the only ones involved in the propagation of light.[107]

The correlation between propagation and polarization

In the same *annus mirabilis*, 1821, Fresnel made his first foray into the theory of propagation in crystals. He knew that Wollaston and Malus had accurately confirmed Huygens's laws for Iceland spar and other crystals having axial symmetry ("uniaxal" crystals). He was also aware of Brewster's and Biot's discovery that for some crystals different laws applied. These crystals are called "biaxal" because there are two axes along which normal incidence does not yield double refraction.[108] Fresnel hoped to obtain a full theory of double refraction by focusing on the interplay between polarization and propagation.[109]

Whereas Huygens's analysis of the uniaxal case required two different media, an isotropic one for the ordinary ray and an anisotropic one for the extraordinary ray, Fresnel traced the distinction between these two rays to their different polarizations. For the ordinary ray, the plane of polarization is empirically known to be the plane containing the ray and the axis of the crystal; and the velocity of propagation should not depend on the inclination of this ray. Fresnel met this condition by assuming that the vibration was perpendicular to the plane of polarization and that the velocity of propagation depended only on the direction of the vibration (see Fig. 5.16). For the extraordinary ray, the same

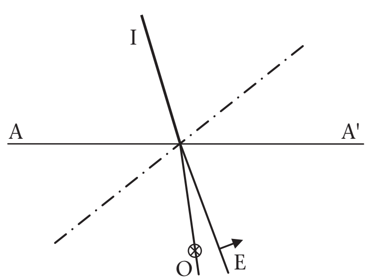


Fig. 5.16. Direction of the vibrations of the ordinary and extraordinary rays according to Fresnel. The incident ray I is refracted at the interface AA′ . The ordinary ray O, vibrates perpendicularly to the plane containing the axis and the ray, while the extraordinary ray vibrates in this plane (as indicated by the arrows).

[107]Fresnel 1823a, pp. 758–62; 1823b, 769–70 (boundary condition), 781–7 (total reflection). As Poincaré noted (1889, pp. 321–2), the discontinuity of the normal component is acceptable if the resistance of the media to plane compression vanishes (as it does in the later labile-ether theory: see pp. 235–6 below). Indeed the discontinuity can then be absorbed by a thin transition layer whose alternative compressions and dilations do not require work. Fresnel's formulas for the amplitude ratios happen to be exactly identical with those provided by the electromagnetic theory of light.

[108]As Hamilton later understood, these are cases of conical refraction.

[109]Fresnel [1821e]; Brewster 1818; Biot 1819, pp. 190–1. Brewster's discovery derived from his interpretation of anomalies in the chromatic polarization of inclined plates of gypsum or mica. On Fresnel's theory of double refraction, cf. Verdet 1866–1870, vol. 1, pp. 45–89; Whittaker 1951, pp. 117–23; Buchwald 1989, chap. 11.

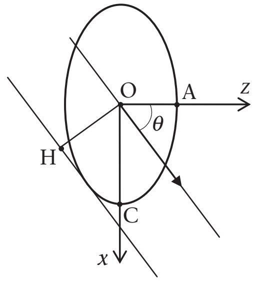


Fig. 5.17. *Diagram for deriving the velocity of plane waves through Huygens's construction.* The ellipse represents the trace at time $t = 1$ of the ellipsoidal wavelet emitted in O at $t = 0$. According to Huygens's construction, a wave plane passing through O at $t = 0$ must be tangent to this wavelet at time $t = 1$. The velocity V of this wave is given by the distance OH, which depends on the angle θ that the vibration (thick arrow) makes with the axis Oz and on the semi-axes OA = a and OC = c. The properties of the ellipse yield $OH^2 = OC^2 \cos^2\theta + OA^2 \sin^2\theta$.

assumptions imply that the vibration occurs in the plane containing the ray and the axis and that the velocity varies with the inclination of the wave planes with respect to the axis of the crystal.[110]

Fresnel used a surface to represent the way in which the velocity of plane waves depended on the orientation of the vibrations. His first choice for this surface, an ellipsoid, only worked for weakly anisotropic crystals, and did not agree with Huygens's construction of the extraordinary waves in the uniaxal case. In his final theory, Fresnel inferred the true surface from this construction. Call θ the angle that the direction of vibration makes with the axis. According to Fig. 5.17, the propagation velocity V is given by

$$V^2 = a^2 \sin^2\theta + c^2 \cos^2\theta, \tag{16}$$

where c denotes the propagation velocity for a vibration parallel to the axis of the crystal, and a the propagation velocity for a vibration perpendicular to the axis. Choosing the axis of the crystal for the z axis, and calling (α, β, γ) the direction cosines of the vibration, the former relation may be rewritten as

$$V^2 = a^2(\alpha^2 + \beta^2) + c^2\gamma^2. \tag{17}$$

In the biaxal case, Fresnel assumed the natural generalization

$$V^2 = a^2\alpha^2 + b^2\beta^2 + c^2\gamma^2, \tag{18}$$

[110]Fresnel [1821e], pp. 280–2.

which is the polar definition of the surface of fourth degree that Fresnel called the *surface of elasticity*.[111]

Fresnel further assumed that for a given orientation of the wave planes, the possible directions of the vibration of the plane waves were those for which the velocity V was an extremum (more on this later). Call **n** a vector directed along the normal of the wave planes and whose length is the inverse of the velocity V (optical index vector). The extremum condition is met if and only if this vector lies on the surface of the fourth degree, which is now called the *index surface*:

$$n^2(b^2c^2n_x{}^2 + a^2c^2n_y{}^2 + a^2b^2n_z{}^2) - [n_x{}^2(b^2+c^2) + n_y{}^2(a^2+c^2) + n_z{}^2(a^2+b^2)] + 1 = 0. \quad (19)$$

This surface is double-sheeted, as should be expected on physical grounds. The two sheets correspond to the two different possibilities of polarization.[112]

Fresnel had no difficulty determining this surface by differentiation and elimination. A modern derivation goes as follows. In vector form, the velocity formula (18) reads $V^2 = \mathbf{x}\cdot\mathrm{K}\mathbf{x}$, where **x** denotes the unit vector in the direction of the vibration and K the operator that is diagonal with the diagonal elements a^2, b^2, c^2 in the crystal's system of axes. This velocity must be an extremum under the constraints $\mathbf{x}^2 = 1$ and $\mathbf{x}\cdot\mathbf{n} = 0$. Equivalently, $\mathrm{K}\mathbf{x}\cdot\delta\mathbf{x} = 0$ must hold whenever $\mathbf{x}\cdot\delta\mathbf{x} = 0$ and $\mathbf{n}\cdot\delta\mathbf{x} = 0$. Or else, the vectors K**x**, **x**, and **n** must be coplanar. Together with $\mathbf{x}\cdot\mathbf{n} = 0$ and $V^2 = \mathbf{x}\cdot\mathrm{K}\mathbf{x}$, this implies that $(\mathrm{K} - V^2)\mathbf{x}$ is parallel to **n**. Hence $(\mathrm{K} - V^2)^{-1}\mathbf{n}$ is parallel to **x** and perpendicular to **n**, so that

$$\mathbf{n}\cdot(\mathrm{K} - V^2)^{-1}\mathbf{n} = 0. \quad (20)$$

In the systems of axes of the crystal, this gives

$$\frac{n_x{}^2}{V^2 - a^2} + \frac{n_y{}^2}{V^2 - b^2} + \frac{n_z{}^2}{V^2 - c^2} = 0, \quad (21)$$

which is equivalent to the equation (19) of the index surface since $V^2 = 1/\mathbf{n}^2$.

In a useful variant of this reasoning, the coplanarity of K**x**, **x**, and **n** implies, together with $\mathbf{x}\cdot\mathbf{n} = 0$, that **x** must be parallel to $\mathbf{n} \times (\mathbf{n} \times \mathrm{K}\mathbf{x})$. Using $V^2 = \mathbf{x}\cdot\mathrm{K}\mathbf{x}$ and $\mathbf{n}^2 = 1/V^2$, this condition becomes

$$\mathbf{n} \times (\mathbf{n} \times \mathrm{K}\mathbf{x}) + \mathbf{x} = \mathbf{0}, \quad (22)$$

[111]Fresnel [1821e], pp. 285–6; [1821f], p. 317; [1821g]; [1822a]. Cf. Verdet 1866–1870, vol. 2, pp. 326n–329n; Buchwald 1989, p. 276–9.

[112]Fresnel [1822b], pp. 351–5; [1822c], 380–2; 1827, pp. 538–45. Fresnel did not explicitly introduce the index surface; instead he gave the second degree equation for the square of the velocity V as a function of the orientation of the wave planes.

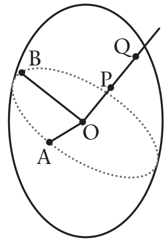


Fig. 5.18. *Fresnel's construction of the wave surface.* The larger ellipse is the trace of an ellipsoid whose axes are equal to the velocities in the directions of the principal axes of the crystal; the dotted ellipse is the trace of the elliptic intersection of this ellipsoid with the wave plane passing through its center O; OA and OB are the semi-axes of this elliptic intersection. On the perpendicular to the wave plane through O, the points P and Q such that OP = OA and OQ = OB describe the two sheets of the wave surface when the orientation of the wave plane varies.

which requires

$$\det\left[\mathbf{n} \times (\mathbf{n}\times\ ) + \mathrm{K}^{-1}\right] = 0. \tag{23}$$

The expression of this determinant in the system of axes of the crystal leads to equation (19) as the sixth-order terms cancel out.

As Fresnel relied on Huygens's wavelets in the uniaxal case, he naturally sought to determine the shape of these wavelets in the biaxal case. For this purpose he cleverly inverted the Huygens construction. Namely, he obtained the wavelet caused at the origin of time by a perturbation localized at point O by taking the envelope of all the plane pulses that pass through O at the origin of time. At time $t = 1$, this envelope is given by another surface of the fourth degree:

$$s^2(a^2 s_x{}^2 + b^2 s_y{}^2 + c^2 s_z{}^2) - [s_x{}^2 a^2(b^2 + c^2) + s_y{}^2 b^2(a^2 + c^2) + s_z{}^2 c^2(a^2 + b^2)] + a^2b^2c^2 = 0, \tag{24}$$

where $\mathbf{s}$ denotes the vector joining a point of the wavelet to the origin O. According to Huygens's construction, the $\mathbf{s}$ vector yields the direction and velocity of the rays in the anisotropic medium. Again, the two possibilities of polarization give two different sheets, which degenerate into a mutually touching sphere and spheroid in the uniaxal case.[113]

Fresnel reached the equation (24) of the wave surface in the spring of 1822. As he judged the analytical determination of the defining envelope to be too difficult, he followed a "synthetic" route based on noticing that the intersections of this surface with the coordinate planes were the conjunction of an ellipse and a circle and seeking the (single) fourth degree surface that had this property. This led him to the prescription given in Fig. 5.18.

[113]Fresnel [1822c], p. 386; 1827, pp. 554–63.

Fresnel then derived equation (24) from this prescription, and verified that this equation was a solution of the elimination problem. The relevant calculations were so "fastidious" that he decided not to include them in his memoir.[114]

Thus, Fresnel implicitly challenged his followers to simplify the determination of the wave surface. His friend Ampère offered the first algebraic proof in 1828. A simple modern derivation goes as follows.[115]

The planes of which the wave surface is the envelope have the equation $\mathbf{n}\cdot\mathbf{r} = 1$, where $\mathbf{n}$ is the index vector. A point $\mathbf{s}$ of the envelope is obtained by satisfying $\mathbf{n}\cdot\mathbf{s} = 1$ and $\mathbf{s}\cdot\mathrm{d}\mathbf{n} = 0$ for any possible $\mathrm{d}\mathbf{n}$. Differentiating equation (22), we get

$$\mathrm{d}\mathbf{n} \times (\mathbf{n} \times \mathrm{K}\mathbf{x}) + \mathbf{n} \times (\mathrm{d}\mathbf{n} \times \mathrm{K}\mathbf{x}) + \mathbf{n} \times (\mathbf{n} \times \mathrm{K}\mathrm{d}\mathbf{x}) + \mathrm{d}\mathbf{x} = \mathbf{0}. \tag{25}$$

Using the properties of the mixed product, the scalar product of this equation with $\mathrm{K}\mathbf{x}$ can be expressed as

$$2\mathrm{d}\mathbf{n}\cdot[(\mathbf{n} \times \mathrm{K}\mathbf{x}) \times \mathrm{K}\mathbf{x}] + \mathrm{K}\mathrm{d}\mathbf{x}\cdot[\mathbf{n} \times (\mathbf{n} \times \mathrm{K}\mathbf{x})] + \mathrm{d}\mathbf{x}\cdot\mathrm{K}\mathbf{x} = 0. \tag{26}$$

Using equation (22) again and exploiting the symmetry of the operator K (following which $\mathbf{x}\cdot\mathrm{K}\mathrm{d}\mathbf{x} = \mathrm{d}\mathbf{x}\cdot\mathrm{K}\mathbf{x}$), we arrive at

$$\mathrm{d}\mathbf{n}\cdot[(\mathbf{n} \times \mathrm{K}\mathbf{x}) \times \mathrm{K}\mathbf{x}] = 0 \quad \text{for any possible } \mathrm{d}\mathbf{n}. \tag{27}$$

Hence the vector $\mathbf{s}$ must be parallel to the vector $(\mathbf{n}\times\mathrm{K}\mathbf{x})\times\mathrm{K}\mathbf{x}$ and perpendicular to the vectors $\mathrm{K}\mathbf{x}$ and $\mathbf{n}\times\mathrm{K}\mathbf{x}$. Consequently, we have

$$\mathbf{s} \times \mathbf{x} = -\mathbf{s} \times [\mathbf{n} \times (\mathbf{n} \times \mathrm{K}\mathbf{x})] = (\mathbf{s}\cdot\mathbf{n})(\mathbf{n} \times \mathrm{K}\mathbf{x}) = \mathbf{n} \times \mathrm{K}\mathbf{x}, \tag{28}$$

and

$$\mathbf{s} \times (\mathbf{s} \times \mathbf{x}) = -(\mathbf{s}\cdot\mathbf{n})\mathrm{K}\mathbf{x} = -\mathrm{K}\mathbf{x}. \tag{29}$$

Comparing this last equation with equation (22), we see that the wave surface can be obtained from the index surface by replacing the operator K with the operator K^{-1}. In other words, equation (24) of the wave surface is obtained by inverting the coefficients *a, b, c* in equation (19) of the index surface.[116]

[114] Fresnel [1822c], pp. 383–7; 1827, p. 560. Cf. Buchwald 1989, pp. 282–5; Grattan-guinness 1990, vol. 2, pp. 884–95. The origin of this prescription is obscure. Some of Fresnel's remarks in an earlier MS ([1822c], pp. 386–7) suggest that it came as a side product of his earlier assumption that the elasticity surface was an ellipsoid. The index surface corresponding to this elasticity surface is indeed identical with the wave surface of the true theory in which the elasticity surface is given by equation (18).

[115] Ampère 1828. See also MacCullagh 1830; Hamilton 1837; Sénarmont 1868, pp. 606–10.

[116] This derivation parallels the geometrical derivation of MacCullagh 1830. It also corresponds to an electromagnetic derivation in which $\mathbf{x}$ would be proportional to the electric displacement $\mathbf{D}$, $\mathrm{K}\mathbf{x}$ to the electric field $\mathbf{E}$, $\mathbf{n}\times\mathrm{K}\mathbf{x}$ to the magnetic field $\mathbf{H}$, $(\mathbf{n} \times \mathrm{K}\mathbf{n}) \times \mathrm{K}\mathbf{x}$ to the Poynting vector.

To sum up, Fresnel's theory involves three surfaces, the *elasticity surface* which yields the velocity of plane waves as a function of the direction of vibration, the *index surface* which yields this velocity as a (double-valued) function of the orientation of the wave planes, and the *wave surface* (*surface de l'onde* now often called "ray surface") which yields the expansion of a pulse from a point-like origin. With his usual thoroughness, Fresnel showed that the consequences of this theory agreed with experiments performed on biaxial crystals. In particular the theory implied that there was no ordinary refraction in such crystals, as Fresnel had discovered a few months before devising it. It gave rules for the polarization and direction of the two refracted rays in agreement with Brewster's and Biot's observations. As we may judge in retrospect, Fresnel's predictions are identical with those of the electromagnetic theory of light. This success rested on an astute combination of well-established principles and empirical data including Huygens's construction, the transverse character of optical vibrations, and the ellipsoidal form of wavelets in uniaxal crystals.[117]

The latter assumption was the weakest link in Fresnel's reasoning, as its validity was only known to a certain approximation. Fresnel supported it with the molecular model he had earlier invented to justify the transversal character of light waves. In this model, the luminiferous medium resists any change of distance between two consecutive layers of molecules infinitely much more than their sliding on each other. Consequently, for a given shifted layer the only active component of the restoring force is its projection on the plane of the layer. Fresnel had this force depend on the direction of the shifts only (not on the orientation of the layers), so that the velocity of waves would depend only on the direction of vibration.[118]

Call **D** the displacement of a molecular layer with respect to the neighboring layers and **E** the restoring force. Fresnel assumed the relation

$$\mathbf{E} = \mathrm{K}\mathbf{D}, \tag{30}$$

where K is a symmetric operator describing the linear response.[119] Granted that the propagation of a wave only involves the projection of such forces on the direction of vibration, the propagation velocity V is given by

$$V^2 = \frac{\mathbf{D}\cdot\mathrm{K}\mathbf{D}}{D^2} \tag{31}$$

in a medium of unit density. As Fresnel proved by elementary means, the symmetry of the operator K implies the existence of a system of axes in which it is diagonal. In this system, the former expression of the velocity reduces to the form

[117]Fresnel [1821e], pp. 261–74 (no ordinary refraction), 290–8 (conformity with Biot's laws); 1822d; 1827, pp. 573–8, 580–4. For a detailed analysis of Fresnel's predictions and verification, cf. Buchwald 1989, chap. 11.

[118]Fresnel [1822b], pp. 343–55; [1822c], pp. 369–79; 1827, pp. 507–38. As will be seen in chap. 6, this assumption is incompatible with molecular models of the ether.

[119]Fresnel only derived this relation in the case of the displacement of a single molecule when all other molecules remain in the same position; but he believed it also applied to the elastic forces called into play by a propagating wave.

$$V^2 = a^2\alpha^2 + b^2\beta^2 + c^2\gamma^2, \tag{18}$$

which Fresnel had earlier induced from Huygens's ellipsoidal wavelets.[120]

The condition (22) for this velocity to be an extremum for a given orientation of the wave planes can be rewritten as

$$\mathbf{n}^2\mathbf{E} - (\mathbf{n}\cdot\mathbf{E})\mathbf{n} = \mathbf{D}, \tag{32}$$

which means that the projection of the restoring force **E** on the wave planes is parallel to the displacement of the layers of the medium. According to Fresnel, the two rectilinear vibrations that meet this condition are the only ones for which the direction of vibration is preserved during propagation and the velocity is well-defined. If any vibration enters the crystal with a different polarization, it must divide itself into two vibrations of the former kind.[121]

Fresnel thus justified the form and use of his elasticity surface. The kind of elastic response he imagined in his molecular medium soon proved incompatible with the general laws of elasticity. Yet, as we will see in the next chapter, his assumptions foreshadowed the correct notion that the velocity of propagation of a plane light wave is determined by a linear response to a vector perpendicular to the plane of polarization and contained in the plane of the wave.

Early reception

From a Laplacian viewpoint, Fresnel's optics had the double inconvenient of challenging the corpuscular concept of light and of injecting physical intuition in mathematical demonstrations. The cumulative successes of his works on diffraction, chromatic polarization, and crystal optics nonetheless shook Biot's, Poisson's, and Laplace's confidence in Newtonian optics. Besides, Laplace and his disciples had gradually lost their intellectual and institutional dominance over French physics to the newer physics of Arago, Ampère, Fourier, and their friends. By the time of Fresnel's death, which occurred prematurely in 1827, French opposition to wave optics had nearly vanished. The old Laplace saluted Fresnel's theory of double refraction as the greatest accomplishment in recent years, and his intellectual kin Augustin Cauchy soon embarked on a rigorous construction of Fresnel's molecular ether.[122]

In Britain, Fresnel benefited from the contemporary zeal of Cambridge and Dublin natural philosophers in importing French mathematical physics. In 1826 the Cambridge astronomer John Herschel, who had entered optics through Biot's mobile polarization, asked Fresnel for details about his theories. His subsequent article on light, privately circulated in 1828 and later published in the *Encyclopaedia Metropolitana*, included a

[120]Fresnel [1822b], pp. 343–51; 1827, pp. 514–26.

[121]Fresnel [1822b], pp. 351–5; 1827, pp. 538–43.

[122]Cf. Frankel 1976, pp. 171–4. Laplace's judgment is reported in Verdet 1866–1870, vol. 1, pp. lxxxvi–lxxxvii.

competent and praiseful description of Fresnel's and Young's work. Herschel left no doubt about his own preference:[123]

> The unpursued speculations of Newton, and the opinions of Hooke, however distinct, must not be put in competition, and, indeed, ought scarcely to be mentioned, with the elegant, simple, and comprehensive theory of Young,—a theory which, if not founded in nature, is certainly one of the happiest fictions that the genius of man ever invented to grasp together natural phenomena, which, at their first discovery, seemed in irreconcileable opposition to it. It is, in fact, in all its applications and details, one succession of *felicities;* insomuch, that we may almost be induced to say, if it be not true, it deserves to be so.

Two other Cambridge luminaries, the Lucasian Professor George Biddel Airy and the philosopher William Whewell soon expressed similar opinions. At Dublin's Trinity College in 1830, James MacCullagh gave a most elegant derivation of Fresnel's wave surface based on the notion of the reciprocal surface. Two years later, his colleague William Rowan Hamilton considered Fresnel's theory of propagation in crystals in the last installment of his "Theory of a system of rays." Hamilton offered his own algebraic derivation of the wave surface, and showed that the conic cusps on Fresnel's index and wave surfaces implied "conical refraction": namely, to a single incoming ray corresponds a complete cone of refracted rays. At Hamilton's request, his colleague Humphrey Lloyd verified the existence of this strange phenomenon, thus providing a striking confirmation of Fresnel's theory.[124]

As MacCullagh noted, conical refraction follows directly from Fresnel's constructions for the index and wave surfaces (see above, Fig. 5.18, for the latter). When the section of the generating ellipse is circular, the two sheets of the index or wave surface intersect and form two opposite conic cusps (see Fig. 5.19). Consequently, the normal to the surface at the point of intersection is ill defined: its possible values describe a cone. In the case of the wave surface, the normal corresponds to the direction of the index vector **n**. Remembering that according to Huygens's construction this vector does not change during refraction at normal incidence and is identical to the ray vector **s** in the isotropic medium, the cusps on the wave surface imply "external" conical refraction in which a single ray in the crystal corresponds to an entire cone of rays outside the crystal. Similarly, a cusp on the index surface implies that the possible values of the ray vector **s** describe a cone for a given value of the index vector **n**. This leads to "internal" conical refraction.[125]

[123]J. Herschel 1845a [1827], p. 456. On Herschel's questions to Fresnel, cf. the reply in Fresnel [1826]. On the British reception, cf. Cantor 1983, pp. 159–65; Buchwald 1989, chap. 12; Chen 2000, chaps. 1–2.

[124]MacCullagh 1830, 1837a; Hamilton 1837 [1832], pp. 130–44. Cf. Graves 1882, vol. 1, pp. 631, 685–9; Hankins 1980, p. 168; O'Hara 1982; Lunney and Weaire 2005. Two surfaces are said to be reciprocal with respect to an origin O if and only if they meet the following condition: *At point* Q *of the first surface, draw the tangent plane and the perpendicular to this plane that passes through the origin* O. *Call* P *the intersection of the perpendicular with this plane, and* R *its intersection with the second surface. For any choice of* Q, *the distances* OP *and* OR *are inversely proportional*. Fresnel's index and wave surfaces are reciprocal in this sense. So are too their generating ellipsoids in Fresnel's construction (cf. Darrigol 2010b, 141–3). On Hamilton's optics and his derivation of Fresnel's wave surface, see below, chap. 7, pp. 269–70.

[125]MacCullagh 1833a, 1833b.

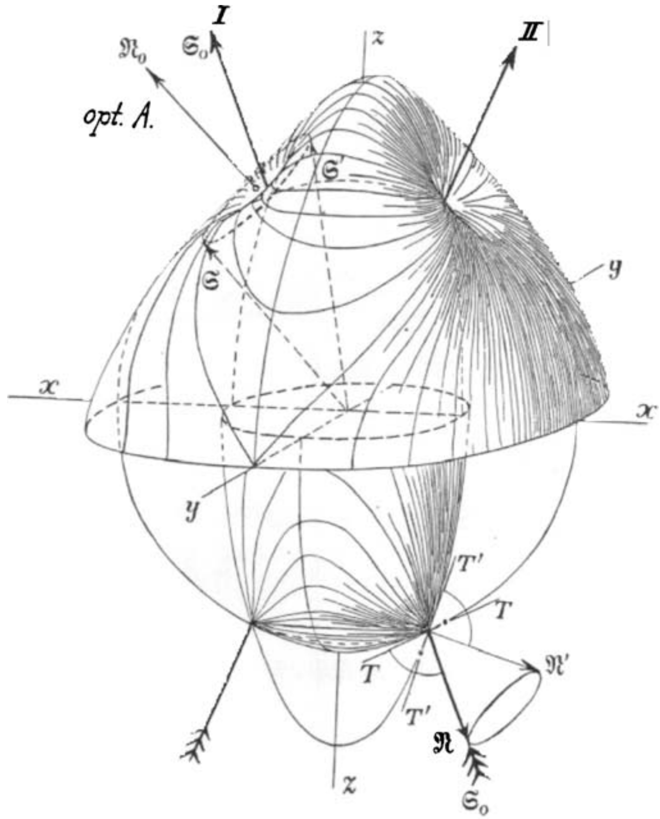


Fig. 5.19. Fresnel's wave surface. From Schaefer 1929–1937, vol. 3, part 1, p. 485. The arrows I and II represent the two axes of the crystal. The traces of the two sheets of the surface on the xz plane are the circle and ellipse visible on the lower part of the figure. The two sheets cross each other at four points, forming conical cusps. At the South-East point, T and T′ are two tangent planes; $\mathfrak{N}$ and $\mathfrak{N}'$ are the corresponding wave vectors. They belong to the cone of all wave vectors compatible with the ray $\mathfrak{S}_0$. At the North-West corner of the figure, the dotted ellipse represents the circle of contact between a plane normal to $\mathfrak{N}_0$ and the outer sheet. The rays $\mathfrak{S}$ and $\mathfrak{S}'$ and all rays passing through this circle correspond to the same wave vector $\mathfrak{N}_0$.

Lloyd's brilliant demonstration of conical refraction marked the Dublin meeting of the British Association for the Advancement of Science in 1833, in stark contrast with Brewster's attacks against the wave theory at the previous meeting. Lloyd's subsequent BA report on optics trumpeted the victory of Fresnel's and Young's theory. By the end of the decade, Brewster was about the only British expert that still supported the Newtonian theory. Although he acknowledged the predictive power of the wave theory, he excluded it for photochemical reasons: he believed that the chemically specific actions of light could only be explained if light itself was a substance. He used his discovery, in 1832, of numerous dark lines in the absorption spectrum of nitrogen dioxide as a further objection

to the wave concept: he could not imagine any acoustic analogy to this phenomenon. Despite his immense authority as an experimenter, these arguments failed to curb the rise of the wave concept of light.[126]

Fresnel's optics enjoyed a similar success in Germany. In 1823 the Bavarian telescope maker Joseph Fraunhofer used his fine work on diffraction by gratings to support the principle of interference and wave optics in general. In 1825 the Leipzig physicists Ernst and Wilhelm Weber published an extensive *Wellenlehre* in which they aimed to provide the general experimental foundation of the concept of wave and thus to ease its application to optics. In the 1830s German physics textbooks began to favor the Young–Fresnel theory. The Königsberg physicist Franz Neumann inaugurated German theories of Fresnel's ether, in parallel with Cauchy's and MacCullagh's contemporary forays into this difficult territory.[127]

No one tried to retrieve Fresnel's most specific results in a Newtonian framework. None of the physicists of the younger generation accepted corpuscular optics. The ultimate blow to this theory came in 1850, when Léon Foucault and Hippolyte Fizeau implemented Arago's plan to measure the velocity of light in water with Charles Wheatstone's method of the rotating mirror. They found this velocity to be smaller than in air, in conformity with the wave theory of refraction. Although this result came too late to create a big stir, it was frequently cited in later optical treatises as the most decisive proof of the failure of Newton's optics.[128]

5.4 Conclusions

Thomas Young's undulatory optics resulted from his improvement of the acoustic analogy in the course of astute, precise, and varied experiments on sound and light. He thereby benefited from his encyclopedic knowledge of British and continental sources, on both sides of past controversies. Although his contemporaries partially recognized the importance and originality of his works, they did not follow him on his adventurous track. On the contrary, Malus's discovery of polarization by reflection and Arago's and Biot's subsequent works on chromatic polarization revived Newton's old concept of polarization as an asymmetry of the light corpuscles. This concept fitted naturally in the Laplacian program which then dominated French physics, as well as in the British program of an optics entirely based on corpuscles and attraction. Biot's bulky treatises impressed everyone by their qualitative and quantitative thoroughness. His analogy between Newton's fits and the periods of mobile polarization seemed to reach the heart of the matter.

The young Fresnel originally ignored polarization and studied diffraction to confirm his inclination toward the wave theory of light. Although his first findings only confirmed Young's earlier results, his attention to quantitative details gradually brought him to the Huygens–Fresnel principle for computing the amplitude of the diffracted light. He then

[126]Lloyd 1835. Cf. Chen 2000, pp. 6–12, 57–61 (Brewster), 21–3 (BA meetings).

[127]Fraunhofer 1823, pp. 358–9, 366n–388n (wave reflection on rough surfaces); Weber and Weber 1825. On Fraunhofer and on Neumann, see below, chap. 6, pp. 226, 231, 245.

[128]Foucault 1850; Fizeau and Breguet 1850a (setup), 1850b (results). Cf. Mach 1921, pp. 37–40; Rosmorduc 1978.

turned to polarization and determined the role it played in interference. This led him to a theory of chromatic polarization that contradicted Biot's results in special cases. Biot saved his theory at the price of ad hoc modifications. In general, the Laplacians admired Fresnel's results without yet espousing the sustaining wave theory. They deplored the lack of a dynamical foundation for the Huygens–Fresnel principle; they denied the possibility of any understanding of polarization in the wave picture; and they speculated that interference was an interaction between rays rather than a wave process.

At some point, Fresnel adopted the purely transverse waves which he and other theorists had believed to be mechanically impossible. He justified this concept through a molecular medium whose elastic response was very large for compression, moderate for the small shears implied in transverse waves, and negligible for macroscopic shears. He used it to perfect his earlier theories of chromatic polarization and of polarization by reflection, and also to derive the intensities of reflected and refracted light. Most importantly, he developed a complete quantitative theory of the interplay between polarization and propagation in anisotropic media. The unprecedented mathematical sophistication and the exquisite empirical accuracy of the latter theory ended up converting the adversaries of the wave theory, both in France and abroad. It was indeed difficult to imagine how the corpuscular theory could account for the wide variety of phenomena that Fresnel's theory now encompassed in a unified, quantitative manner.

There is no doubt that Fresnel's hypothesis of the molecular ether played a role in his theoretical inventions, at least as an indication that the dynamical response of the ether could be widely different from that of known elastic bodies and compatible with the phenomenology of light propagation. However, the quantitative laws that Fresnel derived for the propagation of polarized light did not depend on his precise model of the ether. They originated in experimental results combined with a few principles including the principle of interference, the sinusoidal character of the vibrations associated with simple colors, the Huygens–Fresnel principle, and the ellipsoidal shape of Huygens's wavelets in uniaxal crystals. This explains the persistency of Fresnel's results, as well as their nearly complete acceptance in the first decade following his death. Fresnel's model of the ether did no fare as well, as we are about to see.

6

ETHER AND MATTER

Fresnel's tentative concept of the ether as a molecular lattice resisting small shears and yet fluid at larger scales puzzled his contemporaries and challenged them to find a better mechanical picture. There were many attempts of this kind in the nineteenth century, with a slow decline toward the end of the century owing to the competition of the electromagnetic ether. The form of these attempts varied considerably with the taste of their authors and the scientific subculture in which they were immersed. Before studying them, it is useful to recall some general characteristics of physics and optics in the three countries that most contributed to nineteenth-century optics: France, Britain, and Germany.

Fresnel's duties as an engineer, his fragile health, and his early death prevented him from forming his own disciples. His friend Arago went on carrying the torch of wave optics. Most French physicists recognized the importance of Fresnel's achievements. By the mid-century his theories of interference, diffraction, and polarization had become the core of the new optics taught at the *Grandes Ecoles* and at French universities. Owing to its privileged connection with astronomy and mathematics, optics was the best career opportunity for budding French physicists. The preferred approach was experimental, as French taste for speculative theory dwindled and interest in precision measurement rose. On the theoretical side, Augustin Cauchy and his disciples developed an influential theory of the ether as a molecular lattice. After Fresnel, the most frequently cited French optical investigations probably were Cauchy's theoretical memoirs, Jules Jamin's measurements of reflected light, and various experiments by Hippolyte Fizeau and Léon Foucault concerning the velocity of light and interference. The French were slow in appreciating works from other countries. Notable exceptions were Emile Verdet, whose thorough teaching of French and foreign doctrines marked the mid-century generation, and Eleuthère Mascart and Henri Poincaré, who advocated Maxwell's electromagnetic theory of light later in the century.[1]

In Britain, Fresnel's memoirs arrived at a time when the most prestigious universities were busy integrating the new French mathematical physics. Astronomers such as John Herschel and George Biddell Airy were eager to apply the new wave optics to the theory of optical instruments. A brilliant constellation of natural philosophers including George Green, James MacCullagh, William Rowan Hamilton, George Stokes, William Thomson, and Lord Rayleigh developed mathematical theories of the ether as an elastic body. Although Cauchy's molecular ether originally received much attention, the British soon favored a more phenomenological approach in which the molecular structure of the ether

[1]On the evolution of French physics in the first half of the nineteenth century, cf. Fox 1974. On the hegemony of optics, cf. Davis 1986. On Cauchy's biography, cf. Belhoste 1991.

was blackboxed or simply denied. For instance, Green and MacCullagh developed a Lagrangian method in which the knowledge of the potential of large-scale deformations entirely determined the observable behavior of the ether. Thomson and the next generation of British ether theorists tended to regard the optical ether as a continuum, the rigidity of which resulted from low-scale internal motions. Later in the century, they gradually adopted Maxwell's electromagnetic theory of light in which the optical ether was described through electric and magnetic field whose mechanical meaning could be left open.[2]

The Germans welcomed Fresnel's and Cauchy's theories as part of a new French mathematical physics whose superiority could no longer be doubted. The most active centers of this renewal were Königsberg under Friedrich Bessel and Franz Neumann, and Göttingen under Carl Friedrich Gauss and Wilhelm Weber. Neumann's memoirs paralleled Cauchy's molecular theory of the ether and completed Fresnel's theory of the reflection of light on crystals. Despite his earlier immersion in molecular physics and crystallography, Neumann soon came to prefer a more economical approach in which the ether was treated as a continuum. So too did his disciple Gustav Kirchhoff, who became the leader of German theoretical optics and the champion of what Ludwig Boltzmann called "mathematical phenomenology." In Germany as anywhere else, most optical researches were experimental and pure theory was a rarity. Most influential was Joseph Fraunhofer's work on optical glass, dispersion, and diffraction. Later in the century, Gustav Kirchhoff and Robert Bunsen provided the experimental and theoretical basis for modern spectroscopy and its use in chemical analysis.[3]

The first section of this chapter recounts a few of the numerous theories of the ether as an elastic body, molecular or continuous according to the aforementioned regional preferences. The most popular of those theories were Cauchy's lattice theory, the Green–Stokes jelly theory, and the Cauchy–Thomson labile-ether or foam theory, which all compared the ether to a variety of ordinary matter. In contrast, James MacCullagh defended a rotationally-elastic ether which did not resemble any known kind of matter. Despite its contemporary rejection, this theory is worth special attention because it best fitted the phenomena and because George Francis FitzGerald and Joseph Larmor later revived it to found electromagnetic theory. The second section briefly describes the emergence of the electromagnetic theory of light, the connection with MacCullagh's ether, and the late-nineteenth century persistence of the mechanical ether. The third section is devoted to the phenomena that brought theorists to separate the ether from the embedded matter: dispersion, optical rotation, and the optics of moving body. Although Fresnel, Cauchy, and a few followers originally traced dispersion to the finite spacing of the ether's molecules, the internal difficulties of this approach, MacCullagh's proof of its incompatibility with the existence of optical rotation, and the discovery of anomalous dispersion prompted theorists to develop alternative theories in which the dynamical coupling between ether and matter became responsible for dispersion and

[2]On the evolution of British mathematical physics, cf. Smith and Wise 1989, chap. 6 and further reference there.

[3]On German theoretical physics in the nineteenth century, cf. Jungnickel and McCormmach 1986 (vol. 1, p. 149 for the phenomenological turn in Neumann's optics, pp. 297–302 for Kirchhoff and Bunsen). On nineteenth-century spectroscopy, cf. McGucken 1969; Hentschel 2002.

other optical phenomena dependent on the presence of matter. In this process, any effect of the molecules of matter on the constitutive parameters (density and elasticity) of the ether and on its position in space was gradually given up. Joseph Boussinesq's highly adequate theory of 1867 assumed a completely incorruptible and stationary ether, as Hendrik Lorentz's and Larmor's electron theories later did.

6.1 The ether as an elastic body

The elastic resistance of Fresnel's ether to small-scale shears made it similar to an elastic solid, as Young remarked in 1823. Attempts at building the ether on this analogy began in the 1830s, after Claude Louis Navier, Augustin Cauchy, and Siméon Denis Poisson had given the foundations of a general theory of linear elasticity.[4]

New theories of elasticity

There were two approaches to the theory of elasticity. In the molecular approach initiated by Navier and perfected by Cauchy and Poisson, the elastic solid is regarded as made of molecules interacting through short-range central forces. The macroscopic equation of motion is obtained by summing the molecular forces acting on a given molecule, assuming that the shift of the molecules is small compared with the range of these forces and that this range is small compared with the characteristic length of the deformation. In these approximations, the equation of motion is a partial differential equation of second order. In the most general medium without initial stress, this equation involves fifteen elastic constants. In the case of an isotropic body without initial stress, there is only one elastic constant and the equation of motion is Navier's

$$\rho\ddot{\mathbf{u}} + K[\Delta\mathbf{u} + 2\nabla(\nabla\cdot\mathbf{u})] = \mathbf{0}, \tag{1}$$

where $\mathbf{u}$ is the shift of the elements of the body, ρ the density, and K the elastic constant.[5]

Meanwhile, Cauchy developed the modern macroscopic approach based on the concept of internal stress. In this approach the equation of motion is obtained by taking the resultant of the pressures acting on the surface of a volume element of the body. Owing to the lack of fluidity, these pressures are not restricted to being normal to the surface. The pressure on the arbitrary surface element $\mathrm{d}\mathbf{S}$ is given by the sum over j of $\sigma_{ij}\mathrm{d}S_j$, where σ_{ij} is a symmetric system of stresses now called stress tensor. To first order, the distortion of the body is characterized by the strain tensor

$$e_{ij} = \partial_i u_j + \partial_j u_i, \tag{2}$$

if $\mathbf{u}$ denotes the shift of the elements of the body (namely, this shift causes the variation $e_{ij}\mathrm{d}x_i\mathrm{d}x_j$ of the squared length of the material vector joining x_i and $x_i + \mathrm{d}x_i$). The most general linear relation between the strain and stress tensors involves 21 constants (because

[4]For Young's remark, see above, chap. 5, p. 211.

[5]Cf. Saint-venant 1864; Darrigol 2002; 2005, pp. 109–25, and further reference there. The vector and tensor notations used in this chapter are of course anachronistic.

the potential of the deformation is a quadratic form of the six components of the strain tensor). In the isotropic case, there are only two elastic constants and the induced stress has the form

$$\sigma_{ij} = K'(\partial_i u_j + \partial_j u_i) + K''\delta_{ij}\partial_k u_k, \tag{3}$$

where $_{ij}$ is the unit tensor and summation over repeated indices is understood.

Cauchy's early attempts

In 1829, Cauchy showed that Navier's equation (1) admitted both transverse and longitudinal plane (or spherical) wave solutions. He thus initiated the reduction of optical waves to transverse vibrations in an elastic solid, with the concomitant difficulty of undesired longitudinal waves. The following year, he studied plane-wave propagation in an anisotropic elastic solid, with the hope of retrieving Fresnel's laws of propagation. In a molecular solid devoid of initial stress and symmetric with respect to three orthogonal planes, there are six independent elastic constants. Cauchy found that by imposing three ad hoc conditions on these constants, he could approximately retrieve Fresnel's index surface for the solutions that were approximately transverse. He thereby assumed that the vibration was in the plane of polarization, against Fresnel's opinion. The necessity of this move results from the following argument.[6]

Consider a plane wave such that the wave vector is parallel to one axis of elasticity (in Fresnel's sense) and the vibration is parallel to another axis. According to the theory of elasticity, the velocity of this wave remains the same when the two axes are permuted (because the stress and strain tensors are both symmetrical). If the vibration is taken to be in the plane of polarization, this means that the theory automatically meets the empirical condition that the velocity only depends on the orientation of the plane of polarization. If the vibration is taken to be perpendicular to the plane of polarization, this empirical condition implies a further restriction on the elastic constants, which can only be met when the medium is originally stressed.[7]

In the same year, 1830, Cauchy retrieved Fresnel's sine and tangent laws for the intensity of reflected light at the interface between two transparent media under the following assumptions: equality of the densities but different rigidities of the two media, equality of the stresses across the plane of incidence when the vibration is perpendicular to this plane, equality of the normal pressures across the separating plane and equality of the normal shifts when the vibration is in the plane of incidence. Cauchy probably chose the first assumption (instead of Fresnel's assumption of equal rigidities) so as to ease the generalization to anisotropic media: anisotropic inertia is harder to imagine than anisotropic elasticity. His other conditions were probably inspired by a desire to rely on the basic concepts of his theory of elasticity: stress and shift. He declared them "obvious" on grounds of symmetry. Their consequences only agree with Fresnel's predictions if the direction of polarization is perpendicular to the direction of vibration. Cauchy seems to

[6]Cauchy 1830a, 1831. Cf. Whittaker 1951, pp. 132–3; Buchwald 1980a; Harman 1998.

[7]A similar reasoning is found in Whittaker 1951, p. 155.

have overlooked this contradiction with his slightly earlier theory of anisotropic propagation.[8]

With these early attempts, Cauchy launched the long popular trend of deriving the laws of optics from those of elasticity. Two basic difficulties were already apparent: the need to arbitrarily restrict the overabundant elastic constants in an anisotropic body in order to retrieve Fresnel's laws of propagation, and the need to ignore some of the true boundary conditions in order to retrieve Fresnel's laws of reflection and refraction. In both contexts, longitudinal waves had to be artificially eliminated unless one speculated—as Cauchy did—that they were too weak or too special to be seen.[9]

MacCullagh's theory of refraction by crystals

The most unnatural feature of Fresnel's and Cauchy's theories of reflection and refraction (in isotropic media) is the admission of discontinuities of the vibration at the interface between two media. While studying Fresnel's demonstration, the Irish mathematician and natural philosopher James MacCullagh found it to imply the continuity of the vector $\mathbf{n} \times \mathbf{w}$, wherein $\mathbf{n}$ is the index vector and $\mathbf{w}$ the vibration according to Fresnel. In other words, the total vibration is continuous if one assumes, against Fresnel, that the vibration of a plane wave belongs to the plane of polarization. MacCullagh found the latter choice confirmed in Cauchy's theory of 1830 for the propagation of light in crystals.[10]

One difficulty remained: if the product nw measures the vibration, the corresponding energy flux $V\rho(nw)^2$ is no longer conserved. In 1835, MacCullagh solved this difficulty by adopting Huygens's and Cauchy's assumption of equal density of the two media. Then Fresnel's sine and tangent formulas follow from the continuity of the vibration and the conservation of *vis viva*, because the product $V\rho$ is proportional to the inverse of the index n. MacCullagh also observed that in this new derivation Cauchy's stress condition (that the stress across the plane of incidence should be continuous at the frontier between two isotropic media) could replace the conservation of *vis viva*. This simplification allowed him to compute the polarizations and intensities of reflected and refracted rays at the surface of a crystal. As the results agreed with Brewster's observations for the polarization of reflected light, he published them toward the end of the year.[11]

MacCullagh's procedure required knowledge of the stress components in an anisotropic elastic medium. He relied on a simplified version of the expression that Cauchy had given in 1830:

> Wishing to get rid of [the third longitudinal wave], as well as of the slight deviations from the symmetrical law of Fresnel [for the propagation of transverse waves], I adopted the expedient of altering the equations of pressure [the stress system], in

[8]Cauchy 1830b. On Cauchy's oversight, cf. MacCullagh 1835, p. 57. As Green later showed, the true boundary conditions are the continuity of the shift and the continuity of the pressure across the interface.

[9]On Cauchy's speculations about the longitudinal waves, cf. Whittaker 1951, p. 134; Chappert 2004, p. 75.

[10]Cf. MacCullagh 1838, pp. 87–8.

[11]MacCullagh 1835, 1836. On the history, cf. MacCullagh 1837c, p. 84; 1838, pp. 89–90. Fresnel's proof requires the energy flux to be proportional to nw^2: see above, chap. 5, p. 212.

such a way as to make them afford only two rays, and give a law of refraction exactly the same as Fresnel's.

With respect to Fresnel's axes of elasticity, the resulting stress system is:

$$\sigma_{xx} = -2(c^2\partial_y u_y + b^2\partial_z u_z), \quad \sigma_{xy} = c^2(\partial_x u_y + \partial_y u_x), \text{ etc.} \tag{4}$$

The resulting force density, $f_i = \partial_j\sigma_{ij}$, is easily seen to verify the identity $\partial_i f_i = 0$, which implies that any plane-wave solution of the equation of motion $\mathbf{f} = \rho\ddot{\mathbf{u}}$ is transverse. In the isotropic case ($a^2 = b^2 = c^2 = K$), MacCullagh's stresses reduce to

$$\sigma_{ij} = K(\partial_i u_j + \partial_j u_i) - 2K\delta_{ij}\partial_k u_k. \tag{5}$$

The corresponding equation of motion is

$$\rho\ddot{\mathbf{u}} = K\Delta\mathbf{u} - K\nabla(\nabla\cdot\mathbf{u}) = -K\nabla\times(\nabla\times\mathbf{u}). \tag{6}$$

A slightly longer calculation shows that in the anisotropic case MacCullagh's stresses lead to the equation of motion:[12]

$$\rho\ddot{\mathbf{u}} = -\nabla\times(\mathrm{K}\nabla\times\mathbf{u}). \tag{7}$$

This is a first yield of MacCullagh's strategy of plying the ether dynamics to Fresnel's phenomenology of light propagation.

In 1836 MacCullagh learned from the German experimenter Thomas Seebeck that some predictions of his theory were incorrect. He then gave up the continuity of "lateral pressure" and returned to Fresnel's condition that the *vis viva* should be conserved. The consequences for the polarizations and intensities of reflected and refracted rays in a few particular cases agreed very well with Seebeck's measurements.[13] The calculations were forbiddingly difficult in the general case. MacCullagh greatly simplified them through his "theorem of the polar plane." For a given refracted ray, the *polar plane* is the plane containing the vibration vector and the difference $\mathbf{s} - \mathbf{n}$ between the ray and index vectors. MacCullagh's theorem states that whenever the vector of the incident vibration belongs to this plane, the intensity of the other refracted ray vanishes. For a given angle of incidence, there are two polar planes corresponding to the two refracted rays. Therefore, there are two orientations of the plane of polarization of the incident ray for which this condition is met. In these two cases, the determination of the intensity and polarization of the reflected and refracted rays is easily done in a way similar to that used for ordinary refraction. In the general case, the vibration can be regarded as the sum of two vibrations belonging to the

[12]MacCullagh 1836, p. 76. The translation works only for the homogeneous media in which K is uniform. In the heterogeneous case, MacCullagh's stresses lead to an equation of motion which differs from the modern one and therefore cannot be used to derive the boundary condition (in Lorenz's manner; see below p. 238).

[13]MacCullagh 1837c, 1838, 1842. MacCullagh's formulas agree with the modern electromagnetic theory, since his new boundary conditions correspond to the continuity of the magnetic vector and to the conservation of the electromagnetic energy.

two special cases, and the intensity and polarization of the reflected and refracted rays is obtained by superposition.[14]

MacCullagh read this memoir in January 1837.[15] He soon discovered that the Königsberg physicist Franz Neumann had obtained a very similar theory of double refraction. Neumann's starting point was his earlier theory of 1832 for the propagation of light in crystals, which implied like Cauchy's that the luminous vibrations were in the plane of polarization. Although Neumann recognized that both the total vibration and the total pressure on the separating surface should be continuous in the full mechanical theory, he used the former condition only, together with the conservation of the live force of the transverse vibrations. The truth of the latter assumption, he commented, depended on the truth of its consequences, as the production of longitudinal waves or absorption of the transverse waves possibly impaired it. Neumann thus employed exactly the same boundary conditions as MacCullagh. He worked out their consequences in a thorough algebraic manner, to be contrasted with MacCullagh more geometrical and less exhaustive approach. Read in late 1835, Neumann's memoir appeared in print a little after MacCullagh's results. A quarrel over priority ensued.[16]

Higher principles

In his memoir, MacCullagh deplored the lack of a proper dynamical foundation for his theory. He also pointed to the mysterious compatibility between the boundary conditions he had adopted and Fresnel's laws of propagation. This brought him to suggest:

> Perhaps the next step in physical optics will lead us to those higher and more elementary principles by which the laws of reflexion and the laws of propagation are linked together as parts of the same system.

In December 1839 he proudly announced to the Irish Academy:[17]

> This step has since been made, and these anticipations have been realised. In the present Paper I propose to supply the link between the two sets of laws by means of a very simple theory, depending on certain special assumptions, and employing the usual methods of analytical dynamics.

MacCullagh was aware of Cauchy's first two attempts at reducing optics to the dynamics of a system of molecules interacting through central forces. The first attempt, which has already been mentioned, displeased MacCullagh because it was incompatible with strictly transverse waves and because it implied the production of longitudinal waves. Cauchy's second attempt, published in 1836, involved a return to Fresnel's assumption of a vibration perpendicular to the plane of polarization, which MacCullagh believed to be

[14]MacCullagh 1838, pp. 96–101 (statement and applications of the theorem), 101–9 (proof); 1848, pp. 177–83 (simplified proof). Both proofs are difficult and involve a lot of algebra. The theorem still plays an important role in modern expositions of this topic: cf. Poincaré 1889, pp. 364–8 ("Théorème de Mac-Cullagh … remarquable par son élégance"); Mascart 1889–94, vol. 2, pp. 580–91 (including a simpler proof of the theorem).

[15]MacCullagh 1838; Hamilton, *PRIA* (1837–1838), p. 213 (meeting of 25 June 1838).

[16]Neumann 1832, 1837; MacCullagh 1838, p. 143; 1841a.

[17]MacCullagh 1838, p. 112n; 1848 (read in 1839), p. 145.

incompatible with mechanically acceptable boundary conditions. MacCullagh therefore renounced any recourse to the received theory of elastic solids. Instead he relied on a Lagrangian procedure for deriving both the equations of motion and the boundary conditions. This method presupposes knowledge of the potential energy V as a function of the shift $\mathbf{u}$. MacCullagh selected it in a manner that forbade any compression of the medium and therefore implied the transversal character of any plane wave. Remember that in 1835 he had already given an equation of motion that forbade compression, without yet having adequate boundary conditions. His new approach was meant to solve this difficulty.[18]

MacCullagh justified his choice of the potential in the following manner. He first proved that the three quantities $\partial_y u_z - \partial_z u_y$, $\partial_z u_x - \partial_x u_z$, $\partial_x u_y - \partial_y u_x$ transformed like a vector during a change of coordinates. In modern notation, this vector is $\nabla \times \mathbf{u}$. For a plane wave $\mathbf{u} = \mathbf{u}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$, it is equal to $i\mathbf{k} \times \mathbf{u}$, where $\mathbf{k}$ is the wave vector. It is therefore proportional to Fresnel's displacement $\mathbf{D}$, on which the force $\mathbf{E}$ depends through the elasticity operator K. Knowing $\nabla \times \mathbf{u}$ and this operator, we may draw the plane containing $\mathbf{D}$ and $\mathbf{E}$; and the perpendicular to $\mathbf{D}$ in this plane gives the direction of the wave vector. Therefore, for plane waves of a given wavelength in a crystal, the knowledge of $\nabla \times \mathbf{u}$ is sufficient to determine the wave. From this result, MacCullagh concluded that the potential V was a function of $\nabla \times \mathbf{u}$ alone (for a given crystal). As Stokes later commented, the inference is dubious because a property that holds for plane waves does not need to hold for more general deformations. MacCullagh's choice for V should rather be judged from its consequences, which he developed in a most elegant manner.[19]

For small deformations, MacCullagh approximates the potential by the quadratic form

$$V = \frac{1}{2}(\nabla \times \mathbf{u})\cdot \mathrm{K}(\nabla \times \mathbf{u}), \tag{8}$$

where K is a symmetric operator. He obtains the equations of motion by Lagrange's condition that

$$\int \rho \ddot{\mathbf{u}} \cdot \delta\mathbf{u}\, d\tau + \int \delta V d\tau = 0 \tag{9}$$

for any variation $\delta\mathbf{u}$. The result is

$$\rho\ddot{\mathbf{u}} = -\nabla \times \mathrm{K}\mathbf{D}, \text{ with } \mathbf{D} = \nabla \times \mathbf{u}. \tag{10}$$

[18]Cauchy 1836a, 1839a. On Cauchy's second theory, cf. MacCullagh 1838, p. 92; Whittaker 1951, pp. 133–4. On MacCullagh's dynamical theory, cf. Whittaker 1951, pp. 142–5; Schaffner 1972, pp. 59–66, 187–93; Stein 1981, pp. 310–15; Buchwald 1985, pp. 283–4; Hunt 1991, pp. 9–10; Darrigol 2000, pp. 190–2.

[19]MacCullagh 1840, 1841b, 1842; 1848 (read in 1839), sections 2–3; Stokes 1862, p. 178: "This reasoning, which is somewhat obscure, seems to me to involve a fallacy." The interpretation of $\nabla\times\mathbf{u}$ as twice the rotation of an element of the medium was still unknown; it was first given by Cauchy in 1841, Stokes in 1849, and Helmholtz in 1858: cf. Darrigol 2005, p. 149n.

These equations imply $\nabla \cdot \ddot{\mathbf{u}} = 0$, so that only transverse waves can exist. MacCullagh then solved the equation for transverse plane waves and derived Fresnel's index surface.[20]

In order to derive the boundary conditions at the interface between two different (homogeneous) media, MacCullagh required

$$\int \rho\ddot{\mathbf{u}}\cdot\delta\mathbf{u}_1 d\tau_1 + \int \rho\ddot{\mathbf{u}}\cdot\delta\mathbf{u}_2 d\tau_2 + \int \delta V_1 d\tau_1 + \int \delta V_2 d\tau_2 = 0 \tag{11}$$

for any variations $\delta\mathbf{u}_1$ and $\delta\mathbf{u}_2$ such that $\delta\mathbf{u}_1 = \delta\mathbf{u}_2$ at the interface between the two media (labeled 1 and 2). Integration by parts of the two last integrals yields

$$\int (\rho\ddot{\mathbf{u}}_1 + \nabla \times \mathrm{K}_1\mathbf{D}_1)\cdot\delta\mathbf{u}_1 d\tau_1 + \int (\rho\ddot{\mathbf{u}}_2 + \nabla \times \mathrm{K}_2\mathbf{D}_2)\cdot\delta\mathbf{u}_2 d\tau_2 + \int (\mathrm{K}_1\mathbf{D}_1 - \mathrm{K}_2\mathbf{D}_2)\cdot(\delta\mathbf{u}_1 \times d\mathbf{S}) = 0 \tag{12}$$

where the last integral is taken over the interface between the two media. The two first integrals vanish owing to the equations of motion. The last one vanishes if and only if the component of the vector **KD** parallel to the interface is continuous. MacCullagh then showed that this boundary condition, together with the continuity of **u**, implied the same laws of refraction as those derived in his memoir of 1837. The new derivation was simpler, because the new boundary conditions were linear whereas the earlier ones involved the conservation of a quadratic quantity (the live force).[21]

MacCullagh concluded his memoir with the following remarks:

> In this theory, everything depends on the form of the function V; and we have seen that, when that form is properly assigned, the laws by which crystals act upon light are included in the general equation of dynamics. This fact is fully proved by the preceding investigations. But the reasoning which has been used to account for the form of the function is indirect, and cannot be regarded as sufficient, in a mechanical point of view. It is, however, the only kind of reasoning which we are able to employ, as the constitution of the luminiferous medium is entirely unknown.

MacCullagh thus admitted that his theory could not pass for a full mechanical reduction of optics. A year before he took his life, he confided to John Herschel:[22]

> With respect to the question which you have put regarding my notions of the constitution of the ether, I must confess that I am not able to give any satisfaction—I have thought a good deal (as you may suppose) on the subject—but have not succeeded in acquiring any *definite mechanical* conception—i.e. such a conception that would lead directly to the form of my function V, and would of course include the actual laws of the phenomena. One thing only I am persuaded of, is that the constitution of the ether, if it ever would be discovered, will be found to be quite different from any thing that we are in the habit of conceiving, though at the same time very simple and very beautiful. An elastic medium composed of points acting on each other in the way supposed by Poisson and others, will not answer.

[20] MacCullagh 1848, pp. 157–60. The present V corresponds to MacCullagh' $-V$. Today's physicists use a procedure similar to MacCullagh's to derive the index surface from Maxwell's equations in anisotropic media.

[21] MacCullagh 1848, section 5. MacCullagh restricted his reasoning to the case when one of the media is isotropic.

[22] Ibid., p. 184; MacCullagh to John Herschel, October 1848, in Scaife 1990, pp. 76–7.

Green's ether

In basing optical theory on a macro-scale potential function, MacCullagh was possibly inspired by a powerful memoir that a miller turned mathematician, George Green, had published a few months earlier.[23] Green's method consisted in combining a variant of Cauchy's macroscopic stress/strain approach to elasticity theory with Lagrangian dynamics. Green thus wanted to avoid molecular reduction and to benefit from the manner of deriving boundary conditions that Lagrange had inaugurated in his fluid mechanics. Green showed that the most general form of the potential V for an isotropic elastic solid was

$$V = \frac{1}{4}K'(\partial_i u_j + \partial_j u_i)(\partial_i u_j + \partial_j u_i) + \frac{1}{2}K''(\partial_k u_k)^2. \tag{13}$$

The variation of the total potential in a homogenous medium yields

$$\int \delta V \mathrm{d}\tau = \int \sigma_{ij}\partial_j \delta u_i \mathrm{d}\tau = -\int \delta u_i \partial_j \sigma_{ij} \mathrm{d}\tau + \int \delta u_i \sigma_{ij} \mathrm{d}S_j, \tag{14}$$

with

$$\sigma_{ij} = K'(\partial_i u_j + \partial_j u_i) + K''\,\delta_{ij}\partial_k u_k. \tag{3}$$

According to Lagrange's general law of dynamics, the first integral yields the equation of motion $\rho\ddot{u}_i = \partial_j \sigma_{ij}$; the second integral, together with the condition that the shifts $\mathbf{u}$ should be equal on both sides of a separating surface, yields the boundary condition that the pressure $\sigma_{ij}\mathrm{d}S_j$ across this surface should be the same on both sides of the surface. Green applied these boundary conditions to the determination of the intensities of reflected and refracted rays. Like Fresnel, he assumed equal elasticities of the two media and vibrations perpendicular to the plane of polarization. He eliminated the longitudinal waves by assuming a very large value of their velocity $\sqrt{(2K' + K'')/\rho}$. The result agreed with Fresnel's sine law but not with his tangent law.[24]

MacCullagh's dynamical theory failed to discourage attempts to represent the ether as an elastic solid. In the year 1839, in which it was read, there appeared two influential attempts of this kind by Green and by Cauchy. Green's relevant memoir dealt with the propagation of light in crystals conceived as elastic solids. In conformity with the method of his memoir of 1837, he determined the most general form of the potential, which

[23]Green 1838. Reference to Green is found in MacCullagh 1840, 1841b, pp. 378–9 (possibly written with Hamilton); Whittaker (1951, p. 142) assert the connection; Stokes (1862, p. 177) and Larmor (1893, p. 340; 1894, pp. 435n–436n) deny it. Cf. Darrigol 2010b, p. 152.

[24]Green 1838. Cf. Whittaker 1951, pp. 139–42. On Lagrange's method, cf. Darrigol 2005, pp. 28, 112. Instead of K' and K'' Green used the constants $A = 2K'+K''$ and $B = K'$ on which the velocities of longitudinal and transverse waves depend. His notation for Lagrange's potential V was $-\phi$. MacCullagh preserved Lagrange's letter choice but adopted Green's sign choice.

involves 21 elastic constants (for there are 21 terms in a quadratic form of six variables) and six more if the solid is permanently strained by extraneous pressure.[25]

Like Cauchy, Green restricted this freedom by requiring symmetry with respect to three mutually orthogonal planes. Unlike Cauchy, he did not assume reducibility to central forces acting in pairs of molecules. This additional freedom enabled him to choose the constants so that strictly transverse vibrations could occur. He then considered two possibilities: no initial stress and vibration in the plane of polarization (as in Cauchy's first theory), initial stress and vibration perpendicular to the plane of polarization (as in Cauchy's second theory). In both cases, Green managed to retrieve Fresnel's wave surface (with an additional sheet for the longitudinal waves). As he himself noted, the first option contradicted his own theory of refraction (in the isotropic case). He did not explore the laws of refraction in the second theory.[26]

The labile ether

In a series of notes that he began to publish in the same year 1839, Cauchy showed that for small negative values of the elastic constant $2K'+K''$ (which determines the velocity of longitudinal waves), the boundary conditions at the interface between two isotropic elastic solids could be satisfied by a system of three transverse waves (incident, reflected, and refracted) and two "evanescent" longitudinal waves confined to the vicinity of the interface. He also showed that in the limit in which this constant vanishes, Fresnel's sine and tangent formulas held strictly. He nonetheless favored a small non-vanishing value that he believed to be corroborated by Jules Jamin's measurements of the polarization of reflected light. The limit of vanishing $2K'+K''$ has some resemblance with MacCullagh's theories of 1835 and 1839, for it yields the same equation of motion:

$$\rho\ddot{\mathbf{u}} = K\Delta\mathbf{u} - K\nabla(\nabla\cdot\mathbf{u}) = -K\nabla\times(\nabla\times\mathbf{u}). \tag{6}$$

Cauchy's assumptions were otherwise similar to Fresnel's and Green's: he set the vibrations of optical waves perpendicular to the plane of polarization; he assumed equal elasticities and different densities in different media; and his boundary conditions were equivalent to Green's conditions once the evanescent longitudinal vibrations were taken into account.[27]

[25]When the medium is initially stressed, the potential energy of the total deformation is a quadratic form of the total deformation $e^0_{ij} + e_{ij}$. This form contains a non-vanishing linear term $a_{ij}\, e_{ij}$. The exact expression of the strain tensor e_{ij} being $\partial_i u_j + \partial_j u_i + \partial_i u_k \partial_j u_k$, there is an additional second-order contribution $a_{ij}\partial_i u_k \partial_j u_k$ to the effective potential of the additional deformation, with six independent coefficients.

[26]Green 1839. Cf. Whittaker 1951, pp. 148–51. See below, p. 236, for Kirchhoff's elaboration of Green's first option.

[27]Cauchy 1839c, 1840, 1848, 1849a, 1849b; Jamin 1848a, 1848b, 1850. Cf. Saint-venant 1872, p. 355; Glazebrook 1885, p. 165; Poincaré 1889, pp. 353–8; Wangerin 1909, p. 50 (German developments of Cauchy's idea); Whittaker 1951, pp. 145–7. Cauchy had his own, intricate approach to the boundary conditions, and he rejected the Lagrangian approach: cf. Saint-venant 1872, pp. 346–7; Chappert 2004, pp. 222–9. On Jamin's measurements and their interpretation, cf. Glazebrook 1885, pp. 187–8. Ludvig Lorenz (1860b) later explained the departure from Fresnel's sine and tangent formulas by the finiteness of the translation layer in which the optical index varies from its first-medium to its second-medium value.

In Cauchy's new ether, the pressure $K'' + \frac{2}{3}K'$ induced by cubic compression is negative (tending to increase the contraction). This strange feature seems to imply instability. Green had earlier rejected the choice $2K'+K'' = 0$ for this reason, and so did all British theorists of optics until William Thomson, as late as 1888, argued that for this choice the instability did not occur when the elastic body was unlimited because the total energy of the medium was essentially positive.[28] As this energy then only depends on the curl of the shift (so that equilibrium is indifferent to compressions), he called the resulting ether *labile*. In order to show the compatibility of this ether with Fresnel's sine and tangent formulas, he calculated the relevant amplitude ratios in the case of a small positive value of $2K'+K''$, and took their limit when this value reached zero. In this limit, the energy carried by the longitudinal reflected and refracted waves vanishes. A few months later, Richard Glazebrook extended the labile-ether theory to anisotropic media by tracing the anisotropy to different inertia in different directions. This scheme yielded the same laws as the Neumann–MacCullagh theory, or as the growingly popular electromagnetic theory of light.[29]

The dominance of the Cauchy–Green theory

These developments show that elastic-body theories of the ether à la Cauchy-Green remained popular in Britain long after MacCullagh dismissed them. One reason was the attractiveness of a thoroughly mechanical reduction. Toward the middle of the century, the rising star of British optics, George Gabriel Stokes, defended a view of the ether that made it a hybrid of Fresnel's and Green's ethers: he assumed that the resistance of this medium to compression was much higher than its resistance to distortion, and he restricted the latter resistance to the minute optical vibrations in order to avoid interference with the motion of matter. He illustrated this view with a jelly made of glue dissolved in a little water.[30]

Neumann and MacCullagh were fairly isolated in their assumption that luminous vibrations were in the plane of polarization. Neumann's disciple Gustav Kirchhoff was the only major physicist to share this assumption, in a theory that amounts to the first alternative of Green's theory of propagation in crystals complemented with MacCullagh's boundary conditions. Around 1850, Stokes argued that the polarization-dependence of the intensity of diffracted light confirmed Fresnel's contrary hypothesis as long as the orientation of the vibration was regarded as the direct cause of this dependence. He also

[28]This results from the expression (13) of the potential V and from the identity $(\partial_i u_j + \partial_j u_i)^2 - 4\partial_i u_i \partial_j u_j = 2(\nabla \times \mathbf{u})^2 + 4\partial_i(u_j\partial_j u_i - u_i\partial_j u_j)$ (the volume integral of the derivative term vanishes).

[29]Thomson 1888; Glazebrook 1888. Cf. Whittaker 1951, pp. 146, 157–8. The equation of motion of the labile ether with anisotropic density $[\rho]$ is: $[\rho]\ddot{\mathbf{u}} = -K\nabla \times (\nabla \times \mathbf{u})$. It has the same form as the equation $\mu[\varepsilon]\ddot{\mathbf{E}} = -\nabla \times (\nabla \times \mathbf{E})$ in Maxwell's theory. Therefore, the consequences are the same for the propagation of light in crystals. The boundary conditions are also the same, granted that these equations remain valid when $[\rho]$ or $[\varepsilon]$ vary from place to place (see note 34 below).

[30]Stokes 1845, p. 128; 1848, pp. 12–13.

argued that the observed polarization of light scattered by particles that are small compared with the wavelength agreed with this hypothesis.[31]

In a British Association report of 1862, Stokes struck what then seemed to be the final blow to MacCullagh's theory. In the isotropic case, the variation of MacCullagh's potential during the variation $\delta\mathbf{u}$ of the shift $\mathbf{u}$ may be written as

$$\delta V = \int \varepsilon_{ijk} E_k \partial_i \delta u_j \mathrm{d}\tau, \text{ with } \mathbf{E} = K\nabla \times \mathbf{u}, \tag{15}$$

where ε_{ijk} is the totally antisymmetric unit tensor. As this variation should be equal to the work

$$\delta W = \int \sigma_{ij} \partial_i \delta u_j \mathrm{d}\tau \tag{16}$$

of the pressures acting on the surface of each volume element, the stress system must be

$$\sigma_{ij} = \varepsilon_{ijk} E_k. \tag{17}$$

The antisymmetry of this stress system contradicts the general theory of elasticity. It implies that each volume element is subjected to a torque that cannot be balanced by the torque of any force density (inertial or external), because the former torque is of third order and the latter of fourth order with respect to the size of the element. The equality of action and reaction thus seems violated. MacCullagh's theory, Stokes concluded, "leads to consequences absolutely at variance with dynamical principles." Among other oddities, "the work stored up in an element of the medium would depend, not upon the change of form of the element, but upon its angular displacement in space." Indeed this work depends on $\nabla \times \mathbf{u}$, which Stokes interpreted as the angular displacement of the element (this interpretation was unknown when MacCullagh wrote his memoir).[32]

In 1871, John William Strutt (soon Lord Rayleigh) gave an additional reason to reject MacCullagh's ether: the equality of the ether density in all media led to wrong polarizing angles for light reflected at the interface between two transparent (isotropic) media. This conclusion is incorrect as it involves boundary conditions (inspired from elasticity theory) that are incompatible with those assumed by MacCullagh and Neumann. William Thomson nonetheless approved it in his Baltimore lectures of 1884. There he denounced what he believed to be "MacCullagh's mistake" of using boundary conditions that were not consistent with Green's. After acknowledging the *bête noire* of the longitudinal waves in Green's theory, Thomson complained that "MacCullagh and Neumann killed the animal with bad treatment." In the same vein, he said: "MacCullagh is a very clever and able man, but he ignored dynamics vitally in the most peculiar parts of his work." For Thomson as for Stokes the compliance with dynamics required a consistent mechanical model of the

[31] Kirchhoff 1876; Stokes 1851; 1852, pp. 530–2. On Stokes's arguments, cf. Whittaker 1951, pp. 154–5, and below pp. 275–6. Ludvig Lorenz (1860a) confirmed the diffraction-based argument.

[32] Stokes 1862, pp. 195–6, 197 (citation).

ether in the spirit of Green's theory. Thomson tried many such models during his long life, whereas MacCullagh early felt that no such model would ever be found.[33]

Lorenz's and Poincaré's agnosticism

In 1863, the Copenhagen physicist Ludvig Lorenz astutely remarked that the equation for the propagation of light in a heterogeneous medium could be derived from the convention that the vibration **w** is perpendicular to the plane of polarization and from three quasi-empirical facts: the validity of the wave equation $n^2c^{-2}\ddot{\mathbf{w}}=\Delta\mathbf{w}$ in a homogenous medium, the transversal character of the waves expressed as $\nabla\cdot\mathbf{w} = 0$, and the validity of Fresnel's sine and tangent formulas for the relative intensities of reflected rays. Treating the heterogeneous medium as a stack of homogenous slices and summing the waves multiply reflected and refracted at the various interfaces, he found that the resultant vibration verified the equation

$$n^2(\mathbf{r})c^{-2}\,\ddot{\mathbf{w}} = \Delta\mathbf{w} - \nabla(\nabla\cdot\mathbf{w}) = -\nabla\times(\nabla\times\mathbf{w}) \tag{18}$$

at a scale largely superior to the thickness of the slices. Lorenz then derived the boundary condition at the frontier between two homogenous media by treating this frontier as a zone of rapid but continuous change of the index $n(\mathbf{r})$ in which the former equation still applied. The parallel components of **w** and $\nabla\times\mathbf{w}$ must be nearly equal on both side of this zone in order that both sides of the equation of motion retain the same order of magnitude in the transition layer. As Lorenz remarked, these conditions are equivalent to Neumann's (or MacCullagh's) conditions if only Neumann's shift **u** is identified with $\nabla\times\mathbf{w}$.[34]

In 1888, the French mathematician Henri Poincaré taught "the mathematical theories of light" from the Sorbonne chair to which he had been newly elected. For the sake of clarity, he adopted a molecular approach in which the motion of the ether was deduced from the total potential of molecular forces. Unlike earlier lattice theorists, he did not restrict this potential to two-body forces, so that the macroscopic equation of motion included Green's additional terms and even terms representing optical rotation. Like Lorenz, Poincaré treated the boundary between two homogenous media as a transition layer in which the equation of motion preserved its form (with rapidly variable constants), and derived the boundary conditions from this picture. Further noting that the equations of motion of the various received theories only differed by substitutions that preserved their form as well as the distribution of luminous intensity, he asserted the full equivalence of these theories:

[33]Rayleigh 1871; Thomson 1884, pp. 170, 180, 204. On Rayleigh's criticism, cf. Larmor 1893, p. 342 (who contradicts Rayleigh); Glazebrook 1885, p. 169, and Schaffner 1972, p. 65 (who both approve Rayleigh). On Thomson's ether models, cf. Smith and Wise 1989, pp. 482–7.

[34]Lorenz 1863, 1864. Cf. Kragh 1991. The continuity of **u** corresponds to the continuity of $\nabla\times\mathbf{w}$; the continuity of the parallel component of $K\nabla\times\mathbf{w}$ corresponds to the continuity of the parallel component of $\ddot{\mathbf{w}}$ or **w**. Lorenz's equation of motion and boundary conditions are formally similar to those of the labile ether theory, and they correspond to the electromagnetic equation $\varepsilon\mu\ddot{\mathbf{E}} = -\nabla\times(\nabla\times\mathbf{E})$.

> In the study of each phenomenon, we have expounded in parallel several theories which account equally well for the observed facts ... We should not deplore the impossibility of making a choice. This impossibility shows us that the mathematical theories of physical phenomena must be regarded as research instruments. Precious though they are, these instruments should not enslave us, and we must reject them as soon as they are formally contradicted by experiment.

Poincaré thereby denied that diffraction, scattering, or photochemical phenomena could decide between Fresnel's and MacCullagh's choices for the direction of vibration. As we will see in a moment, he found flaws in Stokes's and Lorenz's relevant reasoning. Closer to the end of the century, he rejected Otto Wiener's claim that the photographic observation of polarized stationary light waves confirmed Fresnel's choice of the direction of vibration.[35]

In a sequel to his optical lectures, Poincaré adopted the macroscopic, Cauchy–Green approach to elasticity theory. He thus made clear that the molecular hypothesis he had earlier adopted was only what he later called "an indifferent hypothesis." Generally, he purported to demonstrate that the bitter conflicts between the adepts of various optical theories depended on purely conventional elements such as the choice of the direction of vibration and unobservable, fine-scale properties of ether or matter. He paid no attention to the fact that MacCullagh's theory was the only one in which the equations of motion and the boundary conditions derived from least action. And he did not worry about the instability of the labile ether. Mathematical structure, rather than metatheoretical principles, guided his exploration of nineteenth-century ether theories.[36]

6.2 The electromagnetic theory of light

Optics and electricity

At the beginning of the century Thomas Young speculated that the same ether could be responsible both for electric action and for light. He even suggested an experimental test based on the electric alteration of refraction by a fluid. Faraday similarly searched for effects of electric and magnetic fields on the propagation of light. He succeeded in 1845 in the magnetic case. He regarded this finding as a proof of the intimate connection between light and magnetism, and he soon speculated that light was a transverse vibration of the lines of force which he believed to be physically responsible for electric and magnetic actions. Around the same time, Wilhelm Weber revived Ampère's older speculation that the (optical) ether would be a neutral compound of positive and negative electricity.[37]

Some ten years later Weber and Rudolf Kohlrausch measured the ratio C between the electrodynamic and electrostatic units of electric charge and found it to be of the same order as the velocity of light. At the same time Kirchhoff proved by theoretical means that

[35]Poincaré 1889, pp. 398–9 (citation); Wiener 1890; Poincaré 1893, p. 62 (on Wiener). Cf. Whittaker 1951, p. 328. Poincaré nonetheless approved Wiener's conclusion that in the electromagnetic theory of light chemical activity depended on the electric vector.

[36]Poincaré 1893. Cf. Chappert 2004, pp. 180–4.

[37]Young 1800a, p. 79 (see chap. 5, p. 169); Faraday 1839–1855, vol. 3, series 19 [1845]; Faraday 1846; Weber 1846, pp. 167–70. Cf. Darrigol 2000, pp. 64, 96–7, 112–13, and further reference there.

the velocity of electric waves along wires was $C/\sqrt{2}$, or 3.1×10^8 m/s according to Weber and Kohlrausch's measurement of C. Kirchhoff noted without comment the coincidence of this number with the known velocity of light in vacuum or air. Weber judged this finding to be an insufficient starting point for a unified theory of light and electricity, as the propagation mechanisms evoked in the two cases were too disparate. Indeed for Kirchhoff and Weber the fundamental interaction between two particles of electricity was instantaneous, and the finite-velocity of propagation of electric disturbance along wires resulted from the combined effect of the distributed inductance and capacitance of the wire; in contrast, the propagation of light was akin to the propagation of waves in elastic media. In 1858 the mathematician Bernhard Riemann privately removed this asymmetry by introducing retardation in the fundamental electric force law.[38]

Maxwell on physical lines of force

While in Germany, Weber, Neumann, and Kirchhoff were developing a mathematical electrodynamics based on direct action at a distance; in Britain William Thomson and James Clerk Maxwell were busy expressing Faraday's fields in a mechanical and mathematical form. By 1861 Maxwell had a mechanical model of the electromagnetic ether that reproduced the known laws of electromagnetic forces and induction (for quasi-stationary currents). This model involves hexagonal cells separated by idle wheels which roll without slipping between the cells (see Fig. 6.1). A translation velocity of the idle wheels means an electric current **j**, a rotation of the cells a magnetic field **H**. The rolling of the wheels between cells of unequal rotation requires

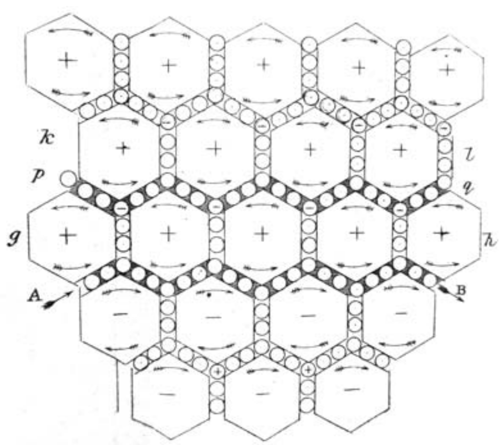


Fig. 6.1. Maxwell's vortex model of the ether. From Maxwell 1861–1862, plate.

[38]Weber and Kohlrausch 1856; Kirchhoff 1857; Riemann [1858]. Cf. Darrigol 2000, pp. 66, 71–2, 211. The electrostatic unit of charge is the unit for which the coefficient in Coulomb's law is *one*; the electrodynamic unit does the same for Ampère's law.

$$\mathbf{j} = \nabla \times \mathbf{H}, \tag{19}$$

which is the differential form of Ampère's theorem. The idle wheels exert a torque $-\nabla\times\mathbf{E}$ on the cells, if $\mathbf{E}$ denotes the average force from the cells on the idle wheels (that is, the electromotive force). Equating this torque with the time derivative of the angular momentum $\mu\mathbf{H}$ (where μ represents the inertia of the matter of the cells) yields

$$\nabla \times \mathbf{E} = -\frac{\partial \mu \mathbf{H}}{\partial t}, \tag{20}$$

which is the differential expression of Faraday's law of induction.[39]

As long as the cells preserve their (hexagonal) shape during rotation, no accumulation of the idle wheels is possible and no electric charge can develop. In symbols, $\mathbf{j} = \nabla \times \mathbf{H}$ implies $\nabla\cdot\mathbf{j} = 0$. Maxwell removed this limitation and included electrostatics a few months later, by allowing an elastic deformation of the cells under the pressure of the idle wheels. This deformation implies an average displacement $-\varepsilon\mathbf{E}$ of the adhering idle wheels, where ε is the elastic constant of the cells. Consequently the relation (19) between magnetic field and (conduction) current must be replaced with

$$\mathbf{j} = \nabla \times \mathbf{H} - \frac{\partial \varepsilon \mathbf{E}}{\partial t}. \tag{21}$$

Equations (20) and (21) are the fundamental equations of Maxwell's electromagnetic theory.

The divergence of equation (21), compared with the conservation of electricity

$$\nabla\cdot\mathbf{j} + \frac{\partial \rho}{\partial t} = 0, \tag{22}$$

leads to $\rho = \nabla\cdot(\varepsilon\mathbf{E})$ for the electric density, and to $(1/4\pi\varepsilon)\rho\mathrm{d}\tau\rho'\mathrm{d}\tau'\,\mathbf{r}/r^3$ for the force between two point charges $\rho\mathrm{d}\tau$ and $\rho'\mathrm{d}\tau'$ separated by the vector $\mathbf{r}$. The centrifugal pressure of the rotating cells leads to $\mathbf{j}\times\mu\mathbf{H}$ for the electromagnetic force density, and to $(\mu\,/4\pi)\mathbf{j}\mathrm{d}\tau\times(\mathbf{j}'\mathrm{d}\tau'\times\mathbf{r}/r^3)$ for the force between two current elements $\mathbf{j}\mathrm{d}\tau$ and $\mathbf{j}'\mathrm{d}\tau'$ separated by the vector $\mathbf{r}$. Consequently, the ratio between the electromagnetic and electrostatic units of charge, which Weber and Kirchhoff had found to be close to the velocity of light, must be equal to $1/\sqrt{\varepsilon\mu}$. Maxwell then computed the velocity of a transverse vibration propagated in the elastic matter of the cells and found it to be also equal to $1/\sqrt{\varepsilon\mu}$. He concluded:[40]

> We can scarcely avoid the inference that *light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena.*

[39]Maxwell 1861–1862. Cf. Siegel 1991.

[40]Maxwell 1861–1862, p. 500.

Electromagnetic waves

This theory is not an electromagnetic theory of light proper, as it still represents light waves as vibrations in an elastic medium. The modern identification of light waves with transverse electromagnetic waves occurs in Maxwell's memoir of 1865, in which he combines the particular case of equation (21) in the absence of conduction currents,

$$\nabla \times \mathbf{H} = \frac{\partial \varepsilon \mathbf{E}}{\partial t}, \tag{23}$$

with other equations equivalent to the expression

$$\nabla \times \mathbf{E} = -\frac{\partial \mu \mathbf{H}}{\partial t} \tag{20}$$

of Faraday's law of electromagnetic induction, to get the wave equation

$$\Delta \mathbf{H} - \varepsilon\mu \frac{\partial^2 \mathbf{H}}{\partial t^2} = 0. \tag{24}$$

The magnetic field therefore propagates at the velocity $1/\sqrt{\varepsilon\mu}$. For plane waves, equations (20) and (23) imply that the electric and magnetic vectors are perpendicular to each other and perpendicular to the direction of propagation of the waves.[41]

This electromagnetic theory of light became known mostly through Maxwell's *Treatise* of 1873 and through a memoir of 1870 in which the great German physicist Hermann Helmholtz reinterpreted Maxwell's equations in terms of direct action at a distance between ordinary electricity and the displaced electricity of an infinitely polarizable ether. In an influential footnote, Helmholtz pointed out that these equations gave the correct boundary conditions for the reflection and refraction of light at the frontier between two media. Maxwell's electromagnetic theory was well established in Britain even before the famous experiments of 1887–88 in which Heinrich Hertz produced and analyzed electromagnetic waves by electric means. It rapidly conquered the continent after these experiments.[42]

The persistence of the elastic-body theory

Yet the elastic-body theory of light survived in several forms until the advent of relativity theory. The main reasons for this persistence are Maxwell's and his disciples' failure to provide a credible mechanical model of the electromagnetic ether, the improved status of the labile ether (thanks to Kelvin's efforts), and the belief that optics, being simpler than electromagnetism, was more likely to provide clues about the deeper dynamics of the ether.

[41] Maxwell 1865.

[42] Maxwell 1873; Helmholtz 1870, p. 68n. Cf. Darrigol 2000, chaps. 5–6, and further reference there.

In 1879, the Trinity Dublin theorist George Francis FitzGerald discovered that the equations of motion of MacCullagh's ether corresponded to Maxwell's equations through the relations

$$\mathbf{H} = \dot{\mathbf{u}}, \quad \mathbf{D} \equiv [\varepsilon]\mathbf{E} = \nabla \times \mathbf{u}, \quad [\varepsilon] = \mathrm{K}^{-1}, \quad \mu = \rho. \tag{25}$$

This correspondence enabled him to find the electromagnetic boundary conditions at the interface between two homogenous dielectrics. FitzGerald used this quasi-mechanical interpretation of Maxwell's equations for devising new theories of two magneto-optical effects: the rotation of the plane of polarization of light in a magnetized medium (Faraday effect), and the effect of a magnetic field on the polarization of metallically reflected light (magnetic Kerr effect). He thereby benefited from two related advantages of MacCullagh's theory over Maxwell's: its field-Lagrangian form (which eased the treatment of the Faraday effect) and its inclusion of the boundary conditions (which needed to be known for the Kerr effect).[43]

In the end, FitzGerald's memoir could be either interpreted as a further success of the electromagnetic theory or as a belated victory of MacCullagh's pseudo-mechanical theory. In his review of FitzGerald's submission, Maxwell recommended the first interpretation:

> If he has succeeded in explaining Kerr's phenomena as well as Faraday's by the purely electromagnetic hypothesis, the fact that he has done so ought to be clearly made out and stated, for it would be a very important step in science.

Maxwell did not live to read FitzGerald's printed compliance to this request:

> The investigation is put forward as a confirmation of Professor Maxwell's electromagnetic theory of light ... If it induced us to emancipate our minds from the thralldom of a material ether, [it] might possibly lead to important results in the explanation of nature.

FitzGerald did not mean to exclude every mechanical explanation of the ether. Rather he wished to condemn any ether theory that drew on analogy with ordinary matter. He later developed a "vortex sponge" theory in which the ether became a dense and complex arrangement of vortex filaments in a perfect liquid. This sponge was supposed to share the MacCullagh kind of rotational elasticity.[44]

In 1889, William Thomson discovered that he could produce purely rotational elasticity by a clever arrangement of gyrostats. In this model, gyrostatic reaction provides for the uncompensated elastic torque that Stokes deplored in MacCullagh's theory. Thomson thus intended to represent the labile ether. He did not mention MacCullagh in this context, presumably because he still believed that MacCullagh's choice of the direction of vibration (in the plane of polarization) was incompatible with the laws of reflection.

[43]Fitzgerald 1880, p. 699. Cf. Hunt 1991, pp. 15–21; Darrigol 2000, pp. 190–2.

[44]Maxwell to Stokes, 6 February 1879, in Stokes 1907, p. 43; Fitzgerald 1880, p. 711. Cf. Stein 1981; Hunt 1991, pp. 21–3.

It was another Irishman, Joseph Larmor, who first connected Thomson's gyrostatic model with MacCullagh's ether.[45]

In 1893, Larmor reported on magneto-optical phenomena for the British Association. He seized this opportunity to review the various theories of light propagation and to promote MacCullagh's "*quasi*-mechanical" theory, "an extremely powerful investigation, which was independent of and nearly contemporary with those of Green, and I think, of at least equal importance."[46] In Larmor's eyes, the only defect of MacCullagh's theory was the lack of a mechanical model of the rotational elasticity. Having found the desired model in Thomson's gyrostatic ether, Larmor was fully convinced that the rotational ether should be the true basis of electrodynamics and optics. Broadly speaking, Larmor shared with William Thomson the idea that optics was more fundamental than electromagnetism and that the latter should be reduced to the former rather than vice-versa. He also admired Thomson's theory of atoms as vortices in a perfect liquid. He therefore built vortices in MacCullagh's medium as a tentative representation of the interaction of ether and matter. He thus hoped to solve a number of difficulties of the electromagnetic theory of light concerning the optics of moving bodies, magneto-optical phenomena, and optical dispersion.[47]

In 1895, the failure of this vortex approach led Larmor to introduce point-like singularities in MacCullagh's ether. These were centers of radial twist, carrying electric charge and freely circulating through the ether. He called them "electrons," the name that George Johnstone Stoney had given to the electrolytic quantum of electricity. Larmor's theory soon became similar to Hendrik Lorentz's contemporary ionic theory, except that Larmor and his disciples regarded MacCullagh's ether and its singularities as the true foundation of the theory, whereas Lorentz did not favor any mechanical interpretation of the electromagnetic field.[48]

6.3 The separation of ether and matter

In the theories described in the previous section, matter intervened only as a circumstance modifying the characteristic constants of the ether. This is also true for Maxwell's electromagnetic theory of light, which treated ether and matter as a single medium characterized by common parameters of dielectric permittivity, magnetic permeability, electric conductivity, and common velocity when moving matter was involved. This simplified picture worked well for the theories of propagation, reflection, and refraction of monochromatic light in bodies at rest. Other phenomena including the optics of moving bodies, dispersion, optical rotation, and magneto-optic effects turned out to require a more refined

[45]Thomson 1890. Cf. Whittaker 1951, p. 145; Lorentz [1901–1902], pp. 40–52; Schaffner 1972, pp. 71–4. Heaviside 1991 referred to Thomson's "rotational ether" without mentioning MacCullagh. He discussed the electromagnetic interpretation of this ether (without mentioning FitzGerald) and judged it impossible to introduce dissipation in this ether in a way that would represent electric conduction (cf. Buchwald 1985, pp. 68–70).

[46]Larmor 1893, p. 340; Larmor 1894, pp. 415–24. Ibid. on p. 426, Larmor interprets MacCullagh's vector as a rotation; on pp. 419–36 he gives a history of optics with emphasis on MacCullagh's role.

[47]Larmor 1894. Cf. Buchwald 1985, pp. 141–53; Hunt 1991, pp. 210–17; Darrigol 2000, pp. 335–7.

[48]Larmor 1895a, 1895b. Cf. Buchwald 1985, pp. 155–71; Hunt 1991, pp. 217–22; Darrigol 2000, pp. 337–43.

consideration of the interplay between ether and matter. This section deals with some of this evolution.

The measure of dispersion

Since John Dollond's invention of the achromatic telescope, the refractive and dispersive powers of various sorts of glass had become quantities of great practical interest. To be true, most instrument makers long relied on crude empirical methods in which a precise determination of the dispersive power was unnecessary. This state of affairs began to change in the early nineteenth century. In 1802, Wollaston constructed a device that automatically measured the optical index of a substance through the angle of total reflection in a flint cube placed on this substance. In 1813, Brewster invented another convenient method based on the disappearance of a sample of the investigated glass when immersed in a liquid mixture of variable index. He also considerably improved the older way of determining the refractive and dispersive powers through a refracting prism. Dispersion measurement remained vague, however, as there was no precise way to isolate one component of the spectrum.[49]

A Bavarian telescope maker, Joseph Fraunhofer, soon made the most decisive progress in this department of experimental optics. While measuring the index of a glass prism for a given simple color selected through a prism of flint glass, he discovered that the spectrum produced by the latter prism involved a doublet of bright lines when he used a burning lamp as a source, and a numerous series of dark lines when he used sunlight as a source. He labeled the most distinct lines with capital letters of the alphabet, as we still do (for instance, he called D what is now regarded as the sodium doublet). He soon found out that electric light involved better defined bright lines, and that the light from the stars had its own dark lines. These discoveries depended on his superior technique of measurement, which involved flint glass of unprecedented homogeneity, and a telescope mounted on a theodolite for the observation of the spectrum. The spectral lines provided a natural reference for measuring the optical index for well-defined simple colors. Fraunhofer published these results in 1817. A few years later he completed them with amazingly precise determinations of the lines' wavelengths through gratings of his own. He adopted the wave theory of light in this occasion.[50]

Fraunhofer's dispersion measurements were only one of the three components of his success in making the best refracting telescopes of his time. The two others were a superior glass technology and a good command of geometrical optics. The latter topic he had learned from the *Analytische Dioptrik* which the German mathematician Georg Simon Klügel had published in 1778. Klügel's aim was to extract from the forbiddingly complex reasoning of his predecessors simple results and rules that could be used by the best instrument makers. Euler, to whom he dedicated his treatise, was his main guide although he reproached him with an inclination to excessive generality. He compared d'Alembert's

[49] Wollaston 1802a; Brewster 1813, pp. 273–399. Cf. Buchwald 1980b, 1989; Jackson 2000, pp. 24–31.

[50] Fraunhofer 1817, 1822, 1823. Cf. Jackson 2000, chap. 3. Wollaston (1802a) had observed a few dark lines in the solar spectrum and used them as a natural separation between the colors of the spectrum.

relevant writings to a "thick, overgrown forest" or even to a *Brouillon*, and he decided that the included study of transversal aberration was useless in practice.[51]

On the British side, in 1822 John Herschel similarly condemned the excessive complexity of Clairaut's and d'Alembert's contributions. He tried to make the theory of lenses more useful to instrument makers and he gave precise directions for making achromatic doublets. His subsequent attempt to improve the determination of dispersive powers through the use of filters did not stand comparison with Fraunhofer's. At any rate, no one in Britain was able to make flint glass as pure and homogenous as Fraunhofer's.[52]

The lattice theory of dispersion

While the improvement of achromatic telescopes motivated new measurements of dispersion, the advent of the wave theory of light permitted new theories of dispersion. In the corpuscular theory, dispersion is easily explained by a specific attraction between matter and light corpuscles of a given color. No such easy explanation was available in the wave theory. Euler's idea that successive pulses of light may affect each other's velocity had long been known to be contrary to the theory of wave propagation in a homogenous elastic medium. Young imagined three ways to escape this difficulty: by coupling ethereal and material vibrations, by implying the heterogeneity of the medium, and by analogy with deep-water waves. Fresnel's own suggestion was akin to Young's second option: he argued that the velocity of propagation of light should no longer be a constant when the wavelength was not much larger than the radius of action of the molecular forces in the molecular model of the ether:

> At first glance [the dispersion of light] seems to contradict the results of Mr Poisson's learned calculations about the propagation of sound waves in elastic fluids of different densities; but it must be observed that these general equations are founded on the hypothesis that each infinitely thin slice of the fluid is only repelled by the contiguous slice so that the accelerating force extends only at a distance infinitely small compared to the length of an undulation. This hypothesis is perfectly admissible for sound waves, of which the shortest are still a few millimeters in length; but it could become inexact for light waves, of which the longest do not reach a thousandth of a millimeter. It is quite possible that the sphere of action of the accelerating force that determines the propagation velocity in a refracting medium, or the mutual dependence of the molecules of which it is made, extends to distances that are not infinitely small compared to a thousandth of a millimeter.

Fresnel went on to suggest that the mechanics of this molecular interaction led to a larger velocity for longer waves.[53]

In 1830, Cauchy briefly indicated how this idea worked in his theory of the optical medium as a molecular solid. In his earlier deduction of the net force acting on a molecule of the medium, he had assumed that within the sphere of action of these molecules

[51] Klügel 1778, *Vorrede*, p. 14. Fraunhofer did not share Klügel's opinion about transversal aberration, which his achromatic doublets corrected: cf. Brewster 1827, p. 6.

[52] J. Herschel 1821, 1822, 1823. Cf. Jackson 2000, pp. 31–8.

[53] Fresnel 1822e, pp. 89–90.

their displacement varied so little that it could be replaced by its second-order Taylor approximation. In the simplest case of a deformation ξ parallel to the x axis and depending on the y coordinate only, this approximation leads to the wave equation

$$\frac{\partial^2 \xi}{\partial t^2} = a_2 \frac{\partial^2 \xi}{\partial y^2}. \tag{26}$$

When the assumption is no longer valid, higher-order terms must be retained, leading to the modified equation

$$\frac{\partial^2 \xi}{\partial t^2} = a_2 \frac{\partial^2 \xi}{\partial y^2} + a_4 \frac{\partial^4 \xi}{\partial y^4} + \ldots \tag{27}$$

(the odd-order terms disappear for reasons of symmetry). For plane waves of the form $e^{i(ky - \omega t)}$, this leads to the relation

$$\omega^2 = a_2 k^2 - a_4 k^4 + \ldots \tag{28}$$

through which the optical index $n = ck / \omega$ depends on the wavelength $\lambda = 2\pi / k$.[54]

In the same article, Cauchy announced a different approach based on solving the equations of motion for the discrete lattice. The heavy calculations, which he published in 1836, are much simplified by considering a one-dimensional linear model in which the molecules are connected to each other through perfect equal springs (as Euler had done in the previous century). The equation of motion of the nth molecule is

$$\ddot{x}_n = -\sum_{p=-\infty}^{p=+\infty} \alpha_p (x_n - x_{n+p}). \tag{29}$$

For a wave-like solution of this equation of the form $x_n = e^{i(nka - \omega t)}$, we must have

$$\omega^2 = \sum_{p=1}^{+\infty} \alpha_p (1 - \cos pka). \tag{30}$$

Replacing the cosine with its Taylor development and assuming that the force coefficients α_p decrease fast enough with p so that the sums $\Sigma_{p=1}^{+\infty} \alpha_p p^{2n}$ converge, this leads to a dispersion relation of the form

$$\omega^2 = a_2 k^2 - a_4 k^4 + a_6 k^6 - \ldots \tag{28}$$

[54] Cauchy 1830b. Cf. Whittaker 1951, pp. 163–4.

in which the a_{2n} coefficients are positive. The three-dimensional character of the real problem does not alter the form of this result, which became known as Cauchy's dispersion relation.[55]

A prolific optical writer based in Oxford, Baden Powell, read Cauchy's remarks of 1830 and developed them in his own manner. He first assumed that a molecule interacted only with its closest neighbors. The resulting dispersion formula reads

$$\omega = \sqrt{2\alpha_1}\ \sin\ (ka/2), \tag{31}$$

as can be seen by retaining only the term $p = 1$ in formula (30). Following Hamilton's advice, Powell later favored the French Laplacians' assumption that many molecules are included in the sphere of action of a given molecule, which leads to Cauchy's dispersion formula (28). In 1835 Powell found his formula (31) to agree with Fraunhofer's index measurements for the lines B to G in various glasses and liquids. The following year he performed his own measurement on more dispersive media with a technique similar to Fraunhofer's. The results now confirmed Cauchy's formula. Cauchy independently verified the agreement of his theory with Fraunhofer's measurements. Powell announced a new victory of the wave theory of light. Brewster disagreed: in his opinion, there were so many flaws in Powell's experiments that the wave-based theory of dispersion remained unfounded.[56]

Double-lattice theories of dispersion

On the theoretical side, Cauchy's and Powell's theories had the evident defect of implying dispersion even in the absence of matter. Cauchy believed he could avoid this unwanted dispersion by making the intermolecular force vary as the inverse fourth power of the distance. He did not explain, however, how the presence of matter could lead to a different force law. In 1833 the French theorist of elasticity Gabriel Lamé inaugurated another kind of dispersion theory in which the interplay between ether and matter played a central role. He represented matter as solid molecules spread within a much finer-grained ether. By attraction, these molecules induced a higher concentration of the ether around them. Lamé then studied the propagation of transverse waves through the interstitial ether thanks to a generalization of Cauchy's molecular theory of elasticity to the case of heterogeneous density. Through sophisticated mathematics he found that high-frequency

[55]Cauchy 1836b. Cf. Buchwald (1980a, pp. 251–7) for a more literal rendition of Cauchy's reasoning. John Tovey independently gave a much simpler execution of this method (Tovey 1836). He later noted that the dispersion relation for the lattice was compatible with a discrete series of complex value of k, which he believed to be related to John Herschel's selective absorption (Tovey 1839b, 1840): cf. Buchwald 1981, pp. 227–30.

[56]Powell 1835a, 1835b, 1836a, 1836b; Cauchy 1836b; Brewster 1838. Cf. Buchwald 1981, pp. 225–8; Chen 2000, chap. 4. In 1831, Airy had tried to explain dispersion by a frequency-dependent heating of the medium: cf. Buchwald 1980a, p. 256. Baden Powell fathered Robert Baden-Powell, founder of the scout movement. On Hamilton's powerful but mostly unpublished development of the lattice theory (*MPH*, vol. 1), cf. Hankins 1980, pp. 155–71.

waves were more slowed down than low frequency ones when traveling in the vicinity of the material molecules.[57]

Lloyd's praise of Lamé's attempt in his British Association report of 1834 plausibly convinced some of his British colleagues to examine the role of matter in dispersion theory. In 1836, the Scottish mathematician Philip Kelland traced the dispersion in material substances to the increased interval of the molecules of the interstitial ether. Despite its separation of ether and matter, this attempt was closer to Cauchy's than Lamé's, in which the molecular structure of the ether played no role. The following year Lloyd announced another theory of dispersion based on two interacting molecular lattices for ether and matter. This prompted Cauchy to examine the general mathematical form of this problem.[58]

In 1841, Franz Neumann treated the particular case of this problem in which the spacing of the two lattices is negligible and the molecules of matter are too heavy to be displaced by the ethereal vibrations. The relevant equation of motion of the ether is

$$\ddot{\mathbf{u}} = c^2 \Delta \mathbf{u} + K \nabla (\nabla \cdot \mathbf{u}) - C\mathbf{u}. \tag{32}$$

The resulting dispersion relation of transverse waves is

$$\omega^2 = c^2 k^2 + C, \tag{33}$$

where the positive constant C depends on the matter that acts on the ether. The following year, the Cambridge-trained Irish mathematician Matthew O'Brien published a more detailed theory of the same kind. In a simplified one-dimensional model and in the quasi-continuum limit, his equations of motion read

$$\frac{\partial^2 \xi}{\partial t^2} = \alpha \frac{\partial^2 \xi}{\partial x^2} - \gamma(\xi - \eta), \quad \frac{\partial^2 \eta}{\partial t^2} = \beta \frac{\partial^2 \eta}{\partial x^2} - \delta(\eta - \xi), \tag{34}$$

where ξ is the displacement of the ether molecules and η the displacement of the matter molecules. Solving for waves of the form $\xi = A\mathrm{e}^{\mathrm{i}(\omega t - kx)}$, $\eta = B\mathrm{e}^{\mathrm{i}(\omega t - kx)}$ leads to the relation of dispersion

$$\frac{\gamma}{\omega^2 - \alpha k^2} + \frac{\delta}{\omega^2 - \beta k^2} = 1, \tag{35}$$

which degenerates into formula (33) in the limiting case $\delta = 0$.[59]

[57]Cauchy 1836b, p. 191. Lamé 1833, pp. 333–4; Lamé 1834b; 1834a, p. 271. Ibid., p. 280, Lamé suggested that the Fraunhofer lines might correspond to a sort of resonance of the interstitial ether.

[58]Lloyd 1835, pp. 391–2; Kelland 1836; Lloyd 1837; Cauchy 1839b, pp. 349–50. Kelland (1836, pp. 174–8) judged that in order to match his results with the measured dispersion, the intermolecular force had to vary as the inverse squared distance of the molecules. See Buchwald 1980a, pp. 259–60 on the ensuing controversies with Samuel Earnshaw and Matthew O'Brien.

[59]Neumann 1843 [1841]; O'Brien 1842. Cf. Whittaker 1951, pp. 166–7. As Briot later remarked, Neumann's dispersion relation is incompatible with Cauchy's.

Anomalous dispersion and selective absorption

In Neumann's and O'Brien's theories, the molecular structures of ether and matter are irrelevant to the dispersion phenomenon. The true cause of dispersion becomes the interaction of ether and matter. These theories nonetheless share with Cauchy's the property of making the optical index decrease with the wavelength. Until the 1860s, this property was generally believed to hold in any spectrum. William Fox Talbot's observation, around 1840, of the opposite behavior in the extraordinary beam from a crystal of chromium salt remained private until 1871. In 1862, the French experimenter François Leroux published his observation of the anomalous dispersion of iodine vapor in the *Comptes rendus*. The Danish physicist Christian Christiansen rediscovered this anomaly with a solution of fuchsin in 1870. This prompted Fox Talbot to publish his own observations, and the German experimenter August Kundt to systematically investigate the phenomenon on various dyes. Kundt found it to be associated with "surface color," namely the bright coloration of a thin layer of the substance when exposed to white light.[60]

"Surface color," first observed by John Herschel on sulfate of quinine in 1845, was the object of a thorough investigation by Stokes. In the resulting hundred-page memoir, Stokes established that light of a narrow frequency range was selectively absorbed by the medium while light of a lower (or equal) frequency was reemitted in every direction. Stokes called this phenomenon "internal dispersion" (meaning scattering) in the main text, and "fluorescence" in a footnote because fluorspar enjoyed the property and because phosphorescence similarly implied a change of frequency. He emphasized the violation of Newton's principle according to which light of a given simple color retained this color after traversing any number and kind of substances. He explained the phenomenon as a special kind of resonance, in which material resonators with a specific frequency selectively absorbed light of this frequency and reemitted light of lower frequencies. He vaguely suggested that the lowering of the frequency had to do with the nonlinearity of the oscillators. More influentially, he argued that selective absorption was generally due to the resonant scattering of a specific component of the spectrum. Two earlier investigators of selective absorption, John Herschel and Fox Talbot, had related this phenomenon to the resonant excitation of material oscillators, but in a contrary manner: they assumed, as Mairan had done long ago, that resonance *facilitated* the transmission of light.[61]

In the same vein, in 1854 Stokes explained the coincidence of the yellow line of sodium and the D line of sunlight "by supposing that a certain vibration capable of being excited among the ultimate molecules of certain ponderable bodies, and having a certain periodic time belonging to it, might ever be excited when the body was in a state of combustion, and thereby give rise to a bright line, or be excited by luminous vibrations of the same period, and thereby give rise to a dark line by absorption." Five years later, Kirchhoff and Bunsen made a systematic study of the spectra of pure salts with Bunsen's burner. In this context,

[60]Leroux 1862; Fox Talbot 1871; Christiansen 1870, 1871; Kundt 1871a, 1871b. Cf. Thomson 1898; Chen 1999, pp. 530–5.

[61]J. Herschel 1845b; Stokes 1852, p. 479n ("fluorescence"); J. Herschel 1833; Fox Talbot 1835. Stokes was also aware of relevant experiments by Brewster.

Kirchhoff explained the correspondence between absorption and emission spectra through the theoretical universality of the ratio between the emissive and absorptive powers of any substance at a given temperature. The importance of this law in the history of spectroscopy and in later studies of the blackbody spectrum is well known.[62]

The point of this digression was to show that the correlation of surface color with anomalous dispersion suggested, in the light of Stokes's memoir of 1852, that the latter phenomenon also depended on selective absorption. Kundt indeed observed spectra in which a region of normal dispersion was separated from a region of anomalous dispersion by an absorption line. In turn, this finding suggested that dispersion generally involved a coupling of the ethereal vibrations with material oscillators. Thomas Young had long ago imagined a mechanism of this kind. In 1830, the Cambridge natural philosopher James Challis had developed Young's idea through a model in which the material atoms interspersed in the free ether were subjected to the combined effect of an elastic restoring force and of the incident ether waves; and, reciprocally, the atoms retarded the waves in a manner depending on the phase of their oscillation. An eccentric application of the laws of mechanics led him to the final formula

$$(n^2 - 1)/n = \alpha\rho(\omega^2 - {\omega_0}^2)/\omega, \tag{36}$$

where n is the optical index, ω the angular frequency of the waves, ω_0 the intrinsic pulsation of the atoms, ρ the density of the medium, and α a coupling constant. Challis assumed the optical frequency to be much larger than the material frequency so that dispersion was always normal in his theory.[63]

In a Mathematical Tripos question of 1869, Maxwell had students investigate the vibrations of an elastic medium whose particles were attracted by the atoms of another substance with a force proportional to the distance and with a resistance proportional to the relative velocity. Granted that the atoms do not interact, the equations of motion are

$$\rho\,\ddot{\xi} - E\partial^2\xi/\partial x^2 = -\sigma[{\omega_0}^2(\xi - \eta) + R(\dot{\xi} - \dot{\eta})] = -\sigma\,\ddot{\eta}, \tag{37}$$

where ξ denotes the shift of the medium, η denotes the displacement of the atoms, and the remaining letters stand for characteristic constants of the medium, atoms, and their interaction. Maxwell directed his students to a solution of the form $e^{-x/l}\sin\omega(t - kx)$. In the case of negligible damping ($R = 0$) the resulting relation of dispersion is

$$Ek^2/\omega^2 = \rho + \sigma\omega_0^2/(\omega_0^2 - \omega^2). \tag{38}$$

Maxwell further asked his students for an optical interpretation of this model. It is not clear whether he was aware of Leroux's slightly earlier discovery. Although the resulting index formula

[62] Stokes to Thomson, 1854, in *SMPP* 4, pp. 370–1; Kirchhoff and Bunsen 1860–61; Kirchhoff 1859a, 1859b. Cf. McGucken 1969, pp. 23–34; Jungnickel and McCormmach 1986, vol. 1, pp. 297–302.

[63] Kundt 1871c, 1872; Challis 1830.

$$n^2 - 1 = (\sigma/\rho)\omega_0^2/(\omega_0^2 - \omega^2) \tag{39}$$

entails anomalous dispersion for $\omega > \omega_0$, Maxwell possibly ignored this case as Young and Challis had done earlier.[64]

The first theorist to explicitly and publicly relate anomalous dispersion with material oscillators was Franz Neumann's former student Wolfgang Sellmeier. In the mid-1860s Sellmeier had privately speculated that the material resonators often invoked in selective absorption could also be responsible for dispersion. From this picture he had inferred that dispersion would be anomalous above the absorption frequency; and he had vainly tried to verify this prediction on a solution of fuchsin. His reasoning was based on the behavior of a harmonically driven harmonic oscillator, of which the equation of motion reads

$$\ddot{\eta} + \omega_0^2\eta = \omega_0^2\eta_0 \quad \text{with} \quad \eta_0 = \mathrm{A}\ \sin\omega t. \tag{40}$$

The forced vibration of the material oscillator satisfies

$$\eta = \frac{\omega_0^2}{\omega_0^2 - \omega^2}\eta_0. \tag{41}$$

Through molecular reasoning, Sellmeier argued that the driving shift η_0 was proportional to the ethereal displacement. Comparing this interaction with the coupling of two pendulums, he then showed that the reaction of the material oscillator on the ether increased or decreased the effective elastic response of the ether according as the frequency ω was superior or inferior to ω_0. Through a murky energetic reasoning published in 1872, he lastly derived a formula of the form

$$n^2 - 1 = \sum_{i=1}^{n} \alpha_i \frac{\omega_i^2}{\omega_i^2 - \omega^2} \tag{42}$$

in which the ω_i's are selective absorption frequencies and the α_i's are constants. Helmholtz simplified and improved Sellmeier's theory in 1875 by directly assuming the system

$$\rho\,\ddot{\xi} - E\partial^2\xi/\partial x^2 = -\alpha(\xi - \eta), \quad \sigma\,\ddot{\eta} + b\,\dot{\eta} + \sigma\omega_0^2\eta = \alpha(\xi - \eta) \tag{43}$$

for the coupled vibrations of plane ether waves and material oscillators. The latter include the damping term $b\dot{\eta}$ and the intrinsic restoring force $-\sigma{\omega_0}^2\eta$ for the material vibrations. Later electromagnetic theories of dispersion by Lorentz (1878) and Helmholtz (1893) yielded similar dispersion formulas as they involved the (now electromagnetic) coupling between elastically bound ions (or electrons) and the ether.[65]

[64]Maxwell 1869. Cf. Rayleigh 1899, 1917 (which includes a comparison with Helmholtz's first theory of dispersion).

[65]Sellmeier 1871, 1872a, 1872b; Helmholtz 1875. The double-pendulum analogy is better expressed in Rayleigh 1872. On the electromagnetic dispersion theories, cf. Buchwald 1985, pp. 237–9; Carazza and Robotti 1996; Darrigol 2000, pp. 320–1, 325. See also Ketteler 1893.

MacCullagh on optical rotation

In a first approximation in which the spatial derivatives of order higher than two are neglected, the elastic-solid theory of light implies the rectilinear polarization of plane waves propagated in crystals. In 1836 MacCullagh modified the relevant wave equation so that the polarization could be circular or elliptical, in conformity with Fresnel's theory of optical rotation. Remember that, in this theory, the rotation is explained by the different velocities of the two circularly polarized waves that propagate through the active medium. The wave-number difference δk must be proportional to the squared frequency of the light in order to comply with Biot's experimental law that the optical rotation is proportional to the inverse squared wavelength of the incoming light. From this information, MacCullagh inferred that the modified wave equation should involve third-order spatial derivatives. For the component $\xi(z, t)$ and $\eta(z, t)$ of a transverse plane wave propagating in the direction of the z-axis, he found that the equations

$$\frac{\partial^2 \xi}{\partial t^2} = A\frac{\partial^2 \xi}{\partial z^2} + C\frac{\partial^3 \eta}{\partial z^3}, \quad \frac{\partial^2 \eta}{\partial t^2} = B\frac{\partial^2 \eta}{\partial z^2} - C\frac{\partial^3 \xi}{\partial z^3} \tag{44}$$

correctly represented the phenomena. When the z axis is an axis of symmetry, the coefficients A and B are equal, and there are two solutions of the form

$$\xi = a\mathrm{e}^{\mathrm{i}(\omega t - kz)}, \ \eta = b\mathrm{e}^{\mathrm{i}(\omega t - kz)}, \quad \text{with } \omega^2 - Ak^2 = \pm Ck^3 \text{ and } b = \pm \mathrm{i}a, \tag{45}$$

corresponding to two circularly polarized waves with opposite rotation and with the wave-number difference $\delta k = (C/A^2)\omega^2$.[66]

As MacCullagh knew, one of Cauchy's British followers, John Tovey, had shown that the lattice-ether problem generally led to elliptically polarized waves and he had related the velocity difference of a pair of such waves to Biot's laws of optical rotation. Powell had applauded this result, and Cauchy had written to Powell to claim priority in the discovery of the elliptical polarization of waves propagated in asymmetrical lattices (for which there do not exist three mutually orthogonal planes of symmetry). MacCullagh himself tried this route but he soon realized that it implied the wrong signs for the third-order terms in the twin plane-wave equations (44). He thus arrived at the following proof of the general incompatibility of Cauchy's theory with optical rotation.[67]

Call $\mathbf{r}_\alpha$ the position of the α molecule in the undisturbed lattice, $\mathbf{u}(\mathbf{r})$ the deformation of the lattice leading to the position $\mathbf{r}_\alpha + \mathbf{u}(\mathbf{r}_\alpha)$ of the α molecule, and $\mathbf{R}_{\alpha\beta}$ the difference $\mathbf{r}_\alpha - \mathbf{r}_\beta$.

[66]MacCullagh 1837b. Cf. Whittaker 1951, pp. 159–60. MacCullagh also treated the case of oblique incidence with respect to the optical axis, which Airy had investigated experimentally. The vector form of MacCullagh's equations is $\ddot{\mathbf{u}} = -\nabla \times [\mathrm{K}(\nabla \times \mathbf{u})] - C\Delta(\nabla \times \mathbf{u})$. FitzGerald's electromagnetic reinterpretation (see above, p. 243) leads to $\mathbf{E} = [\varepsilon]^{-1}\,\mathbf{D} - C\nabla \times \mathbf{D}$ instead of the now accepted $\mathbf{D} = [\varepsilon]\mathbf{E} + C\nabla \times \mathbf{E}$ (cf. Landau and Lifshitz 1960, §83). In conformity with the latter choice, MacCullagh (1837b, p. 72) realized that different positive coefficients for $\partial^3\eta / \partial z^3$ and $-\partial^3\xi / \partial z^3$ led to better results.

[67]Tovey 1838, 1839a; Powell 1841a, pp. 33–4; Powell 1841b (about Cauchy's letter); MacCullagh 1844 [read in 1841]. Cf. Buchwald 1980a, pp. 260–2.

To first order in $\mathbf{u}$, the force between two molecules α and β of the disturbed lattice has the form

$$\mathbf{F}_{\alpha\beta} = \phi_{\alpha\beta}\mathbf{R}_{\alpha\beta} + \phi_{\alpha\beta}[\mathbf{u}(\mathbf{r}_\alpha) - \mathbf{u}(\mathbf{r}_\beta)] + \psi_{\alpha\beta}\mathbf{R}_{\alpha\beta}[\mathbf{u}(\mathbf{r}_\alpha) - \mathbf{u}(\mathbf{r}_\beta)]\cdot\mathbf{R}_{\alpha\beta}, \tag{46}$$

where $\phi_{\alpha\beta} = \phi(R_{\alpha\beta})$ and $\psi_{\alpha\beta} = R_{\alpha\beta}{}^{-1}\phi'(R_{\alpha\beta})$. For a transverse plane-wave disturbance these forces lead to the equations of motion

$$\ddot{\xi} = \sum_\beta f_{\alpha\beta}(\xi_\alpha - \xi_\beta) + \sum_\beta h_{\alpha\beta}(\eta_\alpha - \eta_\beta), \quad \ddot{\eta} = \sum_\beta g_{\alpha\beta}(\eta_\alpha - \eta_\beta) + \sum_\beta h_{\alpha\beta}(\xi_\alpha - \xi_\beta) \tag{47}$$

with $f_{\alpha\beta} = \phi_{\alpha\beta} + \psi_{\alpha\beta}(x_\alpha - x_\beta)^2, g_{\alpha\beta} = \phi_{\alpha\beta} + \psi_{\alpha\beta}(y_\alpha - y_\beta)^2,\ h_{\alpha\beta} = \psi_{\alpha\beta}(x_\alpha - x_\beta)(y_\alpha - y_\beta)$. Assuming the exponential form (45) of the solutions, this system leads to

$$-\omega^2 a = F(k)a + H(k)b,\ -\omega^2 b = G(k)b + H(k)a, \tag{48}$$

with $F(k) = \sum_\beta f_{\alpha\beta}[1 - \mathrm{e}^{\mathrm{i}k(z_\alpha - z_\beta)}]$ and similar expressions for $G(k)$ and $H(k)$. The corresponding vibrations are rectilinearly or elliptically polarized, according as the ratio *a/b* is real or imaginary. This ratio is given by the two roots of

$$(a/b)^2 + [(F - G)/H](a/b) - 1. \tag{49}$$

The product of these roots is -1, instead of 1 in MacCullagh's theory. This implies that in Cauchy's theory elliptically polarized solutions come in pairs that rotate in the same direction. Moreover, in an isotropic Cauchy medium the equality of F and G implies that $a/b = \pm 1$ so that the polarization can only be rectilinear. In MacCullagh's opinion, this argument "swept away the very foundations of [Cauchy's] theory." Its publication in 1844 may indeed have disheartened Cauchy's British followers. But it failed to shake Cauchy's confidence in the molecular ether.[68]

Periodic coefficients

MacCullagh's argument implicitly assumes that the optical medium is homogenous at the wavelength scale (the coefficients *F, G*, and *H* should not depend on position). In 1849 Cauchy observed that, in a double lattice of ether and matter molecules, the latter lattice could cause a (spatially) periodic variation of the coefficients in the equations of motion of the ether molecules. Granted that the matter molecules remain approximately at rest and that the ether molecules are much closer to each other than the matter molecules, the coefficients do not depend on time and the finite spatial differences of the ethereal shifts

[68]MacCullagh 1844, p. 208. As Poincaré later showed (1889, pp. 176–89), optical rotation is possible in a molecular lattice if the total potential of the molecular forces is a function of their mutual distances that cannot be split as sums of terms involving one pair of molecules only.

can be replaced by their Taylor expansion. Consequently, the displacement $\mathbf{u}(\mathbf{r}, t)$ of the ether obeys a partial differential equation of the form

$$\ddot{u}_m = a_{mn}(\mathbf{r})u_n + a_{mnpq}(\mathbf{r})\partial_p\partial_q u_n + a_{mnpqr}(\mathbf{r})\partial_p\partial_q\partial_r u_n + \ldots \tag{50}$$

in a modern tensor notation in which the indices $m, n, p\ldots$ run from 1 to 3. The periodic a coefficients can be expressed by Fourier series

$$a(\mathbf{r}) = \sum_{\boldsymbol{\kappa}} b(\boldsymbol{\kappa})\mathrm{e}^{\mathrm{i}\boldsymbol{\kappa}\cdot\mathbf{r}}, \tag{51}$$

where $\boldsymbol{\kappa}$ runs through the reciprocal lattice of the material lattice. Implicitly using what we now call Bloch's theorem, Cauchy sought a solution of the form

$$\mathbf{u} = \sum_{\boldsymbol{\kappa}} \tilde{\mathbf{u}}(\boldsymbol{\kappa})\mathrm{e}^{\mathrm{i}(\boldsymbol{\kappa}\cdot\mathbf{r}-\omega t)}\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}. \tag{52}$$

This leads to the linear system

$$-\omega^2\tilde{u}_m(\boldsymbol{\kappa}) = \sum_{\boldsymbol{\kappa}'}[b_{mn}(\boldsymbol{\kappa}-\boldsymbol{\kappa}') - b_{mnpq}(\boldsymbol{\kappa}-\boldsymbol{\kappa}')(\kappa'_p + k_p)(\kappa'_q + k_q) - \ldots]\tilde{u}_n(\boldsymbol{\kappa}'). \tag{53}$$

The elimination of all the Fourier coefficients $\tilde{\mathbf{u}}(\boldsymbol{\kappa})$ for which $\boldsymbol{\kappa} \neq \mathbf{0}$ yields an expression of the large-scale amplitude $\tilde{\mathbf{u}}(\mathbf{0})$ as a function of the large-scale wave vector $\mathbf{k}$. Granted that the corresponding wavelength is much larger than the periods of the material lattice, this function can be expressed as a power series of the components of the vector $\mathbf{k}$. By reverse Fourier-transformation, it yields a partial differential equation of the form

$$\ddot{\bar{u}} = A_{mn}\bar{u}_n + A_{mnpq}\partial_p\partial_q\bar{u}_n + A_{mnpqr}\partial_p\partial_q\partial_r\bar{u}_n + \ldots \tag{54}$$

for the large-scale motion $\bar{\mathbf{u}}(\mathbf{r},t)$. Retaining only the three first differential orders in equations (50) and the first harmonics of $a(\mathbf{r})$ and $\mathbf{u}(\mathbf{r})$ in the system (53), Cauchy found that the A coefficients could take the form required by MacCullagh's theory of optical rotation if only the material lattice was not parity invariant.[69]

In these (never published) calculations Cauchy had the ether share the anisotropy of the crystal that it penetrated. When applied to double refraction, this assumption implies that for a crystal of cubic symmetry the elastic response of the ether includes a term proportional to $(\partial_x^2 u_x,\ \partial_y^2 u_y,\ \partial_z^2 u_z)$. This anisotropic term implies a double refraction that does not exist for such crystals. In order to meet this objection, in the early 1860s Cauchy's disciple Charles Briot restricted the effect of the material molecules to a direction-dependent contraction of the ether (a contraction is necessarily isotropic in

[69]Cauchy 1850a, 1850b. Cf. Saint-venant 1872; Buchwald 1980a, pp. 266–71. Cauchy 1850c developed a method for determining the most general differential operator (function of ∇) that is invariant by rotation. This includes the parity-violating operator $\nabla^2(\nabla\times\ )$, which is responsible for optical rotation in MacCullagh's theory.

the cubic case). In 1867 another disciple, Emile Sarrau, further simplified the theory by assuming the isotropy of this contraction. That is to say, he started with the equation of motion

$$\ddot{\mathbf{u}} = -a(\mathbf{r})\nabla \times (\nabla \times \mathbf{u}) \tag{55}$$

involving the rapidly oscillating coefficient $a(\mathbf{r})$; and he generated the anisotropic, dispersive, and rotational terms in the large-scale equation of motion by Cauchy's method of elimination of the harmonics. In Denmark, Ludvig Lorenz had done the same in 1863 in the context of his more phenomenological theory of light. Lorenz simply assumed the rapid spatial oscillations of the local propagation velocity $\sqrt{a}$, without giving any molecular mechanism.[70]

Boussinesq's theory of 1867

In the hands of Cauchy's disciples, the ether became less and less responsive to the presence of matter molecules. In a lucid review of these theories, the great theorist of elasticity Adhémar Barré de Saint-Venant found it difficult to admit that material molecules altered the density of the ether without altering its elasticity. He therefore recommended the theory of his protégé Joseph Boussinesq, in which the distribution of the interstitial ether was completely unaltered by the molecules of matter except perhaps in their immediate vicinity. Boussinesq justified this radical assumption by the unimpeded motion of bulk matter through the ether and by the equal elasticity of the ether in every transparent medium (in Cauchy's theory of refraction). He further assumed that the motion of the ether induced a motion of the immersed molecules, in analogy with particles suspended in a moving fluid. More precisely, he regarded the displacement $\mathbf{v}$ of a molecule of matter as a linear function of the values that the displacement $\mathbf{u}$ of the ether takes in the volume occupied by the molecule at a given instant. This volume being small compared with the wavelength, this relation can be written as

$$v_m = A_{mn}u_n + A_{mnp}\partial_p u_n + A_{mnpq}\partial_p\partial_q u_n + A_{mnpqr}\partial_p\partial_q\partial_r u_n + \ldots \tag{56}$$

The macroscopic equation of motion of the ether is obtained by equating its local acceleration to the elastic force plus the reaction $-\sigma\ddot{\mathbf{v}}$ of the molecules of matter:[71]

$$\rho\ddot{\mathbf{u}} = K\Delta\mathbf{u} + L\nabla(\nabla\cdot\mathbf{u}) - \sigma\,\ddot{\mathbf{v}}. \tag{57}$$

In the isotropic case and to second order, the Taylor expansion of $\mathbf{v}$ reduces to

[70] Briot 1859, 1860, 1864; Sarrau 1867–1868; Lorenz 1863, 1864. On Briot and Sarrau, cf. Saint-venant 1872; Poincaré 1889, pp. 192–208 (for a free interpretation of Sarrau's theory); Buchwald 1980a, p. 272; Chappert 2004, pp. 158–9. On Lorenz, cf. Kragh 1991. Briot carefully investigated how the form of this equation depended on the symmetry of the medium. Sarrau's theory has some analogy with Lamé's theory of 1834 (see above, pp. 248–9).

[71] Saint-venant 1872, pp. 366–7; Boussinesq 1868, 1873. Cf. Whittaker 1951, pp. 167–9; Chappert 2004, pp. 160–2.

$$\mathbf{v} = A\mathbf{u} + B\nabla \times \mathbf{u} + C\nabla(\nabla\cdot\mathbf{u}) + D\Delta\mathbf{u}. \tag{58}$$

The first term increases the effective inertia of the ether in a manner depending on the matter that it permeates. It therefore leads to Fresnel's laws of reflection and refraction at the interface of two isotropic media if longitudinal vibrations are eliminated in the manner of Cauchy's labile-ether theory. The second term leads to MacCullagh's equation for optical rotation in isotropic media. The fourth term leads to the dispersion formula

$$\omega^2 = \frac{Kk^2}{\rho + \sigma A + Dk^2}, \tag{59}$$

which is approximately equivalent to Cauchy's dispersion formula, except that the coefficient D is not necessarily positive. Boussinesq used the negative option to explain the anomalous dispersion of iodine vapor. For anisotropic media, he replaced the constant A in equation (58) with a symmetric operator. This leads to MacCullagh's and Neumann's laws of reflection and refraction at the surface of a crystal, and to a theory of optical rotation in quartz equivalent to the modern one.[72]

Lastly, Boussinesq investigated the case of a transparent body moving through the ether at a constant velocity $\mathbf{U}$. At a given instant t a molecule of the body interacts with the portion of the ether located at $\mathbf{r}+\mathbf{U}t$. Consequently, the first-order part of the relation (58) between ether and molecule displacement becomes

$$\mathbf{v}(t) = A\mathbf{u}(\mathbf{r} + \mathbf{U}t,\ t). \tag{60}$$

The resulting equation for transverse ether vibrations reads

$$\rho\ddot{\mathbf{u}} + A\sigma(\partial/\partial t + \mathbf{U}\cdot\nabla)^2\mathbf{u} = K\Delta\mathbf{u}. \tag{61}$$

The frequency and the wave vector of monochromatic plane-wave solutions of this equation satisfy

$$\rho\omega^2 + \sigma A(\omega - \mathbf{k}\cdot\mathbf{U})^2 = Kk^2, \tag{62}$$

whose first-order approximation in U leads to

$$V = \frac{c}{n} + \left(1 - \frac{1}{n^2}\right) U \cos\theta \tag{63}$$

for the wave velocity $V = \omega/k$ as a function of the velocity c in vacuum, the index $n = \sqrt{1 + \sigma A/\rho}$, and the angle θ between the direction of the wave and the velocity of the

[72]Boussinesq 1868, p. 322 (iodine vapor); Boussinesq 1873, p. 778 (labile ether). Boussinesq's theory of refraction is equivalent to Glazebrook's later theory of a labile ether with anisotropic density (see above, p. 236). It is therefore also equivalent to the electromagnetic theory of refraction (see note 29 above). For optical rotation in quartz, its electromagnetic translation implies the relation $\mathbf{D} = [\varepsilon]\mathbf{E} + \alpha\nabla \times \mathbf{E}$, as in the modern theory.

body. As we will see in a moment, this is the "partial drag" formula which Fresnel had assumed to preserve the law of refraction in a moving optical system and which Fizeau had verified with an interferential device.[73]

Boussinesq's theory anticipated many features of modern optics. Its predictions were very nearly the same as those of Hendrik Lorentz's later theory, except for dispersion. In 1893 Boussinesq corrected this discrepancy by injecting into the equation of motion of the ether an additional force varying linearly with the displacement of the ether. He explained this force by a small modification of the distribution of the ether molecules in the vicinity of the molecules of matter. In the isotropic case, the resulting dispersion formula was the same as Sellmeier's or Lorentz's (without the damping term).[74]

The agreement between the predictions of Lorentz's and Boussinesq's theories is easily explained by their sharing the concept of a stationary ether, whose interaction with the molecules of matter is purely local. The only difference is in the nature (and detailed form) of this interaction; in Boussinesq's case it is inspired by hydrodynamic analogy, in Lorentz's case it is identified with the electromagnetic interaction between the ether and the ions (or electrons) contained in matter. There is no doubt, however, that the ionic or electron theories developed in the 1890s by Lorentz, Larmor, and Emil Wiechert encompassed a wider range of phenomena than Boussinesq's theory. They integrated the rising microphysics of electrolysis and gas discharge, and they included powerful theories of magnetism and magneto-optic phenomena.[75]

Optics of moving bodies

The oldest consideration of the interplay between matter and ether occurred in the context of the optics of moving bodies. In the mid-eighteenth century, Euler argued that the vibrational theory of light explained stellar aberration just as well as the corpuscular theory. His reasoning implicitly assumed that the earth did not disturb the ether while moving through it. In 1804, Thomas Young accompanied this assumption with a poetic metaphor:

> Upon considering the phenomena of the aberration of the stars, I am disposed to believe that the luminiferous aether pervades the substance of all material bodies with little or no resistance, as freely perhaps as the wind passes through a grove of trees.

As we saw, stellar aberration played an important role in Fresnel's early reflections about the nature of light. In the wave theory, he assumed like Young that the ether remained stationary around moving bodies. He is likely to have discussed the matter with Arago, who pressed him to examine whether the undulatory theory could account for his own discovery that the motion of the earth had no effect on the refraction of light.[76]

[73]Boussinesq 1868, pp. 433–8. The better known Potier 1876 (cf. Chappert 2004, p. 259) is Boussinesq's theory in disguise, with "condensed ether" instead of matter.

[74]Boussinesq 1893a, 1893b.

[75]On the rise of ionic theories, cf. Buchwald 1985; Darrigol 2000, chaps. 7–8.

[76]Euler 1746b; Young 1804a, p. 188. Arago's request is reported in Fresnel 1818a, p. 828. On Arago's experiment, see above, chap. 4, pp. 132–3.

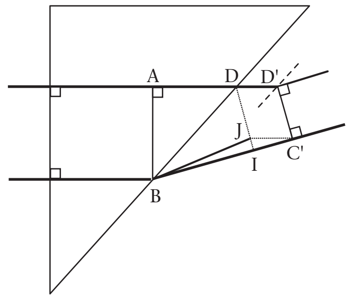


Fig. 6.2. Variant of Fresnel's figure for refraction in a moving prism.

In his letter of reply in 1818, Fresnel considered a prism moving in a direction perpendicular to one side of the prism and two rays entering this side perpendicularly (see Fig. 6.2). When the light of the inferior ray reaches the oblique sides of the prism at B, the light of the superior ray is still inside the prism, at point A. This light exits the prism at a later time for which the facing point D of the oblique side of the prism has traveled to the position D′. According to the Hobbesian rule, at this later time the inferior refracted ray reaches a point C′ such that C′D′ is perpendicular to BC′. Call t the elapsed time, V the velocity of light in the moving glass, c the velocity of light in vacuum (to which Fresnel assimilates the air around the prism), U the velocity of the prism, i the angle of incidence, r' the angle of refraction for an observer at rest in the ether, and I the foot of the perpendicular from D to BC′. Since $i = \mathrm{A\hat{B}D}$ and $r' = \mathrm{B\hat{D}I}$, we have

$$\mathrm{IC'} = \mathrm{DD'}\cos(r' - i), \quad \sin i = \mathrm{AD/BD}, \quad \sin r' = \mathrm{BI/BD}. \tag{64}$$

Together with BC′ = ct, DD′ = Ut, and AD′ = Vt, this implies the relation

$$\frac{\sin r'}{\sin i} = \frac{\mathrm{BC'} - \mathrm{IC'}}{\mathrm{AD'} - \mathrm{DD'}} = \frac{c - U\cos(r' - i)}{V - U}, \tag{65}$$

which implicitly determines r' as a function of i. The refraction angle r measured by a terrestrial observer (moving with the prism) is the angle that the line BJ resulting from the vector composition of BC′ ($\mathbf{c}t$) and D′D ($-\mathbf{U}t$) makes with the normal to the refracting surface. It is therefore given by the relation

$$\sin(r - r') = (U/c)\sin(r - i). \tag{66}$$

The negative result of Arago's experiment requires

$$n \sin i = \sin r. \tag{67}$$

To first order in U/c, this is compatible with equations (65) and (66) if and only if[77]

$$V = \frac{c}{n} + U\left(1 - \frac{1}{n^2}\right). \tag{68}$$

Fresnel interpreted this result through a partial drag of the ether by the moving glass. According to Stokes's later simplification of his reasoning, the equality of the mass fluxes on both sides of the air/glass interface reads $-\rho U = \rho' w$, where ρ and ρ' denote the densities of the ether in air and in glass, and w denotes the velocity of the ether in glass with respect to the glass. Since, according to Fresnel, the elasticity of the ether is the same in any medium, $\rho = \rho'/n^2$. Therefore, the ether in the glass has the absolute velocity $U+w = U - U/n^2$. The relative velocity of waves in this denser ether being c/n, their absolute velocity agrees with equation (68).[78]

Fresnel's optics of moving bodies did not please everyone. In 1845, Stokes denounced "the rather startling hypothesis that the luminiferous aether passes freely through the sides of the telescope and through the earth itself." He offered an alternative theory in which the ether assumed the velocity of the earth in its vicinity. In this case, the negative result of Arago's prism experiment becomes obvious whereas stellar aberration is harder to explain. Stokes met this challenge by showing that for any irrotational motion of the ether the propagation of light was the same as if the ether remained at rest. Fermat's principle of least time allows a simple derivation of this result. Call $\mathrm{d}\mathbf{l}$ an element of the path of light in absolute space, $\mathrm{d}s$ its length, and $\mathbf{v}(\mathbf{r})$ the absolute velocity of the ether. The time needed for light to travel from one end of this element to the other is

$$\mathrm{d}t = \frac{\mathrm{d}s}{c + \mathbf{v}\cdot\mathrm{d}\mathbf{l}/\mathrm{d}s} \approx \frac{\mathrm{d}s}{c} - \frac{\mathbf{v}\cdot\mathrm{d}\mathbf{l}}{c^2}. \tag{69}$$

Stokes's condition of irrotationality makes the second term of this expression an exact differential, so that the motion of the ether has no effect on the ray propagation of light.[79]

The later nineteenth-century developments of the optics of moving bodies may be summarized as follows. In 1851 Hippolyte Fizeau proved by interference that the velocity of light in moving water was altered in the proportion predicted by Fresnel (see Fig. 6.3). This result increased French and German confidence in Fresnel's optics of moving bodies. Around 1870, Wilhelm Veltmann and Eleuthère Mascart multiplied experimental proofs that the ether wind had no effect on optical experiments and showed that Fresnel's assumptions, together with the frequency shift introduced by Christian Doppler,

[77]Fresnel 1818a. Cf. Whittaker 1951, pp. 109–13; Mayrargue 1991. There is an error in Fresnel's figure: HH′ should be parallel to DD′. For a simpler way of reasoning through Fermat's principle, cf. Mascart 1893, vol. 3, chap. 15; also Darrigol 2000, pp. 434–5.

[78]Fresnel 1818a, pp. 631–2; Stokes 1846a.

[79]Stokes 1846a, 1846b.

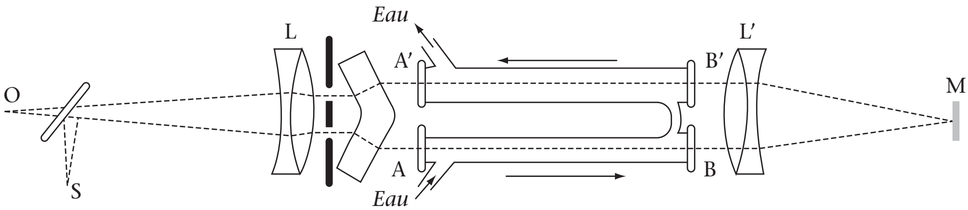


Fig. 6.3. Fizeau's experiment. One portion of the light from the source S travels along running water in the tube AB, and along the running water of the tube A′B′ after reflection on the mirror M. The other portion travels twice against the running water. The two portions interfere in O, with a phase difference depending on the velocity of the water.

explained this negative result as long as a different dragging coefficient was used for different frequencies and for different kinds of rays in anisotropic media. The latter specifications voided Fresnel's intuitive explanation of the partial drag and strengthened Boussinesq's alternative theory of a fully stationary ether coupled with material particles. In Britain, sympathy for Stokes's theory did not diminish, because the precision of Fizeau's experiment could be doubted and because Maxwell's electromagnetic theory of light favored the complete drag of ether by matter. The situation grew more perplexing in the late 1880s, when Albert Michelson and Edward Morley confirmed Fizeau's result on the one hand, and refuted the stationary ether on the other in the interferometric experiment for which they are most famous. In an insightful commentary, Lorentz excluded a return to Stokes's theory because he realized that the condition of irrotational ether motion was incompatible with vanishing relative velocity at the surface of the earth. He subsequently saved Fresnel's stationary ether by assuming the contraction of the longitudinal arm of Michelson's interferometer and by producing an electromagnetic version of Boussinesq's theory of the Fresnel drag.[80]

6.4 Conclusions

Nineteenth-century efforts to develop a consistent and adequate mechanical picture of the ether raised three difficult questions: How can one eliminate the unwanted longitudinal waves? Should the ether be a molecular lattice or a continuum? How does the ether interact with matter?

Cauchy's and Green's early attempts at founding optics on elasticity theory failed to eliminate the longitudinal vibrations in a way compatible with Fresnel's laws of propagation, reflection, and refraction. There were two possible escapes from this difficulty. The first, conceived by MacCullagh in 1839, consisted of imagining a special kind of rotational elasticity that contradicted the received theory of elasticity. Despite its elegant Lagrangian

[80]Fizeau 1851, 1857; Veltmann 1870a, 1870b, 1873; Mascart 1872, 1874; Doppler 1843; Michelson and Morley 1886, 1887; Lorentz 1886, 1892; Fitzgerald 1889. On Fizeau, cf. Frercks 2001, 2005. On the history of the optics of moving bodies, cf. Ketteler 1873; Mascart 1893, vol. 3, chap. 15; Schaffner 1972; Hirosige 1976; Buchwald 1988; Darrigol 2000, pp. 314–19; Janssen and Stachel 2004; Chappert 2004, chap. 10.

formulation, this theory was generally dismissed until FitzGerald found the resulting equations to agree with Maxwell's equations and Larmor understood that Kelvin's gyrostatic ether provided a mechanical realization of MacCullagh's medium. The second escape, inaugurated by Cauchy in 1839, was the labile-ether theory for which the velocity of longitudinal waves is zero or close to zero so that they do not contribute to the balance of incoming and emerging vibrations on a refracting surface. The niche of this theory was mostly French and German until Kelvin argued, as late as 1888, that the labile ether did not suffer from the instability earlier detected by Green. Besides the rotational-ether and labile-ether theories, there were many variants of Cauchy's and Green's theories in which some contradiction with the experimental laws of reflection and refraction was provisionally tolerated. The advent of the electromagnetic theory of light did not signal the death of the mechanical ether theories, as this renewal went along with new justifications of the rotational and labile ether. It did however reinforce the tendency, illustrated by MacCullagh, Lorenz, and Poincaré, to leave the internal mechanics of the ether undetermined and to focus on mathematical structures controlled by the energy principle and by the principle of least action.

The question of the molecular nature of the ether received a variety of answers, depending on local or individual preferences. For ontological reasons, Cauchy believed in a fully discrete universe in which matter and ether were both discontinuous. Even though he invented the modern stress–strain approach to the theory of elasticity, he abundantly relied on intricate lattice calculations in his optics. His first theory of dispersion reified the lattice structure by making the finite spacing of ether molecules responsible for the wavelength-dependence of propagation. Some early followers of Cauchy, mostly French and British, adopted this lattice ether. In contrast, Green and Stokes favored the macroscopic stress–strain approach, which provided six more elastic constant and thus permitted a better match to Fresnel's crystal optics. As the Cauchy lattice failed to explain dispersion and optical rotation, British theorists abandoned it in favor of variants of Green's theory. In France, Cauchy and a few disciples combined the ether lattice with a material lattice, ultimately neglecting the effects of the spacing of ether molecules and tracing anisotropic propagation, dispersion, and optical rotation to effects of the material lattice on a quasi-continuous ether.

This brings us to the third question, about the way in which the presence of matter affects the propagation of light. Cauchy's initial answer was to regard matter as a modifying circumstance for the effective interaction and spacing of the molecules of the ether. At the macroscopic level, this implied a mere modification of the elastic constants of the ether. This approach worked well as long as optical theory was confined to the propagation, reflection, and refraction of monochromatic light in bodies at rest. But it had trouble explaining why matter was needed to produce dispersion, and it completely failed to represent optical rotation. This is why Lamé, Neumann, Cauchy, and a few of his followers ended up treating the material lattice as the cause of deeper modifications of the effective ether dynamics. Most of these theories regarded the molecules of matter as fixed centers of local modification of the ether's properties (at least its density). In contrast, Boussinesq's powerful theory of 1867 had the material molecules follow the local motions of the ether and made the ether's properties completely indifferent to the presence of the molecules. After the discovery of anomalous dispersion, Sellmeier inaugurated a third

kind of theory in which the material molecules (later ions or electrons) were elastically bound to their position of equilibrium. Ether and matter thus became two coupled dynamical systems whose combined motion determined the propagation of light. Boussinesq's assumption that the ether between the molecules of ether had invariable properties became a central tenet of the later electron theories of Lorentz, Larmor, and Wiechert.

In the latter theories, which dominated the end of the century, the deeper nature of the ether became a secondary issue. The central issue became the elucidation of the atomistic assumptions that successfully represented a large variety of macroscopic phenomena, including the various sorts of magnetism, magneto-optics, conduction in metals, electrolytes and rarefied gases, and the optics of moving body. A few theorists including Poincaré and Emil Cohn even came to believe that the ether was no more than a metaphor for the propagation of autonomous fields.[81]

[81]On this late-nineteenth century evolution, cf. Buchwald 1985; Darrigol 2000, chaps. 7–9 and further reference there.

7

WAVES AND RAYS

The previous chapter was devoted to aspects of post-Fresnel optics for which the deeper nature of the ether was believed to be important. There is no doubt, however, that a good deal of nineteenth-century optics dealt with aspects of the propagation of light that only required the macroscopic picture of waves or rays. The present chapter briefly recounts three innovative contributions of this kind. The first is Hamilton's theory of systems of rays, which pioneered a powerful approach to the theory of optical instruments. The second is diffraction theory, which Fresnel had left in need of a better foundation. The third is the application of Fourier synthesis to luminous oscillations, leading to the concept of wave group and to a useful criticism of the concept of white light.

7.1 Hamiltonian optics

The characteristic function

In a December 1824 session of the Royal Irish Academy, the young William Rowan Hamilton read a memoir "On caustics" in which he applied notions of Gaspard Monge's *géométrie analytique* to "systems of rays" depending on two parameters (our rectilinear congruences). The Academicians told Hamilton that Malus had already done something quite similar and declined to publish this memoir. Hamilton's reaction was to recast his theory in a form that made it much more powerful than Malus's. The result was the "Theory of systems of rays" which he published in four installments between 1828 and 1835. Hamilton's crucial innovation was the introduction of the "characteristic function" of a system of rays, which provided an efficient algebraic tool for deriving the geometric properties of the beams emerging from optical systems.[1]

Hamilton presumably arrived at the characteristic function during the following demonstration and generalization of Malus's theorem. First consider the case of a beam of rays issuing from the point O and reflected by a mirror of arbitrary shape (Fig. 7.1). If $\mathbf{n}'$ and $\mathbf{n}$ denote the unit vectors of an incident ray and of the corresponding reflected ray, the laws of reflection imply the condition $(\mathbf{n}+\mathbf{n}')\cdot d\mathbf{r} = 0$ for any $d\mathbf{r}$ belonging to the tangent plane of the mirror at the point of reflection. For the reflected ray passing through the point O+$\mathbf{r}$, the attached unit vector is $\mathbf{n}' = \mathbf{r}/r$ so that $\mathbf{n}'\cdot d\mathbf{r}$ is identical to the exact differential dr. The aforementioned condition then implies that $\mathbf{n}\cdot d\mathbf{r}$ is an exact differential on the tangent plane. Equivalently, the normal component of $\nabla\times\mathbf{n}$ must vanish. As $\mathbf{n}$ remains the same

[1] Hamilton [1824], 1828a, 1830, 1831, 1837. Cf. Hankins 1980, chap. 2.

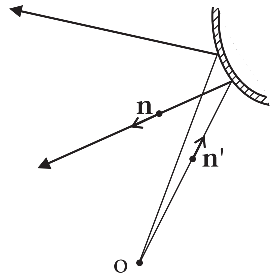


Fig. 7.1. Reflected beam of rays issuing from point O.

along the same ray, we also have $(\mathbf{n}\cdot\nabla)\mathbf{n} = 0$. Combined with $\nabla\mathbf{n}^2 = 0$, this equation implies $\mathbf{n}\times(\nabla\times\mathbf{n}) = 0$. Hence $\nabla\times\mathbf{n}$ must be parallel to $\mathbf{n}$ and belong to the tangent plane. This can only happen if $\nabla\times\mathbf{n}$ vanishes on the mirror. The condition $(\mathbf{n}\cdot\nabla)\mathbf{n} = 0$ further implies that $(\mathbf{n}\cdot\nabla)(\nabla\times\mathbf{n}) = 0$, which means that $\nabla\times\mathbf{n}$ remains the same along a ray (Hamilton overlooked this step of the demonstration). Consequently, $\nabla\times\mathbf{n}$ vanishes everywhere in front of the mirror and $\mathbf{n}\cdot\mathrm{d}\mathbf{r}$ is an exact differential, say $\mathrm{d}V$. This implies that the incident rays are orthogonal to the surfaces V = constant. They define what Hamilton calls a "rectangular system of rays" or what is now called a normal congruence. Integrating $\mathbf{n}\cdot\mathrm{d}\mathbf{r}$ along a ray from a point M before reflection at I to the focus O yields $V = \mathrm{MI}+\mathrm{IO}$. This sum takes a larger value when the point I of the mirror surface differs from the point of reflection, in conformity with Fermat's principle.[2]

Similarly, the laws of refraction imply the condition $(\mathbf{n}+\mathbf{n}')\cdot\mathrm{d}\mathbf{r} = 0$ if $\mathbf{n}$ and $\mathbf{n}'$ now denote vectors parallel to the incident and refracted ray and of lengths equal to the refractive indices n and n' in the two media. The preceding proof of Malus's theorem therefore extends to this case, except that the function V is now given by $V = n\mathrm{MI}+n'\mathrm{IO}$, which is again the quantity to be minimized according to Fermat's theorem. The extension to any number of reflections and refractions is straightforward. In the case of a continuous variation of the optical index, it leads to

$$V(\mathbf{r},\mathbf{r}') = \int n\,\mathrm{d}s, \tag{1}$$

the integral being taken over the path for which it is a minimum for the given ends $\mathbf{r}'$ and $\mathbf{r}$, with the differential

[2]Hamilton 1828a, sections 1–4. Hamilton of course did not use the vector notation. He later condensed some his optical algebra by means of his "quaternions."

$$\mathrm{d}V = \mathbf{n} \cdot \mathbf{dr} - \mathbf{n}' \cdot \mathbf{dr}'. \tag{2}$$

Hamilton confirmed this result by varying Fermat's integral with free path ends:

$$\begin{aligned} \delta \int n\mathrm{d}s &= \int \delta n \mathrm{d}s + \int n\delta(\mathrm{d}s) = \int \nabla n\cdot\delta\mathbf{r}\mathrm{d}s + \int \mathbf{n}\cdot\mathrm{d}\delta\mathbf{r} \\ &= \int \delta\mathbf{r}\cdot\left(\nabla n - \frac{\mathrm{d}\mathbf{n}}{\mathrm{d}s}\right) \mathrm{d}s + \mathbf{n}\cdot\delta\mathbf{r} - \mathbf{n}'\cdot\delta\mathbf{r}', \end{aligned} \tag{3}$$

in which $\mathrm{d}s\delta\mathrm{d}s = \mathrm{d}\mathbf{r}\cdot\ \delta\mathrm{d}\mathbf{r}$ and $\mathbf{n} = n\mathrm{d}\mathbf{r}/\mathrm{d}s$ have been used, and the last integral vanishes for the true ray path.[3]

In Newtonian optics the index n is proportional to the velocity of the light corpuscles and Fermat's integral is Maupertuis's action. This is why Hamilton frequently called his function V the action, although he was open to the possibility that n would rather be inversely proportional to the velocity of the waves associated with the surface of constant V, as it is in Huygens's and Fresnel's theories. He later called V the "characteristic function" because the knowledge of this function for a given optical system completely determined its optical properties (in the limit of validity of geometrical optics). Indeed for a given luminous point situated at $\mathbf{r}'$ in the object space, ∇V yields the direction of the ray passing through the running point $\mathbf{r}$ of the image space. This method however fails in the practically important case for which the point $\mathbf{r}$ is a focus (and for which there is no unique minimum path). For this reason and for computational advantages, Hamilton introduced two Legendre transforms of the function V:

$$W(\mathbf{n}, \mathbf{r}') = \mathbf{n}\cdot\mathbf{r} - V, \quad \text{such that } \mathrm{d}W = \mathbf{r}\cdot\mathrm{d}\mathbf{n} + \mathbf{n}'\cdot\mathrm{d}\mathbf{r}', \tag{4}$$

$$T(\mathbf{n}, \mathbf{n}') = \mathbf{n}\cdot\mathbf{r} - \mathbf{n}'\cdot\mathbf{r}' - V, \text{ such that } \mathrm{d}T = \mathbf{r}\cdot\mathrm{d}\mathbf{n} - \mathbf{r}'\cdot\mathrm{d}\mathbf{n}'. \tag{5}$$

These functions are easier to calculate for simple optical systems, and they directly lead to the outgoing rays as the intersections of two families of planes, for example

$$\mathbf{r}\cdot\frac{\partial\mathbf{n}}{\partial\alpha} - \frac{\partial W}{\partial\alpha} = 0, \quad \mathbf{r}\cdot\frac{\partial\mathbf{n}}{\partial\beta} - \frac{\partial W}{\partial\beta} = 0, \tag{6}$$

if α and β are any two parameters defining the direction of the vector $\mathbf{n}$ (whose length is constrained to be the optical index in the image space).[4]

Optical instruments

With these powerful tools, Hamilton developed the study of caustics, foci, and various kinds of aberrations. In a memoir read at the British Association meeting of 1833, he briefly indicated how to use the T function in the case of optical instruments with axial

[3]Hamilton [1828b], sections 14–16, 22; *MPH* 1, pp. 110–11.

[4]Hamilton 1830, section 4 (W); 1837, section 4 (W and T).

symmetry. If the z-axis coincides with the optical axis, and if we choose $\alpha = n_x$, $\beta = n_y$, $\alpha' = n'_x$, $\beta' = n'_y$, axial symmetry implies that the T function takes the form

$$T = T^{(0)} + T^{(2)} + T^{(4)} + ..., \tag{7}$$

where $T^{(0)}$ is a constant,

$$T^{(2)} = \frac{g}{2}(\alpha^2 + \beta^2) + f(\alpha\alpha' + \beta\beta') + \frac{h}{2}(\alpha'^2 + \beta'^2), \tag{8}$$

$$\begin{aligned} T^{(4)} = {} & A(\alpha^2 + \beta^2)^2 + B(\alpha^2 + \beta^2)(\alpha\alpha' + \beta\beta') + C(\alpha^2 + \beta^2)(\alpha'^2 + \beta'^2) \\ & + D(\alpha\alpha' + \beta\beta')^2 + E(\alpha\alpha' + \beta\beta')(\alpha'^2 + \beta'^2) + F(\alpha'^2 + \beta'^2)^2. \end{aligned} \tag{9}$$

In the case of paraxial optics, α and β are very small and $T^{(2)}$ controls the phenomena. The resulting equations of outgoing and ingoing rays are

$$x - \alpha z = g\alpha + f\alpha',\ y - \beta z = g\beta + f\beta';\ x' - \alpha' z' = -h\alpha' - f\alpha,\ y' - \beta' z' = -h\beta' - f\beta. \tag{10}$$

For two given points $\mathbf{r}$ and $\mathbf{r}'$ in the image and object spaces, they determine the directions of the ingoing and outgoing rays passing through these points as the solutions of the systems

$$\alpha(g + z) + \alpha' f = x,\ \ \alpha f + \alpha'(h - z') = -x';\ \ \beta(g + z) + \beta' f = y,\ \ \beta f + \beta'(h - z') = -y'. \tag{11}$$

Whenever the relations

$$\frac{g + z}{f} = \frac{f}{h - z'} = -\frac{x}{x'} = -\frac{y}{y'} \tag{12}$$

are satisfied, these directions are arbitrary and the point $\mathbf{r}$ is the image of the point $\mathbf{r}'$. The points $z = -g$ and $z' = h$ on the axis are the "principal foci" whose conjugate is infinitely distant, and the planes $z = -g-f$, $z' = h+f$ are the "principal planes" for which the transverse magnification is unity. The T function thus leads to the basic concepts and relations of paraxial optics, although Carl Friedrich Gauss was first to define them in a general manner in a theory conceived in his youth and published belatedly in 1840. In his demonstrations, Gauss relied on the combination of the linear transformations connecting the parameters (inclination and transverse coordinates in a fixed plane) of incoming and outgoing rays for each refracting surface, as is most commonly done nowadays.[5]

The fourth-order contribution $T^{(4)}$ to the characteristic function engenders the various kinds of geometrical aberrations. It depends on the six constants A, … F, which Hamilton called the "radical constants of aberration." Although Hamilton dwelt on some of these

[5]Hamilton 1828a, [1828b], 1830, 1831, 1833a; Gauss 1840. Cf. Conway and Synge in Hamilton 1931–2000, vol. 1, note 25 (pp. 508–10); Synge 1834. The Hamiltonian deduction of Gaussian optics is from Landau and Lifshitz 1951, §55. On Gauss's optics, cf. Schaefer 1929, pp. 152–211, esp. 189–202 (paraxial case).

aberrations (spherical aberration, astigmatism, coma), he did this mostly in manuscript form and he never gave a complete discussion. In 1895, the Leipzig astronomer Heinrich Bruns rediscovered Hamilton's characteristic function by analogy with Hamilton's action in mechanics. Later theorists of optical instruments adopted the name "eikonal" (from the Greek for image) that Bruns gave to this function. In midcentury Jean Baptiste Biot, Joseph Petzval, Ludwig von Seidel, and Ottaviano Mossotti had discovered the various conditions under which optical images in systems of lenses are free of geometrical aberration. Whereas these authors patiently traced the rays through the various refracting spheres, Bruns and his follower Karl Schwarzschild had the higher-order terms of the eikonal directly engender the various aberrations now called spherical aberration, astigmatism, coma, and field curvature.[6]

Anisotropic propagation

In 1830, Hamilton extended his theory to the case of anisotropic media for which the emissionist velocity υ depends not only on the position $\mathbf{r}$ but also on the direction $\mathbf{u}$ of the motion. Like Laplace (see above, chapter 5, pp. 190–1), Hamilton assumed that Maupertuis's principle still applied in this case. While varying the action, care must be taken that the components of $\mathbf{u}$ in the function $\upsilon = f(\mathbf{u}, \mathbf{r})$ are not independent since $\mathbf{u}$ is restricted to the unit sphere $\mathbf{u}^2 = 1$. Hamilton astutely introduced the homogenous extension $\upsilon = uf(\mathbf{u}/u, \mathbf{r})$ which is defined for any $\mathbf{u}$ and for which $\mathbf{u}\cdot\partial\upsilon/\partial\mathbf{u} = \upsilon$. Thanks to this "preparation" of the function υ, varying the action $V = \int \upsilon \mathrm{d}s$ with free path ends gives

$$\delta V = \int \frac{\partial \upsilon}{\partial \mathbf{r}} \cdot \delta \mathbf{r} \mathrm{d}s + \int \left(\upsilon \mathbf{u} \cdot \mathrm{d}\delta \mathbf{r} + \frac{\partial \upsilon}{\partial \mathbf{u}} \cdot \delta \mathbf{u} \mathrm{d}s \right). \tag{13}$$

Varying $\mathrm{d}\mathbf{r} = \mathbf{u}\cdot\mathrm{d}s$, multiplying the result by $\partial\upsilon/\partial\mathbf{u}$, and exploiting the homogeneity of υ, we also have

$$\frac{\partial \upsilon}{\partial \mathbf{u}} \cdot \delta \mathrm{d}\mathbf{r} = \frac{\partial \upsilon}{\partial \mathbf{u}} \cdot [\mathbf{u}(\mathbf{u}\cdot\delta \mathrm{d}\mathbf{r}) + \delta \mathbf{u} \mathrm{d}s] = \upsilon \mathbf{u} \cdot \mathrm{d}\delta \mathbf{r} + \frac{\partial \upsilon}{\partial \mathbf{u}} \cdot \delta \mathbf{u} \mathrm{d}s, \tag{14}$$

which leads to

$$\delta V = \int \left(\frac{\partial \upsilon}{\partial \mathbf{r}} - \frac{\mathrm{d}}{\mathrm{d}s} \frac{\partial \upsilon}{\partial \mathbf{u}} \right) \cdot \delta \mathbf{r} \mathrm{d}s + \frac{\partial \upsilon}{\partial \mathbf{u}} \cdot \delta \mathbf{r} - \left(\frac{\partial \upsilon}{\partial \mathbf{u}} \right)_{\mathbf{r}=\mathbf{r}'} \cdot \delta \mathbf{r}'. \tag{15}$$

[6]Hamilton 1833a; Bruns 1895, pp. 328 (Hamilton's mechanics), 409–10 (derivation of Gaussian optics), 411–21 (higher order); Biot 1843, pp. 1–63; Petzval 1843; Seidel 1856; Mossotti 1855–1861; Schwarzschild 1905. Cf. Atzema 1993b, pp. 60–4, 73; Chappert 2004, pp. 312–21. Max Thiesen anticipated Bruns' approach in 1890. Schwarzschild introduced a more convenient eikonal, which he called "Seidel's eikonal" because it shared with Seidel's theory variables that directly referred to departures from the Gaussian image. Cf. Atzema 1993b, pp. 61–2, 73; Chappert 2004, pp. 312–21. Seidel's conditions are limited to third order in the aperture. James Clerk Maxwell (1858), Rudolf Clausius (1864), and Ernst Abbe (1873) discovered "sine conditions" that are valid at any order for the accurate representation of small objects lying in a plane perpendicular to the axis or on the axis: cf. Whittaker 1907, pp. 34–47. For a Hamiltonian derivation of these conditions, cf. Sommerfeld and Runge 1911, pp. 293–295.

Therefore, for the true ray paths verifying the equation

$$\frac{\partial \upsilon}{\partial \mathbf{r}} - \frac{\mathrm{d}}{\mathrm{d}s}\frac{\partial \upsilon}{\partial \mathbf{u}} = \mathbf{0}, \tag{16}$$

the action V is a function of the origin $\mathbf{r}'$ and end $\mathbf{r}$ of the path such that

$$\frac{\partial V}{\partial \mathbf{r}'} = -\left(\frac{\partial \upsilon}{\partial \mathbf{u}}\right)_{\mathbf{r}=\mathbf{r}'}, \quad \frac{\partial V}{\partial \mathbf{r}} = \frac{\partial \upsilon}{\partial \mathbf{u}}. \tag{17}$$

As Hamilton explained, from a wave-theoretical point of view, $\upsilon(\mathbf{u}, \mathbf{r})$ corresponds to the inverse of the velocity of light passing through $\mathbf{r}$ with the direction $\mathbf{u}$, and $\partial V/\partial \mathbf{r}$ corresponds to the inverse $\mathbf{n}$ of the phase velocity ("normal slowness"). Through the relation $\mathbf{n} = \partial \upsilon/\partial \mathbf{u}$ from (17), $\mathbf{n}$ is a function of $\mathbf{u}$ for a given $\mathbf{r}$. Inverting this function and writing $\mathbf{u}^2 = 1$ leads to an equation of the form $\psi(\mathbf{n}, \mathbf{r}) = 1$, which means that the extremity of the vector $\mathbf{n}$ belongs to a characteristic surface (MacCullagh's "index surface").[7]

Now suppose with Hamilton that this surface is given and that the unknowns are the ray velocity $1/\upsilon$ and its direction $\mathbf{u}$ at a point of the medium. The homogeneity of υ implies $\upsilon = \mathbf{u}\cdot\mathbf{n}$, and $\mathrm{d}\upsilon = \mathbf{n}\cdot\mathrm{d}\mathbf{u}+\mathbf{u}\cdot\mathrm{d}\mathbf{n} = \mathbf{n}\cdot\mathrm{d}\mathbf{u}+(\partial\upsilon/\partial\mathbf{r})$. Hence the ray velocity vector $\mathbf{s} = \mathbf{u}/\upsilon$ is the vector that satisfies the conditions $\mathbf{s}\cdot\mathbf{n} = 1$ and $\mathbf{s}\cdot\mathrm{d}\mathbf{n} = 0$ for all possible values of $\mathrm{d}\mathbf{n}$ at a given point $\mathbf{r}$. Hamilton thus retrieved Fresnel's characterization of the ray surface (wave surface) by purely algebraic means. Most impressively, he derived an expression of $\mathbf{s}$ from the index equation $\psi(\mathbf{n}, \mathbf{r}) = 1$. For this purpose, he noted that this equation implies $(\partial\psi/\partial\mathbf{n})\cdot\mathrm{d}\mathbf{n} = 0$ at a given $\mathbf{r}$. As he made ψ a homogenous function of $\mathbf{n}$ of degree one, he also had $\mathbf{n}\cdot(\partial\psi/\partial\mathbf{n}) = \psi = 1$. Consequently, the ray velocity vector is simply given by

$$\mathbf{s} = \frac{\partial \psi}{\partial \mathbf{n}}. \tag{18}$$

It remains to be shown that the index equation can always be homogenized. This is achieved by putting it in the form $n = f(\mathbf{n}/n, \mathbf{r})$, and setting $\psi = n/f$.[8]

At the end of his last supplement (1832), Hamilton applied the expression (18) of the ray vector $\mathbf{s}$ to the case of Fresnel's theory. His derivation of Fresnel's equation of the wave surface is worth giving here as a sample of his algebraic wizardry. It begins with Fresnel's index equation, written as

$$\frac{n_x^2}{n^{-2}-a^2} + \frac{n_y^2}{n^{-2}-b^2} + \frac{n_z^2}{n^{-2}-c^2} = 0, \quad \text{which is} \quad \mathbf{n}\cdot(1-n^2\mathrm{K})^{-1}\mathbf{n} = 0 \tag{19}$$

in modern vector notation. The corresponding homogenous ψ function verifies

[7] Hamilton 1830, sections 1–3. $\partial f/\partial \mathbf{u}$ denotes the vector of components $\partial f/\partial u_x$, $\partial f/\partial u_y$, $\partial f/\partial u_z$.

[8] Hamilton 1837 [1832], section 2. Cf. O'Hara 1979.

$$\mathbf{n}\cdot[\psi^2(\mathbf{n}) - n^2\mathrm{K}]^{-1}\mathbf{n} = 0. \tag{20}$$

Deriving the latter equation with respect to **n** and setting $\psi = 1$ in the result yields

$$\langle \mathrm{M}^2\rangle\mathbf{s} = \mathrm{M}\mathbf{n} + \langle \mathrm{KM}^2\rangle\mathbf{n}, \tag{21}$$

in which the abbreviations $\mathrm{M}=(1-n^2\mathrm{K})^{-1}$ and $\langle \mathrm{X}\rangle = \mathbf{n}\cdot\mathrm{X}\mathbf{n}$ have been used.

Multiplying this equation by **n** and by **s** and remembering that $\mathbf{s}\cdot\mathbf{n}=1$ and $\mathbf{n}\cdot\mathrm{M}\mathbf{n}=0$ yields the relations

$$\langle \mathrm{M}^2\rangle = \langle \mathrm{KM}^2\rangle n^2, \quad \langle \mathrm{M}^2\rangle s^2 = \mathbf{s}\cdot\mathrm{M}\mathbf{n} + \langle \mathrm{KM}^2\rangle. \tag{22 – 23}$$

Squaring equation (21) yields the third relation

$$\langle \mathrm{M}^2\rangle^2 s^2 = \langle \mathrm{M}^2\rangle + \langle \mathrm{KM}^2\rangle^2 n^2. \tag{24}$$

Using $\mathrm{M}-n^2\mathrm{KM}=1$, equation (21) may be rewritten as

$$\langle \mathrm{M}^2\rangle\mathbf{s} = (1 + \langle \mathrm{KM}^2\rangle)\mathrm{M}\mathbf{n} - n^2\langle \mathrm{KM}^2\rangle\mathrm{KM}\mathbf{n}. \tag{25}$$

By relations (22) and (23), this is equivalent to

$$\mathbf{s} = (s^2 - \mathrm{K})\mathrm{M}\mathbf{n}. \tag{26}$$

Relations (22), (23), and (24) further give

$$\mathbf{s}\cdot\mathrm{M}\mathbf{n} = 1. \tag{27}$$

The vector **n** can therefore be eliminated by forming the product

$$\mathbf{s}\cdot(s^2 - \mathrm{K})^{-1}\mathbf{s} = \mathbf{s}\cdot\mathrm{M}\mathbf{n} = 1. \tag{28}$$

This equation gives the ray velocity s as a function of the direction $\mathbf{s}/s$ of the ray. It is equivalent to the equation

$$\mathbf{s}\cdot(1 - \mathrm{K}^{-1}s^2)^{-1}\mathbf{s} = 0 \tag{29}$$

of Fresnel's wave surface.[9]

[9]This is a free modern rendering of Hamilton 1837, sections 26–7. All operators used in this section are symmetric and commute with each other as they are algebraic functions of the symmetric operator K. See above, chapter 5, pp. 218, for another derivation of Fresnel's wave surface.

Despite the publicity brought by the discovery of conical refraction, Hamilton's theory of systems of rays long remained unexploited. It appeared in the sparingly distributed *Transactions* of the Royal Irish Academy, in an extremely general form with terse commentary and lengthy algebraic developments. The power of Hamilton's method was only recognized after Bruns's aforementioned rediscovery. Hamilton himself seems to have been more concerned with the dissemination of the new form of mechanics that he derived by analogy with his optics.[10]

Another possible reason for the neglect of Hamilton's optics is the lack of connection with the fundamental wave equation, despite the identification of the characteristic function V with the phase of waves. Hamilton wrote the partial differential equation

$$(\nabla V)^2 = n^2, \tag{30}$$

and devoted a whole memoir to generic solutions of this equation. Yet he never tried to connect this equation with a wave equation. The Dutch physicist Peter Debye seems to have been first to do so, in an oral communication to Arnold Sommerfeld in 1910. Ignoring polarization, the optical wave equation in a medium of variable index $n(\mathbf{r})$ reads

$$\Delta\varphi - \frac{n^2}{c^2}\frac{\partial^2\varphi}{\partial t^2} = 0, \tag{31}$$

where c is the velocity of light in vacuum. For a monochromatic wave of angular frequency ω, this equation reduces to

$$\Delta\varphi + \frac{n^2\omega^2}{c^2}\varphi = 0. \tag{32}$$

As Debye remarked, in the approximation of geometrical optics the undulation can be locally approximated by a plane monochromatic wave. The wave φ can then be written in the form

$$\varphi(\mathbf{r}, t) = a(\mathbf{r})\cos[\omega t + \xi(\mathbf{r})], \tag{33}$$

$\varphi(\mathbf{r})$ being a quickly varying phase and $a(\mathbf{r})$ a slowly varying amplitude. The wave equation can be replaced with the equation

$$(\nabla\xi)^2 = n^2\omega^2/c^2, \tag{34}$$

which leads to the eikonal equation (30) for $V = (c/\omega)\xi$. The remark is the basis for a fuller understanding between wave and rays optics.[11]

[10] Cf. Hankins 1980, pp. 86–7. Hamilton's mechanics originated in Hamilton 1833b.

[11] Cf. Sommerfeld and Runge 1911, pp. 289–92; Atzema 1993b, p. 76.

7.2 Diffraction theory

From the beginning, Fresnel's diffraction theory had an ambiguous status. On the one hand, the accurate verification of its empirical consequences seemed to imply the truth of the underlying principle of interfering secondary wavelets. On the other hand, this principle rested on the unproven intuition that the elements of the intercepted wave front acted as secondary sources from which the diffracted light emanated. Most of the later investigators of diffraction phenomena ignored this difficulty and went on applying Fresnel's principle to various kinds of diffracting system.[12]

Airy's application of Fresnel's principle

For instance, in 1834 the Astronomer Royal George Biddell Airy summed the Fresnel wavelets emanating from the surface elements of the truncated spherical wave that emanates from the object lens of a telescope directed toward a point-like star. In the focal plane of the lens at the distance b from the focus, the resulting integral is proportional to

$$I(\alpha) = \int_0^1 \mathrm{d}u\sqrt{1-u^2}\cos\alpha u \qquad \text{with} \qquad \alpha = \frac{2\pi}{\lambda}\frac{ba}{f}, \tag{35}$$

a being the radius of the object lens and f its focal distance. This integral, which Airy evaluated through a power series expansion, produces a series of dark rings, with a central spot of breadth proportional to the ratio a/λ. This result is the basis of the modern theory of the resolving power of optical instruments, which is defined through the separation of the Airy spots.[13]

In 1838, Airy similarly determined the intensity of light in the neighborhood of a caustic by summing the Fresnel wavelets emanating from the reflecting (or refracting) surface. At a small distance, x, from the caustic, the resulting integral reads

$$J(m) = \int_{-\infty}^{+\infty} \mathrm{d}u \cos\frac{\pi}{2}(u^3 - mu) \quad \text{with} \quad m = 4x(3/2\lambda^2\rho)^{1/3}, \tag{36}$$

where ρ denotes the curvature of the caustic. Airy tabulated and plotted the function $J(m)$, which is now called after him. It decreases exponentially on the concave side, and oscillates very quickly with decreasing amplitude on the convex side of the caustic (see Fig. 7.2). Airy found that the (square of) the same integral represented the angular distribution of the intensity of light in the neighborhood of the rays of minimum or maximum deviation in rainbow theory. He thus explained the supernumerary rainbows that Thomas Young had traced to double-ray interference.[14]

[12]For a review of these developments, cf. Verdet 1869–1872, vol. 1, chaps. 7–9.

[13]Airy 1834. Cf. Chappert 2004, pp. 321–3, also for Ernst Abbe's later discussion of the effect of diffraction by a lit object.

[14]Airy 1838. Cf. Hulst 1981, §13.2. As Airy remarked (on p. 392), the double-ray interference theory amounts to retaining only the values $u = \pm\sqrt{m/3}$ for which the phase $(\pi/2)(u^3 - mu)$ is stationary in integral (36).

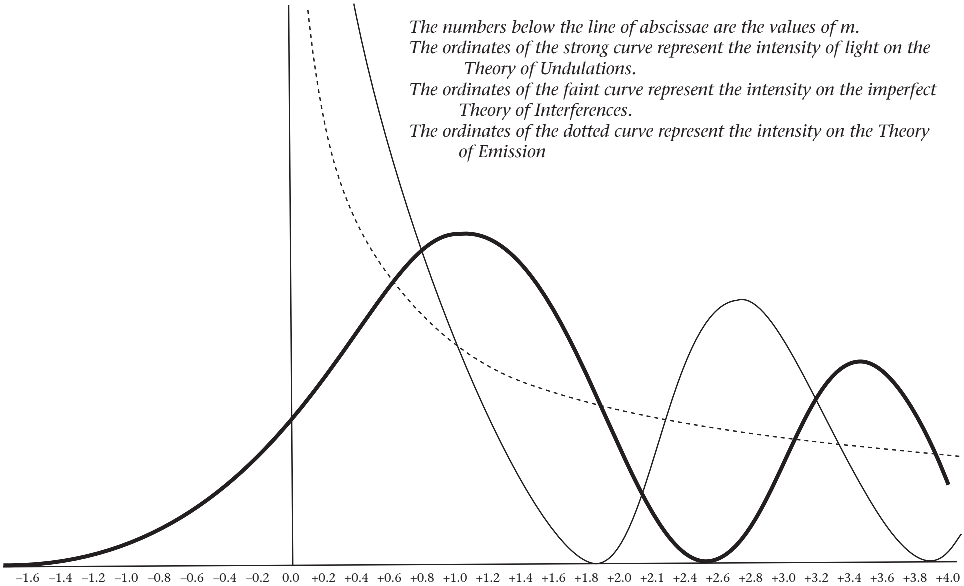


Fig. 7.2. The intensity distribution next to a caustic. From Airy 1838, plate.

Stokes's dynamical theory

The success of Airy's and others' applications of Fresnel's theory of diffraction did not quite extinguish worries about the lack of a rigorous foundation. A few theorists tried to fill the conceptual gap. The first was Stokes, in a "dynamical theory of diffraction" designed in 1849 in the context of the elastic–solid theory of the ether. His starting point was Poisson's integral of the wave equation $\ddot{u}-c^2\Delta u = 0$, which gives the value of the disturbance u at a given point (say $\mathbf{r} = \mathbf{0}$) at a given instant t as the following combination of the averages of the initial values of u and $\dot{u}$ over the sphere of radius ct centered on this point:

$$4\pi u(\mathbf{0}, t) = t\int_{r=ct} \mathrm{d}\Omega\, \dot{u}(\mathbf{r}, 0) + \frac{\mathrm{d}}{\mathrm{d}t}\left[t\int_{r=ct} \mathrm{d}\Omega u(\mathbf{r}, 0)\right], \tag{37}$$

where $\mathrm{d}\Omega$ denotes the element of solid angle in the direction of $\mathbf{r}$. This formula may be rewritten as[15]

$$4\pi u(\mathbf{0}, t) = t\int_{r=ct} \mathrm{d}\Omega\, \dot{u}(\mathbf{r}, 0) + \int_{r=ct} \mathrm{d}\Omega u(\mathbf{r}, 0) + ct\int_{r=ct} \mathrm{d}\Omega \frac{\partial u}{\partial r}(\mathbf{r}, 0). \tag{38}$$

In order to apply this result to the diffraction of the plane wave $U(\mathbf{r},t) = f(ct-x)$ by a flat screen in the plane $x = -a$, Stokes sliced this wave into thin layers of thickness $c\tau$ and

[15] Stokes 1851 [read 1849]; Poisson 1819b, pp. 133–4.

constructed the diffracted wave by summing the contributions of each of these layers at the times $n\tau$ at which they meet the plane $x = -a$; of one of these layers he considered the portion $\mathrm{d}V$ that lies above the unscreened surface element $\mathrm{d}S$ of this plane. For large r, the second term in equation (38) is negligible, and the contribution of the element $\mathrm{d}V$ to the disturbance at $\mathbf{r} = \mathbf{0}$ is

$$4\pi \mathrm{d}u(\mathbf{0}, t) = \sum_n \mathrm{d}\Omega_n \left[\frac{r}{c}\dot{U}(\mathbf{r}, t - r/c) + r\frac{\partial U}{\partial r}(\mathbf{r}, t - r/c)\right], \tag{39}$$

where $\mathrm{d}\Omega_n$ is the solid angle determined by the intersection of the sphere $r{=}nc\tau$ with the volume $\mathrm{d}V$. The identities

$$\mathrm{d}V = c\tau \mathrm{d}S = \sum_n c\tau r^2 \mathrm{d}\Omega_n, \quad \dot{U} = -c\frac{\partial U}{\partial x} = -cU', \quad \frac{\partial U}{\partial r} = -\cos\theta \frac{\partial U}{\partial x}, \tag{40}$$

where θ is the angle that the vector $\mathbf{r}$ makes with the x axis, then lead to the formula

$$u(\mathbf{0}, t) = -\int \frac{\mathrm{d}S}{4\pi r}(1 + \cos\theta)U'(\mathbf{r}, t - r/c) \tag{41}$$

for the total disturbance expressed as an integral over the plane $x = -a$.[16]

As Stokes himself remarked, the slicing up of the incoming wave is a bit problematic because of the discontinuities at the frontiers of each slice. As we may retrospectively judge, it leads to a wrong expression for the $1/r^2$ term of the secondary waves (which Stokes did not bother to calculate). Modern distribution theory allows a more rigorous rendering of Stokes's basic intuition. Namely, the incoming wave can be expressed as the superposition

$$U = f(ct - x) = \int c\mathrm{d}t_0 \delta(ct - x - ct_0) f(ct_0) \tag{42}$$

of the plane pulses $\delta(ct{-}x{-}ct_0)$. Consider the pulse for which $t_0 = 0$. It reaches the plane $x = 0$ at the time $x = 0$, with the amplitude $\delta(x)$ and the velocity $-c\delta'(x)$. The latter disturbance may be regarded as the surface integral of point-like disturbances distributed over the plane $x = 0$. In this integral, the contribution of the point $x = y = z = 0$ has the amplitude $\delta(x)\delta(y)\delta(z)$ and the velocity $-c\delta'(x)\delta(y)\delta(z)$. By time-integration of the Fourier transform of the wave equation, this contribution generates the elementary disturbance

$$e(\mathbf{r}, t) = \frac{1}{8\pi^3}\int \mathrm{d}^3k e^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}} \cos kct - \frac{1}{8\pi^3}\int \mathrm{d}^3k(c\mathrm{i}k_x)e^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}\frac{\sin kct}{kc}. \tag{43}$$

[16]Stokes 1851, p. 31 (footnote, in which Stokes indicates how the formula for sound differs from his formula for transverse diffracted waves; there is an erroneous factor cos θ in the resulting formula). This expression gives a non-zero backward radiation for a single surface element but a negligible backward radiation when integrated over a surface that is large compared with the wavelength.

This may be rewritten as

$$e(\mathbf{r},t)=\left(\frac{\partial}{c\partial t}-\frac{\partial}{\partial x}\right)\int\frac{\mathrm{d}^3k}{8\pi^2}\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}\frac{\sin kct}{k}. \tag{44}$$

Integration in spherical coordinates then yields

$$e(\mathbf{r},t)=\left(\frac{\partial}{c\partial t}-\frac{\partial}{\partial x}\right)\frac{\delta(r-ct)}{4\pi r}. \tag{45}$$

Similarly, the incoming disturbance $\delta(ct-x-ct_0)$ generates the elementary disturbance $e(|\mathbf{r}-\mathbf{r}'|,\ t-t_0)$ when reaching the point $\mathbf{r}'$ of the surface $x=0$. Integrating all these disturbances over t_0 with the weight $cf(ct_0)$ and summing over the surface elements, we finally get

$$u(\mathbf{r},t)=\int\mathrm{d}S'\left(\frac{\partial}{c\partial t}-\frac{\partial}{\partial x}\right)\frac{f(|\mathbf{r}-\mathbf{r}'|-ct)}{4\pi|\mathbf{r}-\mathbf{r}'|}, \tag{46}$$

whose leading terms are identical with Stokes's formula (41) after a change of the origin of coordinates.[17]

Stokes assumed that a diffracting screen in the plane of integration simply restricted this integral to the unscreened part of this plane. For a monochromatic plane wave, the result agrees with Fresnel's theory since the angular dependence, which Fresnel left undetermined, does not affect the value of the integral at distances much larger than the wavelength. Strictly speaking, the above calculations only concern the diffraction of compression waves. Although Stokes privately began with this case, in his published memoir he directly treated the more difficult problem of the diffraction of waves propagated in a bi-constant elastic medium. In this case, the divergence and the curl of the vibration satisfy the d'Alembertian wave equation and can therefore be subjected to the former analysis. Stokes then obtained the longitudinal and transverse parts of the vibration by inverting the div and curl operations (to put it in modern terms).

For the transverse part of the secondary vibration emitted by the surface element dS, which is the only one of optical interest, Stokes found that the vibration occurred in the plane defined by the vibration of the incoming wave and the vector $\mathbf{r}$ joining the point of observation to the surface element, and that its amplitude contained an additional factor $\sin\phi$, where ϕ is the angle that the vector $\mathbf{r}$ makes with the incoming vibration. As Stokes emphasized, the resulting distribution of the polarization of the diffracted light as a function of the polarization of the incoming light depends on whether the optical vibrations occur in the plane of polarization or in the perpendicular direction. By carefully studying the large-angle diffraction from a fine grating, Stokes decided in favor of the second alternative (Fresnel's).[18]

[17]The latter formula is a particular case of Kirchhoff's formula (61) given below.

[18]Stokes 1851, pp. 4–5, 35–61.

This consequence of Stokes's theory of diffraction was important in deciding between the competing theories of the optical medium. In 1860 Ludvig Lorenz confirmed Stokes's experimental result as well as its theoretical interpretation. His own theory of diffraction was based on the continuity of displacement and stress across the opening of the screen, together with the requirement that the transmitted and reflected vibrations should be transverse and should be represented by retarded integrals over the surface of the opening. Its predictions were equivalent to Stokes's for the distribution of the polarization of the diffracted light. In his lectures of 1888 and 1892, Poincaré denied the crucial character of this sort of experiment. As Stokes had himself mentioned, refraction by the glass of the gratings could contribute to the polarization of the diffracted light. Most fatally, Poincaré showed that the large angle of the implied diffraction was incompatible with the approximation implied in Stokes's and others' theories of diffraction.[19]

There are two other weaknesses in Stokes's diffraction theory. The first, of which Stokes was aware, is the neglect of any alteration of the incoming vibration when passing near the edges of the screen. A more serious flaw, which Rayleigh detected later in the century, is that the decomposition of the incoming disturbance with respect to the various surface elements of the screen's opening is not unique. Rayleigh surmised that this indeterminacy could only be avoided by specifying the mode of action of the substance of the screen.[20]

Kirchhoff's theory

Another approach, inaugurated by Kirchhoff in 1882, consists in specifying plausible boundary conditions on the screen and solving the propagation problem by purely mathematical means. Kirchhoff's starting point was a theorem found in a memoir by Hermann Helmholtz on the pitch of organ pipes. Eighteenth-century theories of open organ pipes were based on the idealization of a cylindrical stationary wave with a compression node at the (closed) bottom of the pipe, and an antinode at the opening of the pipe. In contrast, Helmholtz assumed a cylindrical wave toward the bottom of the tube only, and he related this wave to the spherical wave far outside the tube by means of the theorem that caught Kirchhoff's attention. A modern proof of this theorem follows.

For monochromatic sound waves of frequency kc, the wave equation has the form

$$\Delta u + k^2 u = 0. \tag{47}$$

It admits the Green function

$$G(\mathbf{r}) = -\frac{e^{ikr}}{4\pi r} \tag{48}$$

[19] Lorenz 1860a; Poincaré 1889, pp. 400–1; 1892, pp. 213–26; Baker and Copson 1939, p. 149; Whittaker 1951, p. 154. Carl Holtzmann (1856) gave a simple proof of Stokes's angular dependence as well as contradictory experiments leading to Neumann's choice for the direction of vibration (i.e., in the plane of polarization).

[20] Rayleigh 1890, pp. 452–4.

such that

$$\Delta G + k^2 G = \delta(\mathbf{r}). \tag{49}$$

For any two functions f and g of $\mathbf{r}$, we have

$$f\Delta g - g\Delta f = \nabla\cdot(f\nabla g - g\nabla f), \tag{50}$$

whence follows Green's theorem for the volume V delimited by the closed surface ∂V:

$$\int_{\mathrm{V}} (f\Delta g - g\Delta f)\mathrm{d}\tau = \int_{\partial \mathrm{V}} (f\nabla g - g\nabla f)\cdot \mathrm{d}\mathbf{S}, \tag{51}$$

or else

$$\int_{\mathrm{V}} [f(\Delta + k^2)g - g(\Delta + k^2)f]\mathrm{d}\tau = \int_{\partial \mathrm{V}} (f\nabla g - g\nabla f)\cdot \mathrm{d}\mathbf{S}. \tag{52}$$

Helmholtz specialized this identity to $f = u$ and $g = G$, where u satisfies the free wave equation (47) within the volume V and the pole $\mathbf{r} = \mathbf{0}$ of the Green function G belongs to this volume. The resulting identity,

$$u(\mathbf{0}) = \int_{\mathrm{S}} (G\nabla u - u\nabla G)\cdot \mathrm{d}\mathbf{S}, \tag{53}$$

relates the vibration at a given point of space to the vibrations on a closed surface S around this point when all the sources are outside this surface (the integral vanishes when the reference point is outside the closed surface). As Helmholtz's puts it, "the sonorous motion in a finite volume can be represented as emanating from sources confined to the surface of this volume and spread on a single layer or two infinitely close layers." In this electrostatic analogy, the single layer corresponds to the G term of the integral, and the double layer to the ∇G term.[21]

In 1882 Kirchhoff obtained the following generalization of Helmholtz's theorem to non-periodic disturbances in a medium containing the sources $j(\mathbf{r}, t)$. The disturbance u satisfies the equation

$$\ddot{u} - c^2\Delta u = j. \tag{54}$$

The theorem stipulates that for any bounded volume V containing the point $\mathbf{r} = \mathbf{0}$,

$$4\pi u(\mathbf{0}, t) = \int_{\mathrm{V}} \frac{[j]}{rc^2}\mathrm{d}\tau + \int_{\partial \mathrm{V}} \left\{\frac{1}{r}[\nabla u] - \nabla\left(\frac{1}{r}\right)[u] + \frac{\nabla r}{cr}[\dot{u}]\right\}\cdot \mathrm{d}\mathbf{S}, \tag{55}$$

[21]Helmholtz 1859, p. 6. Cf. Darrigol 1998. Green (1828, pp. 23–6, 29) had given the electrostatic counterparts ($k = 0$) of equations (51) and (53).

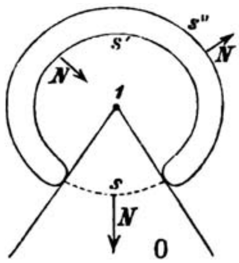


Fig. 7.3. Kirchhoff's diffraction problem. The point source 1 is included in the crescent-shaped cavity with fully absorbing walls s′ + s″ and the opening s. The observation point is at O, outside the cavity. The vector **N** gives the orientation of the normal. From Kirchhoff 1891, p. 80.

where the square brackets are used to indicate the retarded value at $t-r/c$.[22]

Kirchhoff applied this theorem to a component of the vibrations occurring in the diffracting device of Fig. 7.3. For the integration volume, he took the volume between the closed surface $\mathrm{s}'\cup\mathrm{s}''$ and a large sphere containing the whole setup. As long as the light from the source 1 has not had time to reach this sphere, it does not contribute to the surface integral. Kirchhoff further assumed that, for a perfectly black screen,

$$u=0,\ \partial u/\partial n=0 \quad \text{on the internal surface s}'', \tag{56}$$

$$u=u_1,\ \partial u/\partial n=\partial u_1/\partial n \quad \text{on the internal surface s}', \tag{57}$$

where $\partial/\partial n$ denotes the normal derivative and u_1 denotes the wave created by the source 1 in the absence of the screen. For periodic waves, the theorem then gives

$$u(\mathrm{O})=u_1(\mathrm{O})-H(u_1,\mathrm{s}'), \tag{58}$$

where $H(u, \mathrm{s})$ denotes the Helmholtz surface integral (53) for the disturbance u and the surface s. In addition, the theorem yields the relation

$$u_1(\mathrm{O})=H(u_1,\mathrm{s})+H(u_1,\mathrm{s}'), \tag{59}$$

when applied to the volume enclosed by the surface $\mathrm{s}\cup\mathrm{s}'$ in the absence of the screen. Hence we have

$$u(\mathrm{O})=H(u_1,\mathrm{s}), \tag{60}$$

[22]Kirchhoff 1882, §2. Kirchhoff's proof is based on Green's theorem and on something like the Green function $\delta(ct-r)/2r$ of the wave equation. For a modern rendering, cf. Darrigol 2012.

namely, the vibration at point O is given by the Helmholtz integral taken over any surface s covering the opening of the screen. Using the expression $u_1 = -(A/4\pi r_1)e^{i(\omega t - kr_1)}$ of the periodic wave created by the point source 1, this gives

$$u(\mathrm{O}, t) = -A\int_{\mathrm{s}} \frac{\mathrm{i}k}{4\pi r_0 r_1}(\cos\theta_0 + \cos\theta_1)\mathrm{e}^{\mathrm{i}k(ct - r_0 - r_1)}\mathrm{d}S, \tag{61}$$

where r_0 and r_1 denote the distances of the points 0 and 1 from the running point of the surface s; and θ_0 and θ_1 are the angles that the joining lines make with the normal at this point. Stokes's diffraction formula (for sound) corresponds to the limiting case in which the surface s and the waves emitted by the source 1 are both plane and parallel.[23]

Poincaré's criticism

Kirchhoff's diffraction formula proved to be an excellent representation of physical phenomena in almost every case of interest. Yet it is based on mathematically impossible boundary conditions. Poincaré pointed to this difficulty in his optical lectures of 1888 and 1892, after he himself had arrived at a theory of diffraction very similar to Kirchhoff's. Consider with Poincaré the Helmholtz surface integral of the disturbance u at a point located within the substance of the screen and for the surface $\mathrm{s}' \cup \mathrm{s}''$. By Kirchhoff's theorem and under his boundary conditions, this integral yields

$$H(u_1, \mathrm{s}') = u_1. \tag{62}$$

In the absence of the screen, the same theorem gives

$$H(u_1, \mathrm{s}') + H(u_1, \mathrm{s}'') = u_1. \tag{63}$$

Therefore, the integral $H(u_1, \mathrm{s}'')$ must vanish for any surface s″ if Kirchhoff's boundary conditions hold for any screen. This is easily seen to be impossible. As Sommerfeld later pointed out, the boundary condition $u = 0$, $\partial u/\partial n = 0$ on s″ is by itself impossible because, according to a theorem of 1869 by Heinrich Weber, it implies that the function u should vanish in the whole (connected) domain in which Helmholtz's equation (47) holds. The part of the screen invisible from the source must be slightly illuminated, as can be verified experimentally.[24]

Poincaré rescued Kirchhoff's theory by showing that his boundary conditions were approximately valid if the wavelength was sufficiently small. He first showed that if the Kirchhoff integral $H(u_1, \mathrm{s})$ would vanish whenever the point O is within the surface $\mathrm{s}' \cup \mathrm{s}''$,

[23]Kirchhoff 1882, §5. Kirchhoff directly assumed that the vibration in the opening was the same as in the absence of the screen. The proof that this assumption follows from his definition of a perfectly black screen is from Poincaré 1893, pp. 184–7.

[24]Poincaré 1889, pp. iv (his theory independent from Kirchhoff's), 115–16 (boundary conditions); 1893, pp. 187–8; Sommerfeld 1950, p. 202. In 1888, Poincaré only showed the impossibility that u=0, $\partial u/\partial n$=0 on the external surface s″ together with u=u_1, $\partial u/\partial n$=$\partial u_1/\partial n$ on the surface s of the opening.

then its values when O is outside this surface would define a solution of Helmholtz's equation that meets the Kirchhoff boundary conditions. The reason is that the discontinuity of the Kirchhoff integral (respectively, the discontinuity of its normal derivative) when crossing the surface of integration is equal to the intensity of the double layer of superficial sources (respectively, the intensity of the simple layer) in the integrand. This intensity being zero on s″, and the internal value of the integral being zero by assumption, the external value next to s″ must be zero in agreement with Kirchhoff's condition (56). On s this intensity is u_1, so that the external value of the integral must also be equal to u_1, in agreement with Kirchhoff's equating the Helmholtz integrals of u and u_1 over s. In general, the integral $H(u_1, \mathrm{s})$ does not vanish for every choice of the point O within the surface $\mathrm{s}' \cup \mathrm{s}''$. Poincaré was able to prove, however, that it approximately vanished in the circumstances of ordinary diffraction. He thus explained the otherwise astonishing success of Kirchhoff's diffraction formula.[25]

Another difficulty of Kirchhoff's theory is that it does not properly take into account the transverse character of the luminous vibrations or their electromagnetic nature. A full theory of diffraction should be based on the correct boundary conditions for electromagnetic waves, which unfortunately depend on the nature of the screening matter. This difficult problem has only been solved for simple geometries and ideal screens. Most famously, in 1896 Sommerfeld provided an exact solution of the diffraction by a perfectly reflecting half-plane, based on an ingenious use of multivalued solutions of Helmholtz's equation. Results become ambiguous for absorbing screens. From Newton to the present, the diffraction problem has never ceased to be a ground for conflict between physical intuition and mathematical rigor.[26]

7.3 Fourier synthesis

Young and Fresnel associated Newton's simple colors with sinusoidal vibrations, as Daniel Bernoulli had done in the previous century. Cauchy legitimized this choice as the one leading to the simplest solutions of the equations of motion of the ether and as the one that was sufficient to generate the most general solution by superposition. In order to know the solution corresponding to an arbitrary initial disturbance of the ether, one only had to build the superposition of monochromatic plane-wave solutions that agreed with the Fourier analysis of this disturbance. Although Poisson had early done so in the case of sound, in optics there was a general tendency to discuss plane or spherical monochromatic waves only and to regard their kinematic attributes (frequency, amplitude, wavelength, and velocity) as the sole quantities of physical interest. This attitude may have been a belated consequence of the Newtonian individuation of simple colors, as Louis Georges Gouy noted in 1889:[27]

[25] Poincaré 1893, pp. 187–8. Kirchhoff (1882, §3) had already proved the vanishing of the internal values of $H(u_1, \mathrm{s})$ in the limit of geometrical optics. On various attempts at justifying Kirchhoff's result, cf. Baker and Copson 1939, chaps. 2 and 3.

[26] Sommerfeld 1896. For a historical overview, cf. Chappert 2004, pp. 219–32. For modern discussions, cf. Born and Wolf 1964, chap. 8; Jackson 1975, chap. 9.

[27] Gouy 1889, p. 266.

> The principle of the individuality or independence of the [simple] waves, which no one ever tried to demonstrate, seems to me a sort of compromise between the wave theory and the emission theory. These waves, being individually endowed with invariable properties that they carry along, are surely similar to the luminous particles of the old theory.

Group velocity

The first theorist who overcame this prejudice was Hamilton, in a note read in June 1839 to the Royal Irish Academy. In a development of Cauchy's lattice theory, Hamilton discussed a solution for which the disturbance at time $t = 0$ is the sine wave e^{ik_0x} confined to the half space $x<0$. Once stripped of the intricacies of the discrete lattice and translated into modern notation, this solution reads

$$f(x,t) = \int_{-\infty}^{+\infty} \tilde{f}(k)e^{i[kx-\omega(k)t]}dk, \tag{64}$$

$$\text{with } \tilde{f}(k) = \frac{1}{2\pi}\int_{-\infty}^{0} e^{i(k_0-k)x}dx = \frac{1}{2}\delta(k-k_0) - \frac{i}{2\pi(k-k_0)} \tag{65}$$

(the Cauchy principal part being understood around the pole). It is simply obtained by superposing monochromatic plane-wave solutions of frequency $\omega(k)$ for the wave number k. Hamilton then argued that only the vicinity of the pole $k = k_0$ contributed to this integral because of the rapid oscillations of the exponential. In this vicinity, he used the approximation

$$kx - \omega t \approx k_0x - \omega_0 t + (k-k_0)(x-\omega_0' t) \tag{66}$$

as long as x was not too close to $\omega_0' t$. This leads to

$$f(x,t) \approx e^{i(k_0x-\omega_0t)}\int_{-\infty}^{+\infty}\tilde{f}(k)e^{i(k-k_0)(x-\omega_0't)}dk = [1-\theta(x-\omega_0't)]e^{i(k_0x-\omega_0t)}, \tag{67}$$

where θ is the step function that is zero for negative values of its argument and unity for positive values of its argument. This means that the disturbance penetrates the $x>0$ half space at the velocity

$$\omega_0' = (d\omega/dk)_{k=k_0}. \tag{68}$$

For monochromatic light obeying Cauchy's dispersion formula, Hamilton concluded:[28]

[28]Hamilton 1841a (linear row of molecules); 1841b (general case), p. 349. Cf. Hankins 1980, p. 162. In unpublished manuscripts Hamilton gave a finer description of the disturbance in the vicinity of its front, for which the former approximation is not valid: *MPH* 2, pp. 527–75. Cf. Levin 1978.

> *The velocity wherewith light of this colour conquers darkness* … by the *spreading of vibration into parts which were not vibrating before*, is something less than [ω/k], being represented by [$\mathrm{d}\omega/\mathrm{d}k$].

Hamilton's brilliant analysis was quickly forgotten. Group velocity, as Rayleigh later named the differential ratio $\mathrm{d}\omega/\mathrm{d}k$, was rediscovered in the 1870s in the context of water waves in which the high dispersion makes it more evident. The naval engineers John Scott Russell and William Froude had observed that groups of waves traveled slower than the individual waves of the group. In 1876, Stokes related this phenomenon to the slower propagation of the beats in the superposition of two waves of neighboring wavelengths. So too did Rayleigh in his *Theory of sound* of 1877 and in brief articles of 1881 in which he argued that the velocity of light measured by Fizeau's and Foucault's methods was the group velocity. Rayleigh also showed that the group velocity corresponded to the velocity of the energy traveling across the medium.[29]

In 1880, the Lyon-based physicist Louis Georges Gouy reinvented group velocity for the second time, directly in the optical context. His purpose was to show that the velocity measured by Fizeau's method of the toothed wheel differed from what we now call the phase velocity. Like Stokes, Gouy described the traveling modulation of the superposition of two waves of neighboring wavelengths and asserted without proof that any slow modulation of a periodic series of waves traveled with the same velocity $\mathrm{d}\omega/\mathrm{d}k$. The chief authority on Fizeau's method, Alfred Cornu, protested immediately and vigorously. In his view, waves only had a well-defined velocity when they traveled without any change of their shape; in addition, Gouy's superposition of two monochromatic plane waves had nothing to do with the actual motion between the toothed wheels of Fizeau's device. Gouy replied that he had never meant to directly represent the latter motion by the former.[30]

The following year Gouy proved the generality of the concept of traveling modulation by approximating the Fourier integral that represents a "sensibly homogenous" or nearly periodic wave. This integral reads

$$f(x,t) = \int_{-\infty}^{+\infty} \tilde{f}(k)\mathrm{e}^{\mathrm{i}[kx-\omega(k)t]}\mathrm{d}k, \tag{69}$$

in which the Fourier transform $\tilde{f}(k)$ has a sharp maximum at $k = k_0$. Gouy expanded the phase around this maximum as

$$kx - \omega t = k_0 x - \omega_0 t + (k-k_0)(x-\omega_0' t) - (k-k_0)^2 \omega_0'' t - \ldots, \tag{70}$$

and treated the terms of order two and higher as small quantities, so that

[29]Stokes [1876]; Rayleigh 1877–78, vol. 1, pp. 246–7; Rayleigh 1881a, 1881b. Cf. Darrigol 2000, pp. 85–8. In 1839 John Herschel had privately told Hamilton that the group velocity could be seen on the waves formed by the impact of a stone on a water surface: Herschel to Hamilton, 8 February 1839, in Graves 1882, vol. 2, pp. 290–1.

[30]Gouy 1880, 1881a, 1881b; Cornu 1880, 1881. Cf. Chappert 2004, pp. 244–7.

$$e^{-i[(k-k_0)^2\omega_0''t+...]} = 1 - (k-k_0)^2\omega_0''t - ...$$

This leads to

$$f(x,t) = e^{i(k_0x-\omega_0t)}\int_{-\infty}^{+\infty}[1-(k-k_0)^2\omega_0''t - ...]\tilde{f}(k)e^{i(k-k_0)(x-\omega_0't)}dk. \tag{71}$$

Introducing the slowly varying amplitude

$$\varphi(x) = e^{-ik_0x}f(x,0) \text{ for which } \tilde{f}(k) = \tilde{\varphi}(k-k_0), \tag{72}$$

the former integrals lead to

$$f(x,t) = e^{i(k_0x-\omega_0t)}[\varphi(x-\omega_0't) - t\omega_0''\varphi''(x-\omega_0't) - ...]. \tag{73}$$

This means that for moderate times (such that $t\omega_0''\varphi'' << \varphi$) the disturbance is a plane monochromatic wave with a modulation that travels at the group velocity[31]

$$\omega_0' = (d\omega/dk)_{k=k_0}. \tag{68}$$

White light

Gouy's interest in chopped or modulated waves brought him to discuss the received view of white light as a random mixture of wave trains of various lengths, origins, and frequencies (as one would expect if the source is made of randomly excited vibrators). In an influential memoir of 1886, he argued that whatever be the detailed mechanism of the production of light, the ethereal motion $s(t)$ on a plane far from the source before entering an optical system could always be represented by a Fourier integral

$$s(t) = \int \tilde{s}(\omega)e^{i\omega t}d\omega \tag{74}$$

and that the time-averaged illumination at the exit of the optical system could be obtained by superposing the illuminations caused by incoming plane monochromatic waves $e^{i(\omega t-\mathbf{k}\cdot\mathbf{r})}$ with the weights $|\tilde{s}(\omega)|^2$. Although this result is a trivial consequence of the linearity of the equations of propagation and of Parseval's theorem for Fourier's transforms, it has the counterintuitive consequence that no optical experiment can decide between concepts of natural light that lead to the same frequency distribution $|\tilde{s}(\omega)|^2$. In particular, there should be no way to decide whether the disappearance of fringes in an interference device with

[31]Gouy 1882. Gouy was aware of the inverse proportionality of the widths of a function and of its Fourier transform. Besides, he discovered the phase change of light when passing through a focus (Gouy 1891).

large path difference is due to the "complexity" (varying frequency) or to the "irregularity" (disrupted wave trains) of the incoming light.[32]

As Gouy knew, in 1845 Fizeau and Foucault believed to have excluded the second alternative. They used a spectrometer to analyze the light issuing from a double-ray interference device (Fresnel's mirrors) fed by white light. At a given point of the zone of interference, the spectral component of angular frequency ω_0 has an amplitude proportional to $1+e^{i\omega_0\tau}$, where τ denotes the delay caused by the path difference. Therefore, the observed spectrum has periodic dark lines whose number increases with the path difference (*spectre cannelé* or band spectrum). Fizeau and Foucault were able to distinguish these lines for path differences as large as seven thousand wavelengths. They concluded:

> The very restricted limits of path difference beyond which one could not [heretofore] produce the mutual influence of two rays depended only on the complexity of light. By using the simplest light that one might obtain, these limits are considerably shifted. –The existence of these phenomena of the mutual influence of two rays in the case of a large path difference is interesting for the theory of light, for it reveals in the emission of successive waves a persistent regularity that no phenomenon earlier suggested.

Gouy flatly rejected this conclusion, since in his view any irregularity in the source was equivalent to a spread in the Fourier spectrum of the vibration:[33]

> Thus, the existence of interference fringes for large path differences does not at all imply the regularity of the incoming luminous motion. This regularity exists in the spectrum, but it is the spectral apparatus that produces it by separating more or less completely the various simple motions which heretofore only had a purely analytical existence.

Poincaré took Fizeau's defense in a note of 1895 for the *Comptes rendus*. He first argued that Gouy's reasoning led to the absurd consequence that a source of light, when seen through a spectroscope, should appear permanently illuminated even if the source is turned off. Indeed, according to Gouy the spectroscope separates the Fourier components of the vibration, and by definition Fourier components do not depend on time. In order to avoid this paradox, Poincaré introduced the finite resolution of the spectroscope. The amplitude of vibrations at a point of the interference zone can then be written as

$$a(t) \propto \int (1 + e^{i\omega\tau})\chi(\omega - \omega_0)\tilde{s}(\omega)e^{i\omega t}d\omega, \tag{75}$$

where $\chi(\omega-\omega_0)$ characterizes the frequency selection by the spectroscope. In the case of infinite resolution, $\chi(\omega-\omega_0) = \delta(\omega-\omega_0)$ so that

$$a(t) \propto e^{i\omega_0 t}\tilde{s}(\omega_0)(1 + e^{i\omega_0\tau}) \tag{76}$$

[32]Gouy 1886. Cf. Chappert 2004, pp. 247–51.

[33]Fizeau and Foucault 1849; 1850, p. 159; Gouy 1886, p. 362. Cf. Chappert 2004, pp. 240–2.

and the corresponding intensity does not depend on time. In reality, the resolution of the spectroscope is limited by its finite aperture, and the characteristic function has the form

$$\chi(\omega - \omega_0) = \tilde{H}(\omega - \omega_0), \tag{77}$$

with $H(t)=1$ for $t_1 \leq t \leq t_2$ and $H(t)=0$ for $t<t_1$ or $t>t_2$, t_1 and t_2 being the times that light takes to travel from each extremity of the aperture to the point of observation. The resulting exit amplitude is

$$a(t) \propto \mathrm{e}^{\mathrm{i}\omega_0 t}\left[\int_{t-t_1}^{t-t_2} s(t')\mathrm{e}^{-\mathrm{i}\omega_0 t'}\mathrm{d}t' + \mathrm{e}^{\mathrm{i}\omega_0\tau}\int_{t+\tau-t_1}^{t+\tau-t_2} s(t')\mathrm{e}^{-\mathrm{i}\omega_0 t'}\mathrm{d}t'\right]. \tag{78}$$

This expression avoids the aforementioned paradox, since it vanishes if the time t is far enough from the period of activity of the source. As Poincaré regarded the oscillating factor $1+\mathrm{e}^{\mathrm{i}\omega_0\tau}$ as empirically established by Fizeau and Foucault, he required that

$$\int_{t-t_1}^{t-t_2} s(t')\mathrm{e}^{-\mathrm{i}\omega_0 t'}\mathrm{d}t' = \int_{t+\tau-t_1}^{t+\tau-t_2} s(t')\mathrm{e}^{-\mathrm{i}\omega_0 t'}\mathrm{d}t' \tag{79}$$

for any τ at which the band spectrum is still seen, and he concluded:[34]

> *The experiment of Fizeau and Foucault teaches us … that the luminous motion enjoys a certain kind of permanence expressed in equation* [(79)] … Thus, a complete analysis leads to exactly the same consequences that M. Fizeau's clear-sightedness had guessed in advance.

Gouy soon protested: Poincaré had failed to appreciate that the lines of the spectrum could only be separated when the interference delay was smaller than the time t_2-t_1 that determines the resolving power of the spectroscope. In this case, the condition (79) is trivially satisfied, no matter how irregular the original motion might be. In sum, the spectroscope is able to produce by itself all the regularity needed to observe interference. Lord Rayleigh and Arthur Schuster had independently come to the same conclusion. This made the nature of white light a matter of speculation. Gouy preferred complete irregularity:[35]

> To consider an extreme case, by way of example, we may regard white light as made of a series of quite irregular pulses, or of unceasingly perturbed vibrations analogous to the trepidation that some physicists take to be the calorific motion. This fairly plausible assumption permits a simple explanation of the complete continuity of the spectrum, which is rather difficult to understand if one regards the luminous source as producing distinct series of regular vibrations.

[34] Poincaré 1895, p. 761.

[35] Gouy 1895; Rayleigh 1890, p. 425; Schuster 1894; Gouy 1886, p. 62. Cf. Chappert 2004, pp. 250–1.

In the following years, opinions varied on the nature of white light until the whole issue died away. It became clear, however, that Hooke had been right in rejecting Newton's proof of the preexistence of simple colors in white light. In his euphemistic manner, Rayleigh judged that "the assertion that Newton's experiments prove the colors to be already existent in white light, is usually made in too unqualified a form." He went on to compare the periodic waves created by a prism to the periodic waves formed behind a fishing line moved at a small constant speed across a calm water surface. In both cases the periodicity is a consequence of the dispersion and the period depends on the angle of observation, because the boundary conditions select a specific value of a projection of the phase velocity. Had Newton been Rayleigh's fisherman, he would never have needed the doctrine of the heterogeneity of white light.[36]

7.4 Conclusions

Old issues of seventeenth-century optics such as the heterogeneity of white light, ray propagation, and diffraction received satisfactory and nearly definitive answers only in the post-Fresnel phase of the history of nineteenth-century optics. The source of this progress was not any deeper insight into the nature of the ether. Rather, it was the availability or the invention of more advanced mathematical methods including Hamilton's algebrized geometry, Fourier analysis, approximation techniques for Fourier integrals, and generalizations of Green's theorem. Until Fresnel's intervention, the mathematics of optics had been confined to ray geometry occasionally supplemented with Newton's dynamics of the light corpuscles or with Huygens's consideration of normal waves. Fresnel's theories of diffraction and of anisotropic propagation brought unprecedented mathematical sophistication. So too did the later forays into optics of powerful mathematicians like Hamilton, Poincaré, and Sommerfeld; or of mathematically creative physicists like Airy, Stokes, Kirchhoff, and Gouy. In their hands, the laws of the propagation of light became better founded and better adapted to instrumental concerns. In this symbiotic evolution, mathematics enriched the predictive and conceptual apparatus of optics while optical concerns engendered new mathematics.

In the eyes of contemporary physicists, some of these developments must have looked like the drowning of physical ideas in an ocean of abstruse mathematics. This is certainly the case of Hamilton's theory of systems of rays, whose instrumental power remained unrecognized until the end of the century. Kirchhoff's, Poincaré's, and Sommerfeld's theories of diffraction were more immediately appreciated, for they implemented a degree of mathematical rigor that seemed legitimate to the rising generation of theoretical physicists, and they yielded finer details of diffraction patterns that eluded Fresnel's more intuitive approach. Gouy's and others' optical exploitation of Fourier analysis was closely tied to important physical notions such as optical coherence and the composition of white light. It soon became part and parcel of modern optical theory; it may also have inspired the mathematical metaphors of "Fourier spectrum," or, by extension, the "spectral analysis" of operators.

[36]Rayleigh 1905, p. 401. On the history of the fishing line problem, cf. Darrigol 2000, pp. 88–91.

Some of the mathematical methods that permitted the optical theories expounded in this chapter were invented for this purpose. For example, Hamilton introduced the characteristic function of normal congruences and the method of stationary phase for the approximate evaluation of rapidly oscillating functions; and Kirchhoff discovered the extension of Green's theorem to solutions of the d'Alembertian equation. For the most part, however, the mathematics needed originated in physical contexts that were different from optics. Hamilton's and Malus's representation of congruences through the intersection of two families of developable surfaces originated in Monge's theory of the optimal transport of earth for construction purposes; Green's theorem in a study of the potential of a system of electric charges; Helmholtz's extension of this theorem to sinusoidal waves in a study of the pitch of organ pipes; harmonic analysis in Bernoulli's and Lagrange's acoustics, and in Fourier's investigation of heat propagation.

By the end of the nineteenth century, there was a growing sense that shared mathematical methods and structures played an essential role in unifying physics. In earlier times optical theory had depended on more conceptual strategies of unification such as mechanical reduction, direct analogies with acoustic or fluid mechanics, or Newton's analogy of nature. It now implied other kinds of reduction based on electromagnetic or energetic concepts, and other kinds of analogy based on shared mathematical structures. On the one hand, this evolution permitted the exquisite theoretical and experimental precision of late nineteenth-century wave optics. On the other hand, it brought the inter-theoretical tensions that motivated relativity theory and quantum theory in the next century.

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ABBREVIATIONS

ACP *Annales de chimie et de physique.*
AHES *Archive for the history of exact sciences.*
AP *Annalen der Physik und der Chemie.*
BAR British Association for the Advancement of Science, *report.*
BJHS *British journal for the history of science.*
CR Académie des Sciences, *comptes-rendus hebdomadaires des séances.*
EO i.j Leonhard Euler, *Opera omnia* (Leipzig, 1911 on), series *i*, vol. *j.*
FO i Augustin Fresnel, *Œuvres complètes*, ed. *H. de Sénarmont, E. Verdet, and L. Fresnel*, 3 vols. (Paris: Imprimerie impériale, 1866–1870), vol. *i.*
HAS Académie Royale des Sciences, *Histoire.*
HSPS *Historical studies in the physical sciences.*
JMPA *Journal de mathématiques pures et appliquées.*
JP *Journal de physique.*
JRAM *Journal für die reine und angewandte Mathematik.*
MAB Académie Royale des sciences et des belles-Lettres de Berlin, *Mémoires.*
MAS Académie Royale des Sciences, *Mémoires (de physique et de mathématiques).*
MCW James MacCullagh, *The collected works*, ed. by J. Jellett and S. Haughton (Dublin: Hodges, Figges & Co).
MPCSA Société d'Arcueil, *Mémoires de physique et de chimie.*
MPH i William Rowan Hamilton, *The mathematical papers*, 4 vols. (Cambridge: Cambrige University Press, 1931–2000), vol. *i.*
MSE *Mémoires de mathématique et de physique, présentés à l'Académie Royale des Sciences, par divers savans, et lûs dans ses assemblées.*
NBSP Société Philomatique de Paris, *Nouveau bulletin des sciences.*
NRRS Royal Society of London, *Notes and records.*
OCC i.j Augustin Cauchy, *Œuvres complètes*, 2 series, 27 vols. (Paris: Gauthier-Villars, 1882–1974), series *i*, vol. *j.*
PM *Philosophical magazine*.
PRIA Royal Irish Academy, *Proceedings.*
PT Royal Society of London, *Philosophical transactions.*
RHS *Revue d'histoire des sciences et des techniques.*
SHPS *Studies in the history and philosophy of science.*
SMPP George Gabriel Stokes, *Mathematical and physical papers*, 5 vols. (Cambridge: Cambridge University Press, 1880–1905).
TAPS American Philosophical Society, *Transactions.*
TCPS Cambridge Philosophical Society, *Transactions*.
TRIA Royal Irish Academdy, *Transactions.*
YMW Thomas Young, *Miscellaneous works*, 3 vols. (London: Murray, 1855).

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INDEX